Input-Output tables

An input-output model is used to attribute the emissions that arise during production, to final demand and its constituents. To that end, input-output tables are needed for every year. We use the observed input-output table for the year 2000 (CSO, 2006). For all other years, we use the RAS method (e.g., Parikh, 1979) to create an input-output table.

The basic input-output model is given by:

(1) $$X = AX + Y \Leftrightarrow (I - A)X = Y \Leftrightarrow X = (I - A)^{-1}Y = LY$$

where X is the vector of production, Y is the vector of final demand, A is the matrix of production coefficients and L is the Leontief inverse.

The input-output table A is observed for a single year, but is needed for all years. It is updated as follows. Let A^0 be the original input-output table, and let Y^0 and P^0 be the row and column totals, respectively. Let Y^1 and P^1 be the desired row and column totals, respectively, which follow from observations or projections of sectoral demand and production. The updated input-output table A^1 follows from the following algorithm.

<u>Initialisation</u>

(2) $$Y^{(0)} = Y^0$$

(3) $$R^{(0)} = \begin{bmatrix} Y_1^1 / Y_1^{(0)} & 0 & \cdots & 0 \\ 0 & Y_2^1 / Y_2^{(0)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Y_n^1 / Y_n^{(0)} \end{bmatrix}$$

(4) $$B^{(0)} = R^{(0)} A^0$$

Let $P^{(0)}$ be the vector of column totals of $B^{(0)}$.

$$(5) \qquad S^{(0)} = \begin{bmatrix} P_1^1 / P_1^{(0)} & 0 & \cdots & 0 \\ 0 & P_2^1 / P_2^{(0)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P_n^1 / P_n^{(0)} \end{bmatrix}$$

$$(6) \qquad A^{(0)} = B^{(0)} S^{(0)}$$

Iteration

Let $Y^{(i)}$ be the vector of column totals of $A^{(i-1)}$, as in Equation (2). Define $R^{(i)}$ as the diagonal matrix with the ratios of desired column totals and actual column totals, as in Equation (3). Compute $B^{(i)}=R^{(i)}A^{(i-1)}$, as in Equation (4). Let $P^{(i)}$ be the vector of column totals of $B^{(i)}$. Define $S^{(i)}$ as the diagonal matrix with the ratios of desired row totals and actual row totals, as in Equation (5). Compute $A^{(i)}= B^{(i)}S^{(i)}$, as in Equation (6).

Termination

Stop iteration if $A^{(i)}=A^{(i-1)}$. Set $A^1=A^{(i)}$

References

CSO (2006), *2000 Supply and Use and Input-Output Tables*, Central Statistics Office, Cork. Click here for the report, and here for the data.

Parikh, A. (1979), 'Forecasts of Input-Output Tables using the RAS Method', *Review of Economics and Statistics*, **61** (3), 477-481.