Final demand

An input-output model is used to attribute the emissions that arise during production, to final demand and its constituents.

Goods and services are produced either for consumption or for use in further production. That is,

(1)
$$\begin{aligned} X_1 &= X_{1,1} + X_{1,2} + \ldots + X_{1,n} + Y_1 \\ X_2 &= X_{2,1} + X_{2,2} + \ldots + X_{2,n} + Y_2 \\ &\ldots \\ X_n &= X_{n,1} + X_{n,2} + \ldots + X_{n,n} + Y_n \end{aligned}$$

where X_i is the production of good i, and $X_{i,j}$ is the use of good i in the production of good j; Y_i is the consumption of good i, which, for convenience, includes exports and build-up of inventories. Equation (1) can be rewritten as:

(2)
$$\begin{aligned} X_1 &= a_{1,1}X_1 + a_{1,2}X_2 + \ldots + a_{1,n}X_n + Y_1 \\ X_2 &= a_{2,1}X_1 + a_{2,2}X_2 + \ldots + a_{2,n}X_n + Y_2 \\ &\ldots \\ X_n &= a_{n,1}X_1 + a_{n,2}X_2 + \ldots + a_{n,n}X_n + Y_n \end{aligned}$$

where

(3)
$$a_{i,j} := \frac{X_{i,j}}{X_j}$$

In matrix notation,

(4)
$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,2} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} + \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}$$

or

(5) $$X = AX + Y \Leftrightarrow (I - A)X = Y \Leftrightarrow X = (I - A)^{-1}Y = LY$$

Equation (5) specifies how production *X* would respond to a change in demand *Y*, including all intermediate production. *L* is commonly referred to as the Leontief inverse. Equation (5) constitutes an input-output model.

Emissions *M* of substance *l* equal

(6) $$M_l = b_{l,1}X_1 + b_{l,2}X_2 + \ldots b_{l,n}X_n \forall l$$

where $b_{l,i}$ are the emission coefficients, that is, emission per unit of production. In matrix notation,

(7) $$M = BX = BLY$$

Equation (7) relates emissions to production (via *B*) and to final demand (via *BL*).

Final demand Y can be decomposed as follows:

(8) $$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} Y_1^H \\ Y_2^H \\ \vdots \\ Y_n^H \end{bmatrix} + \begin{bmatrix} Y_1^C \\ Y_2^C \\ \vdots \\ Y_n^C \end{bmatrix} + \begin{bmatrix} Y_1^G \\ Y_2^G \\ \vdots \\ Y_n^G \end{bmatrix} + \begin{bmatrix} Y_1^I \\ Y_2^I \\ \vdots \\ Y_n^I \end{bmatrix} + \begin{bmatrix} Y_1^K \\ Y_2^K \\ \vdots \\ Y_n^K \end{bmatrix} + \begin{bmatrix} Y_1^E \\ Y_2^E \\ \vdots \\ Y_n^E \end{bmatrix}$$

where Y^H is household consumption, Y^C is consumption by non-profit institutions servicing households, Y^G is government consumption, Y^I is consumption for the build-up of inventories, Y^K is consumption for investment, and Y^E is export. Combining (7) and (8), emissions are readily allocated to the constituent components of final demand.

Note that the environmental accounts also attribute a part of the emissions directly to households. This is added to the indirect household emissions Y^H.