A Note on Obsolescence

by

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Armchair economists (those who evolve their hypotheses as to the behaviour of businessmen from their inner consciousness, instead of going to the market place and finding out) are wont to be impatient at the slowness of change for the better in economic behaviour, in particular at the length of time which elapses in the general adoption of improved technology, the reluctance of industrialists to change old machines and old ways. The present note is also an armchair exercise, but at least it is informed by the philosophy that there are usually good reasons why things are as they are; that we have no right rationally to advocate change "for the better" until we are aware of these reasons.

After such a portentous opening, what follows may appear trivial. However, it may help a little in propounding the problem properly. The simplest possible case is considered. Later we may be able to generalize it somewhat, by relaxing the restrictive conditions with which we start.

I borrow 1 at interest rate r at time $t = 0$ to purchase a machine to make a certain product. I anticipate that the machine will yield a constant income m during its lifetime of T years. For what follows the definition of income is important. It equals expected value of product less employee compensation (including my salary if I am a working entrepreneur) less non-factor input (including normal repairs and maintenance) but, for the moment, excluding depreciation. The lifetime of T years is realistic; in planning, I draw on my experience and perhaps increase the makers' or tax authority figure by 50 or 100%. This anticipated lifetime leaves obsolescence out of account. It purports to represent the number of years the machine will yield an income. While constant income during lifetime, with sudden collapse at end of T years, is postulated here, more realistic treatment would involve one's assuming decreasing income in later years, mainly perhaps as a result of increasing cost of repairs and maintenance. Income, as defined, will be seen to be particular to the machine, leaving aside financial charges: it will exceed the income I shall receive, which, as will appear, are net of these charges.

The constant period for output, interest and income, described here as a "year" might of course be any constant period, e.g. a month. For simplicity, all payments and receipts are deemed to occur at exact time points, 1,2,...

I propose to liquidate my loan of 1 by the creation of an amortization fund involving a constant annual payments of s accumulating at interest i to 1 at end of year T. My total annual financial charges are therefore (r + s) and my net income is $\bar{m}$=m - r - s. At the end of T years the machine has no value - any scrap value is here ignored. I can repay my loan: in theory the enterprise is over. In practice I start again, buying another machine of the same kind and so on. If my credit and the basic interest rate i are unchanged my payments will continue to be (r + s) ad inf. or to some suitably remote time horizon. No question of obsolescence arises.

Even in this very simple case we note already the appearance of no fewer than three rates of interest, i, r, $\bar{m}$. Normally $i < r \ll \bar{m}$. The interest i is that which I can earn on my amortization fund, without thought, worry or expense; it is safe as hoarding and more profitable. It may therefore be conceived as the interest on consols, here regarded as unchanged to the time horizon. Interest i is used uniformly for the calculation of present values in what follows.

We now consider the obsolescence situation, arising under particular conditions. A new machine also costing 1 appears on the market at year $\tau < T$. Its income (corresponding to m above) is calculated as m' and its (realistic) lifetime is T! I deem it superior to my "old" machine. Do I retain my old machine changing to the new machine only at the end of year T (situation A) or do I change straight away at time $\tau < T$, selling my old machine at price S_τ (situation A')? It is assumed that the new machine involves no change in price of product, hence demand. The superiority of the new machine will be conceived as operating with lower production costs, longer lifetime etc. If I change at $t, \tau \leq t < T$, I have to raise a loan of $(1 - S_\tau)$ with its charge $(1 - S_\tau)(r + s')$, wher is the fixed annual contribution to an amortization fund to liquidate a loan of 1 in T' years. In addition, during the period τ to T I have to pay $(r + s)$ on the earlier loan.

If A and A' are the respective present values at time τ of my anticipated future flows, we find:-

$$(1)\ i\ (A' - A) = R(1 - a^{T'})S_\tau - (R - M)(1 - a^{T - \tau}),$$

where

$$R = r + s'$$
$$M = m' - m$$
$$s' = i/(v^{T'} - 1),\ v = 1 + i$$
$$a = 1/v$$

Formula (1) is surprisingly simple, having regard to the amount of algebra involved in its derivation (see Appendix). If the foregoing expression is positive I change to the new machine. If negative, I retain the old until year T. At year $\tau = T$, $S_\tau = o$ and hence $A' = A$, as it should be.

Obviously the larger the value of S_τ, the selling price of the old machine, the greater the profitability of change, <u>ceteris paribus</u>. So, of course, with $M = m' - m$, the improvement in income. This selling price is therefore crucial. It is, however, circumscribed. It clearly depends on (i) the age of the old machine and (ii) the availability of the new. Its formula might be something like:-

$$(2)\quad S_\tau = \alpha(T - \tau)^\beta \left(\frac{m}{m'}\right)^\gamma,$$

where $o < \alpha < 1, \beta, \gamma > o$, immediately after the appearance of the new machine.

Even though formula (1) is simple, it involves several variables. We will probably be justified in regarding i, and hence a and v as "given". It will also be helpful to note that T' and s' are functionally related: in fact, when i is small, $s' = 1/T'$. Loan interest r may also be regarded as given: perhaps it is .01 or .02 greater than i. It is fortunate that m and m' are subsumed in M and r and s' in R. Our best course may be to study the breakeven value of S_τ, say $\bar{S}_\tau$, i.e. when $A' = A$ for various sets of values of the variables. We have

$$(3)\quad \bar{S}_\tau = (R - M)(1 - a^{T - \tau})/R(1 - a^{T'}).$$

Since a main object of this note is to study the precession of S_τ, the selling value of the old machine at time τ, with τ, it may be observed that, in the expression on the right of (3), only $(1 - a^{T - \tau})$ is a function of τ, which makes calculation easy.

A table is appended giving values of breakeven values of $\bar{S}_\tau$ for a number of values of T' and M given i = .06 and T = 10 uniformly: if actual second-hand value $S_\tau \leq \bar{S}_\tau$ I do not change; if $S_\tau > \bar{S}_\tau$, I buy the new machine. In the extreme case of S_τ zero (i.e. scrap value deemed zero here) M for all τ, the income excess, must be at least equal to R, for change to be desirable. In any particular case, knowing S_τ, there would be no difficulty in making a decision, using the actuaries' standard annuities - certain table for different interest rates. No more general theory is derivable until we can establish a formula (perhaps on the lines of (2) above) for S_τ. In the meantime, some Institute colleague might care to investigate prices of secondhand cars in good condition in Ireland, for which there must be ample data.

On any reasonable assumption as regards S_τ the Table suggests that the new machine must be markedly superior to the old before immediate change can be expected. To be positive, we would require a corresponding table for the given asset. There should not be any difficulty in entrepreneurial decision in any particular case, using formula (3) with actual S_τ

In only one case in the Table is the new machine, on our criteria, inferior to the old, namely the column for M = o, T' = 8, the case of no increase in gross income and shorter lifetime. Then after a year in use selling price S_1 would have to exceed its price as new, an inconceivable situation. From the T' = 8 section, i.e. postulated shorter lifetime of the new machine, one has the impression that the anticipated income excess M should exceed the .06 shown before change would be justified.

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Table. Value of $\bar{S}_{\tau}$ for Various Values of M and T' and when T = 10, i - .06 and r = .08.

τ	M: 0	.01	.02	.03	.04	.05	.06
	T' = 8						
1	1.10	1.03	.97	.91	.85	.79	.73
3	.90	.85	.80	.75	.70	.65	.60
5	.68	.64	.60	.57	.53	.49	.45
7	.43	.41	.38	.36	.34	.31	.29
9	.15	.14	.14	.13	.12	.11	.10
	T' = 10						
1	.92	.86	.81	.75	.69	.63	.57
3	.76	.71	.66	.61	.56	.52	.47
5	.57	.54	.50	.46	.43	.39	.35
7	.36	.34	.32	.29	.27	.25	.22
9	.13	.12	.11	.10	.10	.09	.08
	T' = 12						
1	.81	.75	.69	.64	.58	.52	.46
3	.67	.62	.57	.52	.47	.43	.38
5	.50	.47	.43	.39	.36	.32	.29
7	.32	.30	.27	.25	.23	.20	.18
9	.11	.10	.10	.09	.08	.07	.06

Appendix

Situation A

Income flows at points of time $\tau + 1$, $\tau + 2, \ldots,$ $T = m - r - s$

Income flows at points of time $T + 1$, $T + 2, \ldots,$ $= m' - r - s'$

Present value at τ of all income flows is given by:-

$$i\,A = (m - r - s)(1 - a^{T-\tau}) + (m' - r - s')a^{T-\tau}$$

$$= (m - r - s) + (m' - s' - m + s)\,a^{T-\tau}$$

Situation A'

Suppose that $\tau + T' \geq T$. If I sell old machine for S_τ at point of time τ, I have to borrow $(1 - S_\tau)$ to purchase new machine for 1 but I must continue the liquidation of the old loan until point of time T.

Income flows at times $\tau + 1$, $\tau + 2, \ldots,$ $T = m' - r - s - (1 - S_\tau)(r + s')$

Income flows at times $T + 1$, $T + 2, \ldots,$ $\tau + T' = m' - (1 - S_\tau)(r + s')$

Income flows at times $\tau + T' + 1$ to $\infty = m' - r - s'$

Present value at τ of all income flows is given by:-

$$i\,A' = [m' - r - s - (1 - S_\tau)(r + s')](1 - a^{T-\tau}) + [m' - (1 - S_\tau)(r + s')](a^{T-\tau} - a^{T'}) + (m' - r - s')\,a^{T'} = m' - 2r - s - s' + S_\tau(r + s') + (r + s)a^{T-\tau} - S_\tau(r + s')a^{T'}$$

Hence

$$i(A' - A) = m' - m - r - s' + (r + s' - m' + m)a^{T-\tau} + S_\tau\,(r + s')(1 - a^{T'}),$$

giving (1). The formula also applies if $\tau + T' < T$.