The Cattle-Fodder Problem

R. O'Connor has propounded the following problem.
A farmer has 54 tons of fodder which he proposes to feed
to either (1) 18, or (2) 12 cattle during the winter
months, i.e. (1) a store cattle or (2) a fat cattle
policy. It is known that the live weight price per live
cwt. rises by variable amounts during the winter months.
It is fairly obvious that policy (2) should be adopted
if this quasi-seasonal price rise is small and (1) when
the price rise is large. The problem is: find the
break-even price rise.

The initial weight per beast is $6\frac{1}{2}$ cwt., price
£6.5 per live cwt. so that the initial price per head
is £42.25. Policy (1) is expected to result in a live
weight gain of $\frac{1}{2}$ cwt. per head and (2) by $1\frac{1}{2}$ cwt., so
that final live weights per head are (1) 7 cwt. and (2)
8 cwt. We now have all the data we need.

Let £x be the rise in price per live cwt. and
"gross profit" the difference between the final and the
initial values. Then

£ gross profit, policy (1): $18[7(6.5 + x) - 42.25]$

£ gross profit, policy (2): $12 \; 8(6.5 + x) - 42.25$

Equating these expressions, the break-even rise in price
per cwt. x_o of x is found to be x_o = £1.95.

More generally, let initial weight per head be
w cwt. and initial price per cwt. £c. Assume, further-
more, that, in the range in which we are interested, the
relationship between feed in tons and weight gain g in
cwt. is linear, i.e. $g = \alpha + \beta q/n$, where q is tonnage of

fodder and n number of cattle. The break-even value $£x_o$ of the increase in price per live cwt. of cattle is found to be

$$(1) \qquad x_o = -c\alpha/(w + \alpha),$$

an extremely simple expression, independent of the tonnage of fodder and the number of cattle. However, the value of α may be influenced by the quantity of fodder used.

If one had two pairs of observations for $q/n = f$ and g, say (f_1, g_1) and (f_2, g_2), the linearity assumption implies that $\alpha = (f_1 g_2 - f_2 g_1)/(f_1 - f_2)$. If we had more than two pairs, we would fit the closest straight line. If the linearity assumption be abandoned, e.g. if $g = \alpha + \beta f + \gamma f^2$, the break-even price increase x'_o is given by

$$(1)' \qquad x'_o = -c(\alpha - \gamma q^2/n_1 n_2)/(w + \alpha - \gamma q^2/n_1 n_2)$$

where n_1 and n_2 are the numbers of cattle involved in the respective policies (e.g. 18 and 12 in R. O'Connor's example). To determine γ we require at least three pairs of observations (f_i, g_i). Of course, α has now a different value than in the linear case.

Actually, in the foregoing arithmetical example $\gamma = -1.5$. With $c = £6.5$ and $w = 6.5$ cwt., it follows that $x_o = 1.95$, as already found checking the algebra.

Conclusion: if the anticipated quasi-seasonal price increase per live cwt. $x > x_o$ adopt store cattle policy (1); if $x < x_o$ adopt fat cattle policy (2), (assuming the linear relationship). It is emphasized that, in what has gone before, total quantity of fodder available is regarded as given.

The farmer may not like the foregoing formulation since choice (2) involves his using only a fraction of his land and byre-space, (in R. O'Connor's example to the extent of one-third). His deeper instincts alone may favour full stocking with cattle using more or less fodder (which need not involve a storage problem if he can purchase or otherwise acquire his requirements of feed at near opportunity, or production, cost per ton). In addition, we assume that the seasonal increase in price per live cwt., i.e. x above, is given.

Let n = number of cattle, f = tonnage of feed per beast (e.g. 3 or $4\frac{1}{2}$ tons in R. O'Connor's example), q (as above) = total tonnage of feed, p_0, p = initial and final prices per beast in £, £r = price of fodder per ton. Then, assuming linearity of relationship within the ranges of (p, f) involved,

$$(2) \qquad\qquad p = A + Bf.$$

Given x, A and B will be assumed known. Profit (or "gross profit", other costs being assumed constant) π is

$$(3) \qquad\qquad \pi = n(p - p_0) - qr.$$

Since $f = q/n$, from (3), using (2),

$$(4) \qquad \pi = (A - p_0)n + (B - r)q = A'n + B'q.$$

If we have two pairs of observations for (f, p) say (f_1, p_1) and (f_2, p_2) then $A = (f_2 p_1 - f_1 p_2)(f_2 - f_1)$ and $B = (p_2 - p_1)/(f_2 - f_1)$. In (4), the coefficients A' and B' can in theory assume positive or negative values, (so in fact can A while B is always positive).

The farmer is committed, by hypothesis, to full stocking so that n = N, given. A little reflection will show that, to maximize gross profit π, he must then adopt <u>in toto</u> either a store cattle policy (1), or a fat cattle policy (2): he cannot maximize with a mixed policy. He will not embark on the enterprise at all if $\pi \leqslant 0$; nor will he if π is not so much larger than zero as to compensate him for his labour and interest on capital. If he decides on the enterprise he will adopt store cattle policy (1) if $(B - r) < 0$ and fat cattle policy (2) if $(B - r) > 0$, thus highlighting the role of r, price per ton of fodder, in his decision.

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