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THE IRISH GOVERNMENT BOND YIELD

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As will appear, the quite remarkable showing of the time trend for the ratio:-

Government bond yield (GBY): consumer price index (CPI)

impelled the writer towards a series of experiments reported in this memorandum on methods of analysis of nearly stationary time series. These include spectral analysis which he approaches with some skepticism as to practical utility, however, much he admires its mathematical elegance. A start is made with a short series of annual averages the idea being that if methods seem successful with these they can be extended to the more realistic shorter term (quarterly, monthly) series.

The experimental annual series used was derived from International Financial Statistics. Yields were those of a single stock, in later years on 6% Exchequer Loan 1985-90, no doubt deemed representative, though the stock quoted for was changed at intervals of years.

Chart 1 shows how the yields varied with CPI. Clearly the graphs were similar up to and including 1973; in fact the percentage rise in both in the period 1953-1972 were almost identical. Indeed the most remarkable feature of the chart is the break in the period 1974-1977. It may be added that the low yield continues into 1978. Despite our enormous and rapidly increasing national debt Irish Government credit is good.

[Chart 1]

Correlation between absolute values of time series in recent years is trivial because all are rising because of inflation and the inevitable positive correlation ratio is without significance. It is different with correlation between year-to-year changes, the deltas. We find no significant relationship between Δ GBY and Δ CPI during the period 1959-1977. This is clearly due to the 1974-1977 aberration, in turn seen from Chart 1 to be due mainly to the vast inflation of those years. When these years are omitted

and we add data for 1953-1958 to the beginning, the correlation coefficient between Δ GBY and Δ CPI is .59, significant with 18 d.f. at NHP = .01.

The interweaving of the graphs on Chart 1 suggested the possibility of a cyclical relation in the GBY series. Chart 2 shows real GBY, indexes of the quotient of GBY by CPI with 1975 as 100, 1953-1977. To the eye, there is some suggestion of a cyclical phenomenon. Defining maxima as "up-down" and minima as "down-up" points, distances between these in years are as follows:-

Maxima years	Interval in years	Minima years	Interval in years
1957, 1961	4	1955, 1959	4
1961, 1966	5	1959, 1963	4
1966, 1969	3	1963, 1967	4
1969, 1974	5	1967, 1972	5

These results were encouraging enough to prompt correlogram analysis. In Chart 3 the first fifteen correlation coefficients are displayed. While the c.c. for the four year interval is the highest of the twelve, none is formally significant at NHP = .05,* having regard to number of pairs of observations, no statistical validity cannot be claimed for a periodicity thesis. Nor does appeal to tau (applies to changes in sign of deviation from mean 105.58) improve matters much. Chart 2 shows 8 sign changes out of a possible

[Chart 2]

22: the one-sided NHP for this is .067 equivalent to a two-sided probability of .134, not to be regarded as significant. It is worth remarking that the process of deflation by CPI has converted a very highly autoregressed series for GBY**

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* Figures for years 1976 and 1977 were omitted from the calculations for reasons obvious from Chart 2.

** The tau test shows that there is but one change in sign in the deviations from mean 7.49 of GBY during the period 1953-1977, overwhelmingly indicative of autoregression.

(see Chart 1) into practically a random sequence.

[Chart 3]

Despite the foregoing qualifications about significance the overall showing of the correlogram is overwhelming, far clearer as indicating periodicity than Chart 1 relating to the raw data for real GBY. Maxima and minima are now more clearly defined:-

Maximum at lag -	Interval in years	Minimum at lag	Interval in years
4		2	
8	4	6	4
13	5	11	5
		15	4

By the way the lag 15 ordinate is a minimum, the lag 16 c.c. value (not shown on the chart) being -.02. There can be no doubt that the time series had a major periodicity of a little over four years in the near quarter century 1953-1975, a fact of little value, however, in forecasting values in 1976 and 1977.

Spectrum

The spectral function used on real GBY 1953 -1975

$$f(w) = \sum_{a=1}^{A} c_a \cos \pi a w$$

Number of observations is n; $A = n - 3$; c_a is the quasi-covariance

$$c_a = \sum_{t=1}^{n-a} u_t u_{t+a} / (n - a)$$

each of the u's being measured from the mean. The c_a's were calculated from their correlogram values supplied by the computer. In the first instance f (w) was calculated for values of w from 0 to 1 at intervals of .05:-

w	f (w)	w	f (w)
0	-14.55	.55	-30.53
.05	-11.09	.60	6.38
.10	- 4.17	.65	-25.78
.15	17.32	.70	32.63
.20	4.73	.75	-10.66
.25	-30.67	.80	-2.88
.30	5.98	.85	3.81
.35	10.79	.90	-27.75
.40	-3.08	.95	-13.09
.45	79.23	1.00	-21.17
.50	76.92		

Except from w values 0.45 and 0.50 of w the values of f (w) plunge about in a quite irregular way and we see no point in examining them further. But may not the f (w) values in the range $0.45 \leqslant w \leqslant 0.50$ also fluctuate wildly? This is not the case:-

w	f (w)
.45	79.23
.46	96.09
.47	105.87
.475	107.24
.48	106.48
.49	96.17
.50	76.92

There can be no doubt about the reality of the maximum of f(w) about half way between w = 0.45 and 0.50. But it is known* that the value of w indicating a high point in the spectrum also indicates the probable periodicity of the original u_t series. This periodicity is $2\pi/0.475\pi = 4.2$ years, in excellent accord with the showing of the original series and the correlogram.

* Time Series by M.G. Kendall, Charles Griffin and Co. Ltd, 1973, page 98.

Real GBY on an annual basis, represented (to repeat) by 6% Exchequer Loan 1985-90, the series used as representative for Ireland by IFS has had a period of a little over four years. Unfortunately the regularity was broken c.a. 1975, so rendering this periodicity of doubtful value for forecasting on the basis of a single series. We resume consideration of this point at the end of the section.

Aberration of real GBY

Was the price - behaviour of the Government stock selected by IFS abnormal in the years 1975-1978? To examine this point we set out the simple average £% gross, redemption yields for all the Government stocks quoted by the Central Bank in his Report for 1978 for the last Thursday in December of the years 1975-1977.

Table 4.1 Actual and real average redemption yields for Ireland, December 1975 - 1977

Years to maturity of stock	No. of stocks	Actual average gross redemption yields (£ %) December -			Real average gross redemption yields (values in £1975) December -		
		1975	1976	1977	1976	1977	% f all 1975-77
1. 3 or less	2	11.68	13.24	6.50	11.22	4.85	58.5
2. 3 - 10	4	13.42	14.16	8.84	12.00	6.59	50.9
3. 10 or over	13	14.69	15.35	10.87	13.01	8.11	44.8
6% Exch. Loan	1	14.57	15.49	10.34	13.13	7.71	47.1

Basic source: Central Bank of Ireland Report 1978.

The selected stock is one of the 13 in the long maturity series and its three values are very close to the general averages of group 3, except for a slight widening in 1977: if the group 3 averages were used in Chart 1 for the years 1975-1977 instead of those for the single stock the picture would be little changed.

A remarkable feature of Table 4.1 is its showing the marked widening of the yields in time in relation to date of maturity. The group 3 excess over group 1 moved from 26 per cent in 1975 to 67 per cent in 1977. One suspects that the reason is to be found in the corresponding UK figures. Table 4.2 shows that the change in the IFS series 61 between 1975 and 1977 was almost identical. In fact, over all seven years the trends in the two countries were very similar, the Irish yield being about £1 higher than the UK yields, the aberration to £2 in 1976 being restored in 1977.

Table 4.2 Actual Government Bond Yield, UK (long-term) and Ireland, 1971-1977

	1971	1972	1973	1974	1975	1976	1977
UK	8.90	8.91	10.72	14.77	14.39	14.43	12.73
Ireland	9.71	9.63	11.59	15.98	15.44	16.49	13.99

Source: International Financial Statistics, International Monetary Fund, June 1978, series 61

Note

The Irish series quoted are for the 6% Exchequer Loan 1985-90. They differ from those in Table 4.1, mainly because the latter are gross redemption yields for a single date in the year whereas the figures in Table 4.2 are monthly averages.

The break in trend in the value of actual GBY in Ireland in the last three years was obviously a UK phenomenon. The inflationary trends in the two countries were similar enough to permit the inference that the same is true of real GBY.

Quarterly data

Attention was confined to the period 1959-1975 (68 quarters). Data for 1976 and 1977 were omitted since the foregoing analysis has indicated their aberrant character, due mainly to UK influences.

[Chart 4]

In Chart 4 the interweaving between GBY and CPI is clearer than in annual Chart 1. This is the fundamental difference between the showing of the two series that whereas in the annual series 1953-1975 there is a significant positive correlation between the deltas of GBY and CPI this is entirely absent from the quarterly series (in fact r = .03). This apparent contradiction would be consistent with there being a true relationship (inflation probably being causal) but with adjustment taking place at irregular intervals during the year.

[Chart 5]

Chart 5 quarterly corresponds to Chart 2 annual, though the latter is in index number form. But the maxima and minima are far less decisive than in the annual case though, to the eye, the periodicity is still clear enough. Based on the arrows (some speculative), intervals are as follows:-

Maxima quarters	Interval in quarters	Minima quarters	Interval in quarters
I'62, IV '66	19	III '59, III 64	20
IV '66. I '70	12	III '64, IV 67	13
I '70, IV 74	20	IV '67, I '72	17
		I '72 III '75	14

The average for the maxima intervals is 17 quarters and for the minima 16 quarters, consistent with the corresponding result for the annual data.

Despite appearance, the quarterly real GBY series cannot be regarded as a stationary time series. Dividing the series equally into earlier and later series of 34 quarters each the following means and variances are found:-

	Mean	Variance
Earlier 34 quarters	16.34	1.1077
Later 34 quarters	17.27	2.4297

By the t - test applied to the difference of the means, this is significant at NHP = .01 and the ratio of the variances, namely F = 2.193 is significant at NHP = .025.

Before curve-fitting to real quarterly GBY it is necessary to show formally that the 68 successive time observations in Chart 5 are not random to one another. On the chart is shown the mean which, it may be observed, is an OLS fitting to the 68 observations. The mean line crosses the line of the observations 8 times which equals the number of sign changes in the OLS residues. By the tau test (8, 68)* is overwhelmingly significant: the observations are not random to one another. That the same test applied to the annual series failed to register significance is now seen to be due to the fewness of the latter series.

The correlogram shown on Chart 6 is obviously satisfactory in its regularity, especially the part of the curve up to about 30 lags. The peaking and wider amplitude with longer lags is not understood. It may have something to do with the fact that the number of pairs of observations correlated is diminishing; the number of pairs is the total number of observations, i.e. 68 less the number of lags; thus the number of pairs in 67 for the first lag and only 7 for the last. It is remarkable how well-behaved the curve is with so few pairs in the c.c. calculations at its right side. Deeming the original data for real GBY to be expressible as a harmonic series, i.e. a series of trigonometrical terms (say sines), the correlogram should indicate the periodicity of the leading term, i.e. the term with the largest coefficient. From Chart 6 it is obvious that real GBY has a very strong leading harmonic term.

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* NHP $\ll$ .001

The correlogram emphatically confirms the earlier rudimentary analysis that the main period must be of 16 or 17 quarters. We propose removing by OLS regression since terms of these periodicities and studying the residues for other periodicities. The only remaining difficulty was the phasing of the sine terms. These were determined experimentally from the earlier part of the charts graph. The sine curves designed as indvars were as follows, in the first instance:-

Periodicity

(i) 16 quarters : $\sin[\pi\ (t - 9)/8] = X_1$

(ii) 17 " : $\sin[2\pi(t - 7)/17] = X_2$

Separately OLS regressions were produced for real GBY (= Y) on (i) and (ii). We have seen earlier that the depvar has a significant upward trend so, partially to remove the trend and give the series the appearance of stationarity, time in quarters t was introduced as an indvar in each case:-

(i) $Y_c = 16.32 + \underset{(6.67)}{1.24}\ X_1 + \underset{(2.28)}{0.015}\ t,\ \bar{R}^2 = .48,\ F = 32.4$

(ii) $Y_c = 16.36 + \underset{(7.42)}{1.31}\ X_2 + \underset{(2.01)}{0.013}\ t,\ \bar{R}^2 = .48,\ F = 32.4$

The number of sets of observations was 68. Even if one does not know the number of degrees of freedom absorbed in deriving the period and phase of the since curves from the data, it is obvious that both regressions are, by the F - test overwhelmingling significant (e.g. NHP = .005 for F (2,60) = 5.80). As (ii) seems slightly to give the better fit we select it for further analysis; its Y_c values are graphed on Chart 5. Its residuals have 17 sign changes, indicating very considerable autoregression, for the one-sided NHP for r (17, 68) is far less than .001.

In an effort to eliminate auto-regression from the residuals, Y was regressed on X_2 and five orthopols, equivalent to fitting a trend polynomial of the 5th degree in t, instead of the 1st degree as at (ii) above. The residuals resulted in a r of 24/68, clearly an improvement on (ii) in this respect but still with autoregressed (.OKNHP $\angle$ .05):-

(iii) $Y_c = 16.80 + 1.36\,X_2 + P_5(t)$, $\bar{R}^2 = .59$, $F = 16.76$,
(8.24)

where $P_5(t)$ indicates the fitted orthopols. The constant term and the coefficient of X_2 have nearly the same value as in (i) and (ii) and the fit, as indicated by $\bar{R}^2$ improved somewhat. What is remarkable is the strength of the harmonic term. The value of the coefficient (and its improved t-value) is exactly that which would have been found if a simple regression of remainder of Y on remainder of X_2 after removal of in both cases trend represented by a polynomial of the fifth degree in t.

Our subsequent efforts to discover a second harmonic term were not successful. These efforts entailed the production of a correlogram and a spectral function from the remainders from OLS regression (ii) above. The principal term in the spectral function series of values seemed to indicate a period on which a sine term (X_3) with appropriate period and phase (as above) was constructed. But OLS regression of the original series Y = GBY/CPI on X_2, X_3 and the five orthopols failed to endow the coefficient of X_3 with a value significantly different from zero; nor was autoregression of residuals perceptibly improved. We must be content to have discovered one significant harmonic term but not to have explained the whole variation in the Y time series.

But there remained the 1974-1977 aberration in the GBY. We consulted our colleague, B.R. Dowling, who kindly supplied the following comment:-

> I don't know of any theory that should explain why gross bond yield (GBY) and the CPI level should be related. However the rate of interest (GBY) and the rate of inflation (Δ CPI %) ought to be related in the long run. Investors have some notion of the expected inflation rate $\dot{p}^e$ and the yield (GBY) less the expected inflation rate is the expected real return on investment. If inflationary expectations are formed with a lag then $\dot{p}^e$ may lag behind actual inflation, $\dot{p}$ was high and accelerating (to 1976) are right expect $\dot{p}^e$ to lag behind (especially if some part of $\dot{p}$ was due to 'once off' tax charges). Thus observed real bond yields (GBY - $\dot{p}$) will fluctuate much more than expected yields (GBY - $\dot{p}^e$) because of the smoother adjustment of $\dot{p}^e$.
>
> Another explanation for the observed changes in the ratio was the lack of profitable alternative investment opportunities in 1974-76 for banks etc. So that demand for gilts reflected an excess supply of funds rather than any fundamental change.

The last paragraph of the foregoing provides a clue for the break in trend, namely the recent recession, the most severe post war. The following comparison would appear relevant:-

Prices (1975 as 100)

	1971	1972	1973	1974	1975	1976	1977
1. Irish Government bond prices	159.0	160.3	133.2	96.6	100.0	93.6	110.4
2. Irish share prices	88.7	130.6	154.2	103.6	100.0	106.9	132.5

Source: IFS

Note

1. is based on the reciprocal of GBY (IFS No. 61)
2. is IFS No 62

It must be confessed that the foregoing figures do not support the thesis: the picture is lower prices for both series in 1974-76 compared to 1972-1973 with recovery in 1977. The comparison, however, is mildly interesting in itself. It does not disprove the Dowling thesis, a matter of fact. Perhaps the periodicity phenomenon of 1953-1975 will resume its relevance in more normal times.

1 October 1979 R. C. Geary

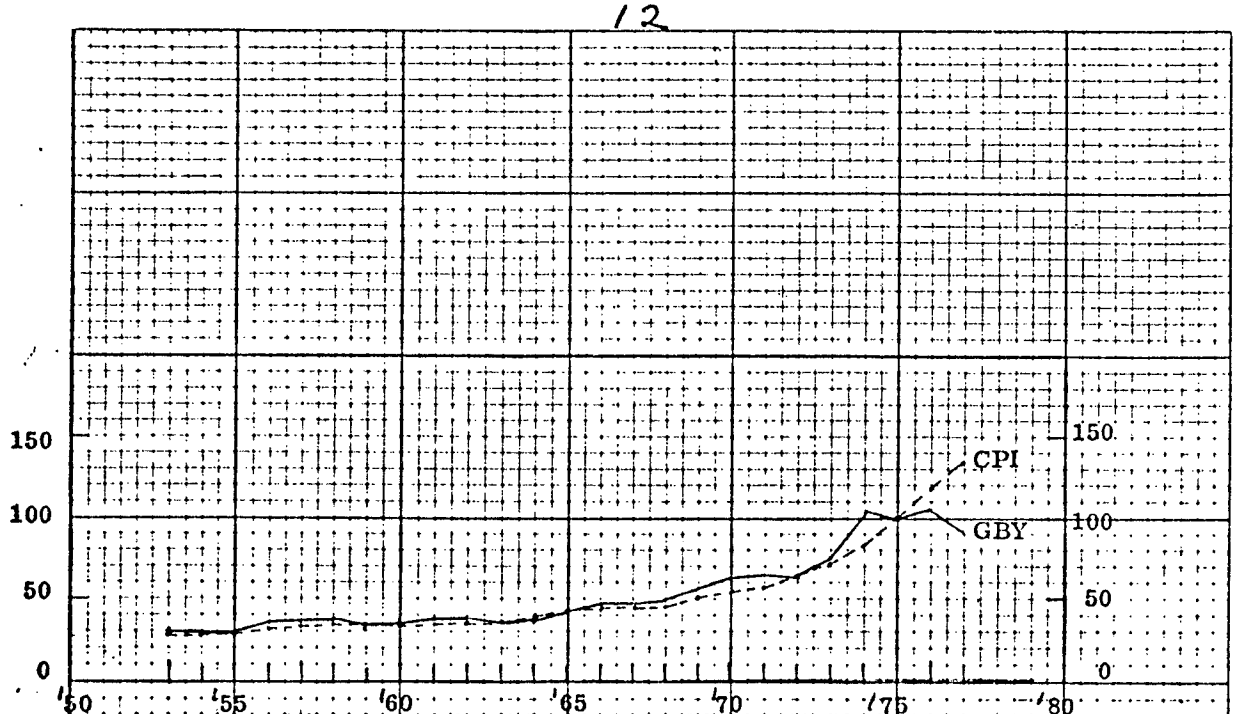

Chart 1. Irish Government Bond Yield (GBY) and Consumer Price Index (CPI), annual averages 1953-1977, 1975 as 100.

Chart 2. Ratio of GBY to CPI 1953-1976, 1975 as 100

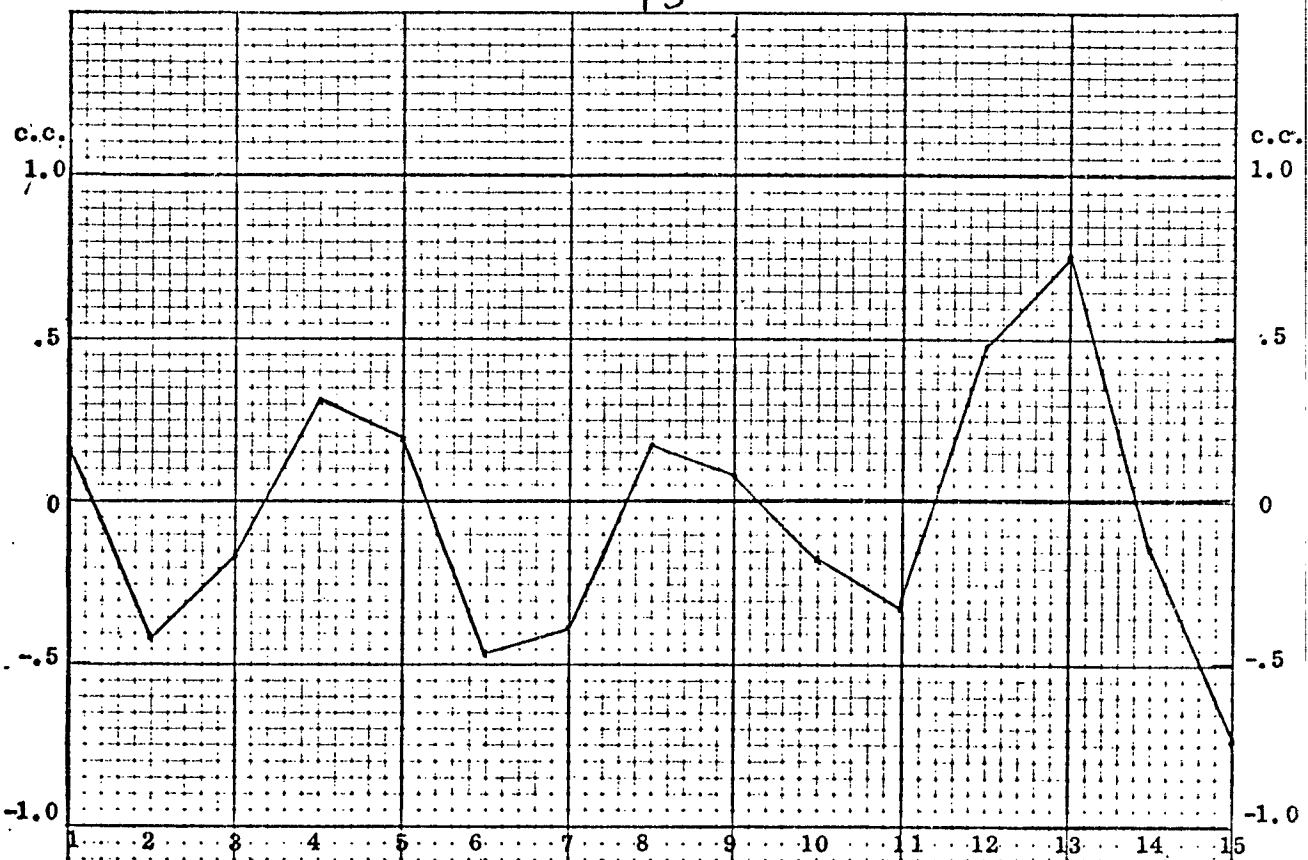

Chart 3. Correlogram for average annual real GBY 1953-1975, 15 lags.

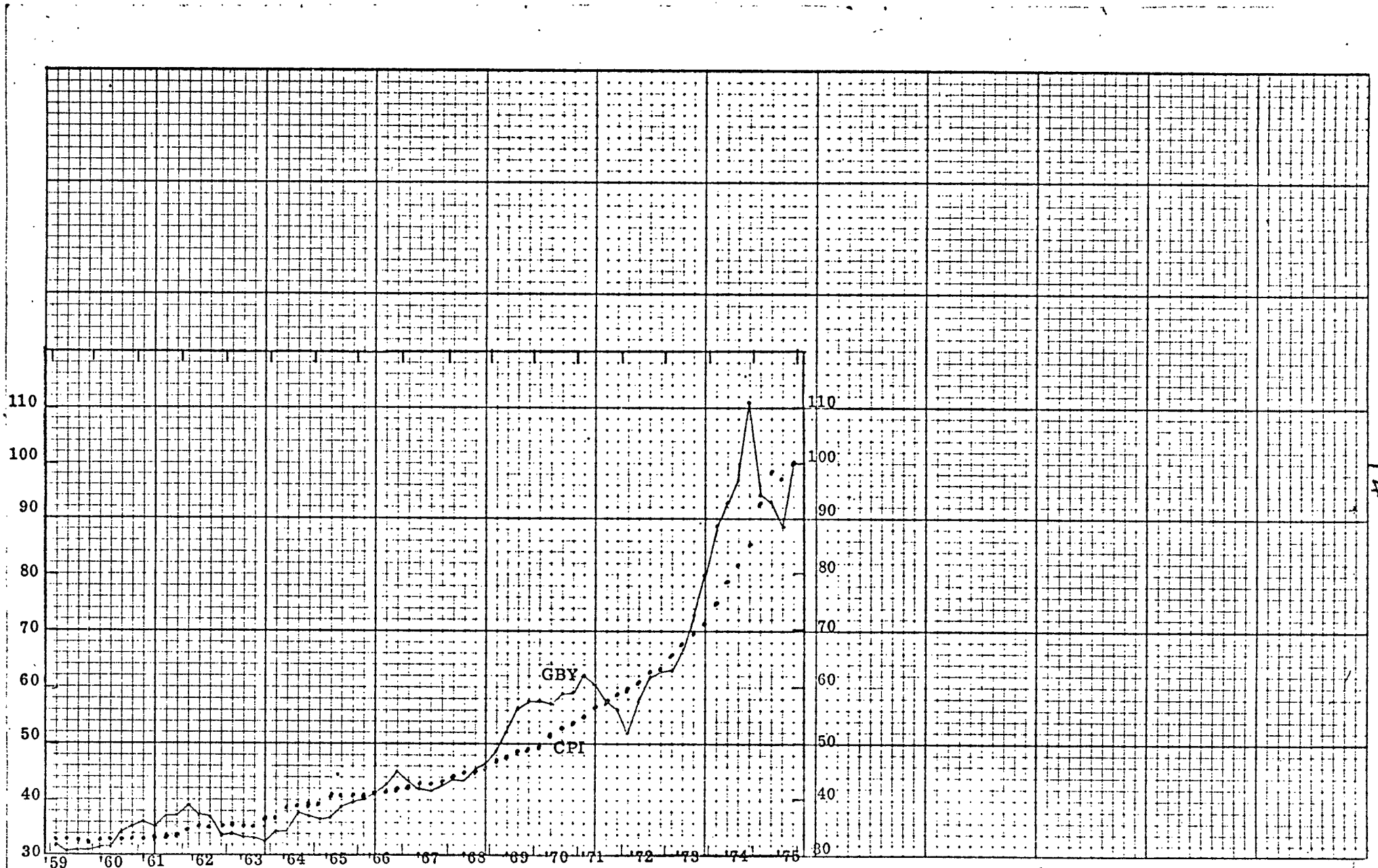

Chart 4. Government Bond Yield (GBY) and Consumer Price Index (CPI)

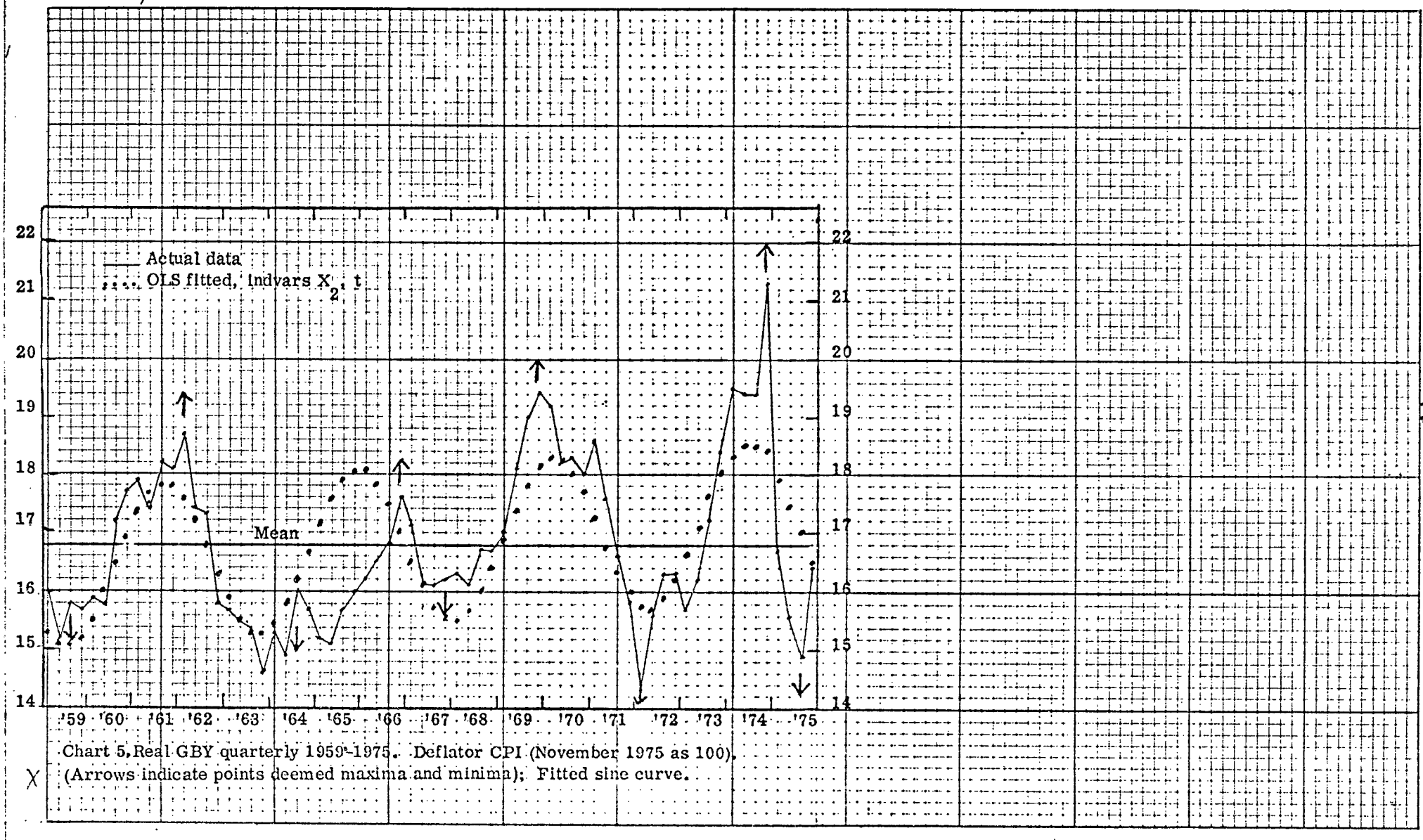

Chart 5. Real GBY quarterly 1959-1975. Deflator CPI (November 1975 as 100). (Arrows indicate points deemed maxima and minima); Fitted sine curve.

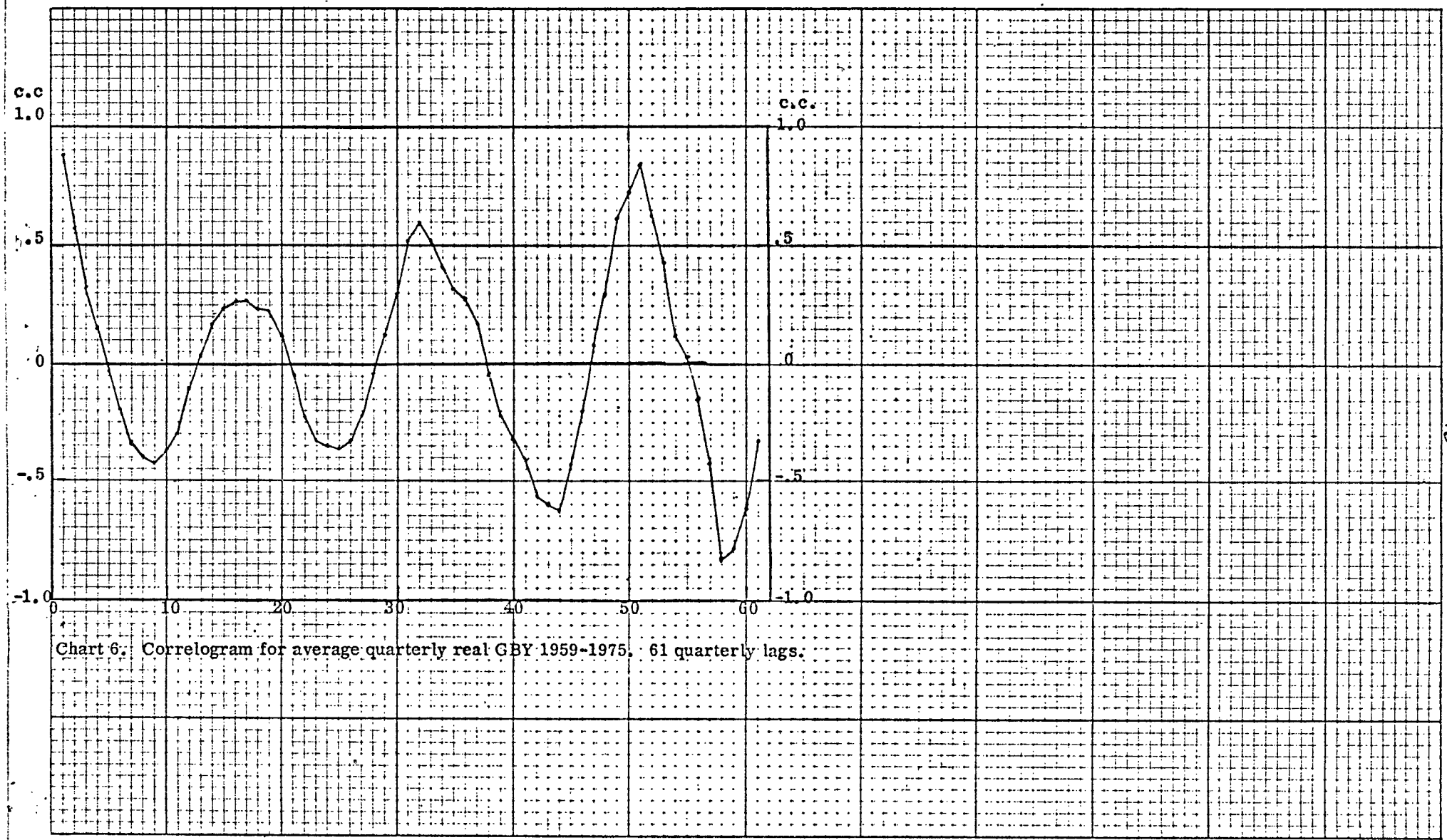

Chart 6. Correlogram for average quarterly real GBY 1959-1975. 61 quarterly lags.