The Updating of Certain Econometric Models

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SECTION 4: THE UPDATING OF CERTAIN ECONOMETRIC MODELS

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§4.1 *Introduction*

The editors of the *Quarterly Economic Commentary* make use of a variety of econometric models, developed in the past by the Institute, as a guide to forecasting. Such models indicate the implications of different assumptions about the course of the economy, the impact on the economy of extrapolated trends, and the consistency of the forecasts of the National Accounts components made in Section 2.2, both with themselves and with the experience embodied in the models.

Following normal practice in the use of working models, it was decided to re-run the equations using the latest figures, and where possible to attempt an improvement in the models. Updating the models not only keeps them relevant to current conditions, but also serves as a check on the stability over time of the implied relationships.

LESER'S CONSISTENCY MODEL

§4.2 *The Original Model*

In *The Irish Economy in* 1964 *and* 1965 (ERI Paper No. 27) C. E. V. Leser presented what has become known as his "consistency" model. This is strictly an empirical model, to be used as a forecasting tool, and does not attempt to analyse fundamental structural relationships. It is a small scale macro-model involving a limited number of equations designed to predict percentage changes in National Accounts items: Personal Expenditure on Consumers Goods and Services (C); Public Net Current Expenditure (G); Gross Domestic Fixed Capital Formation (I); Exports of Goods and Services (X); Imports of Goods and Services (M); and Gross National Product (Y); all in current price terms. The main predictor variable is Final Demand (D) defined as follows:

$$\begin{aligned} D &= C+G+I+X \\ &= M+Y-B \end{aligned}$$

where B is stockbuilding.

The analysis is conducted in terms of year-to-year percentage changes for all variables (except B which appears as a residual once M, Y have been obtained), and c, g, i, x, m, y and d are used to indicate percentage changes in the variables. For stylistic reasons, Leser chose to use the subscript $_p$ to indicate the predicted value of each dependent variable, rather than to include an error term in each equation. His notation has been followed here.

Using current price data from 1948/49 to 1962/63 inclusive the following six prediction equations were estimated and presented by Leser.

			R^2
(2.1)	c_p	$= 1.12+0.640d-0.157(c_{-1}-y_{-1})+k$ (4.211) (1.256)	0.635
(2.2)	g_p	$= -0.61+1.150d-0.076(g_{-1}-y_{-1})+k$ (3.134) (0.398)	0.450
(2.3)	i_p	$= -2.41+1.041d+0.563i_{-1}+k$ (1.355) (3.060)	0.615
(2.4)	x_p	$= -0.39+1.139d-0.043(x_{-1}-m_{-1})+k$ (2.848) (0.402)	0.410
(2.5)	m_p	$= -3.45+1.808d-0.895(p_y-p_m)$ (3.217) (3.496) $-0.410(m_{-1}-d_{-1})+k$ (2.278)	0.759
(2.6)	y_p	$= 1.33+0.700d+0.374(p_y-p_m)$ (4.046) (5.123) $-0.356(y_{-1}-d_{-1})+k$ (2.312)	0.774

The t-values of each coefficient are shown in brackets. It should be noted that in the case of the coefficients of d the t-ratio is not a true test of significance. D is an aggregate containing the components C, G, etc., and so d is a weighted average of the values of c, g, etc.

The model contains several interesting characteristics. As D is defined to be the sum of C, G, I and X,

$$d = \frac{C_{-1}c_p+G_{-1}g_p+I_{-1}i_p+X_{-1}x_p}{C_{-1}+G_{-1}+I_{-1}+X_{-1}}$$

and it is necessary that the calculated values c_p, g_p, i_p, x_p be consistent with a given value of d inserted in the equations. This consistency is maintained by the inclusion of a correction factor k, which is the difference between the value of d calculated as above and the inserted value of d. Thus k is the same for all equations, but varies from year to year.

Each equation also contains a lagged term involving (except in the case of the investment equation) the difference between the percentage changes of variables which on *a priori* grounds might be expected to move roughly in line with each other. The expectation here is that any considerable divergence between two such variables in a year will lead to an adjustment in the opposite direction in the following year. The investment equation does not contain any such "adjustment variable" and the lagged value of i is included to reflect the cyclical nature of investment.

The Import and Gross National Product equations also contain a price variable consisting of the difference between the percentage changes in the implied price indices for Gross National Product (p_y) and Imports (p_m).

§4.3 *Updated Consistency Model*

Initially, using data for the period 1948/49–1967/68, providing 20 observations as opposed to 15 in Leser's study, the equations were recalculated. The results are shown below.

			R^2
(3.1)	c_p =	$0.87+0.696d-0.157(c_{-1}-y_{-1})+k$ (6.770) (1.440)	0.749
(3.2)	g_p =	$-1.60+1.342d-0.035(g_{-1}-y_{-1})+k$ (5.645) (0.216)	0.652
(3.3)	i_p =	$-2.45+1.023d+0.546i_{-1}+k$ (1.960) (3.351)	0.563
(3.4)	x_p =	$-0.16+1.118d-0.03(x_{-1}-m_{-1})+k$ (4.261) (0.333)	0.519
(3.5)	m_p =	$-4.39+1.886d-0.855(p_y-p_m)$ (5.364) (4.417) $-0.409(m_{-1}-d_{-1})+k$ (2.648)	0.785
(3.6)	y_p =	$1.36+0.714d+0.350(p_y-p_m)$ (6.439) (5.897) $-0.341(y_{-1}-d_{-1})+k$ (2.530)	0.826

With the exception of the investment equation, the fit as measured by R^2 is higher taking the longer period of years. The lagged terms in the government expenditure and export equations are again not statistically significant, though the lagged term $(c_{-1}-y_{-1})$ in the consumption equation performs slightly better—being significant at the 10 per cent level.

What is immediately apparent from comparing the updated model with the original is the constancy exhibited by the coefficients over the longer period of years. The equation for g does show some variation both in the coefficient of d (though the new coefficient of 1.342 is not significantly different from that previously estimated of 1.150), and in the coefficient of the lagged term (which itself is not significant). For the rest the differences are slight. A consequence of this is that the associative structure implied by the model has remained relatively constant over time and thus the use of the model in the past (as in several issues of the *Quarterly Economic Commentary*) for checking the consistency of forecasts has not been too misleading.

§4.4 *Amending the Model: Lagged Terms*

While the results of updating the model are satisfactory in themselves it was felt that some improvement could be made while preserving the basic structure of the equations. Experiments were therefore carried out on the lagged terms used by Leser. In the consumption equation for instance rather than using $(c_{-1}-y_{-1})$ two new terms were introduced $(c_{-1}-\beta_1 y_{-1})$ and $(c_{-1}-\beta_2 d_{-1})$ where β_1 is the ratio of the average percentage change in c and y, and β_2 is the ratio of the average percentage change in c and d over the period. The lagged term thus involves the difference between the actual percentage change and an "expected" percentage change. Similar terms were introduced for the remaining equations. In the re-estimated equations involving the new variables the results, with the exception of the investment equation, were not markedly different in terms of R^2. The results of some of the regressions run are given below.

			R^2
(4.1)	$c_p =$	$1.320+0.650d-0.243(c_{-1}-0.85d_{-1})+k$ (5.327) (1.021)	.735
(4.2)	$c_p =$	$1.004+0.696d-0.189(c_{-1}-0.86y_{-1})+k$ (6.853) (1.587)	.755
(4.3)	$g_p =$	$-1.402+1.307d-0.224(g_{-1}-1.10d_{-1})+k$ (5.959) (0.963)	.669
(4.4)	$g_p =$	$-1.619+1.34d-0.038(g_{-1}-1.11y_{-1})+k$ (5.650) (0.240)	.653
(4.5)	$i_p =$	$1.340+1.268d+0.420(i_{-1}-1.55y_{-1})+k$ (2.347) (2.409)	.481
(4.6)	$x_p =$	$-0.190+1.121d-0.019(x_{-1}-1.02m_{-1})+k$ (4.264) (0.201)	.517
(4.7)	$m_p =$	$-4.45+1.872d-0.853(p_y-p_m)-0.399(m_{-1}-1.07d_{-1})+k$ (5.264) (4.349) (2.540)	.779
(4.8)	$m_p =$	$-4.70+1.915d-0.850(p_y-p_m)-0.292(m_{-1}-1.08y_{-1})+k$ (5.298) (4.280) (2.43)	.841
(4.9)	$y_p =$	$1.363+0.728d+0.356(p_y-p_m)-0.377(y_{-1}-0.99d_{-1})+k$ (6.957) (6.264) (2.933)	.841

The results hardly justify the increased labour involved in recalculating yearly the changes in the predetermined parameters of the lagged terms. The lagged terms of the export and government expenditure equations remain statistically insignificant while that of the consumption equation—in the second equation presented above using $(c_{-1}-\beta_1 y_{-1})$ rather than $(c_{-1}-\beta_2 d_{-1})$—is marginally better. In general the equations agree fairly closely with those calculated previously.

A further attempt to modify the model was made by taking final supply (d+B) rather than final demand (d) as the main independent variable. In this case the correction factor has to be calculated from the predicted values of m and y, rather than from the demand components. The results, calculated for both 20 year and 15 year periods, showed no improvement over those based on final demand, except in the case of imports. As imports are dealt with separately in §4.10 in a model based on final supply, it is felt that the general consistency model is best left based on final demand.

§4.5 *Amending the Model: Investment Equation*

The revised formulation of the investment equation given in §4.4 performs rather worse than the original formulation derived by Leser. Even in its original form however the equation is not entirely satisfactory, and appears to require some modification. While investment does seem to display cyclical characteristics over the period, the model as formulated does not pick up turning points. A comparison of actual percentage changes, i, and calculated changes (excluding the adjustment term k which tends to be small), i_p, indicates that the equation consistently fails to "predict" both upturns and downturns.

Year	Actual i %	Predicted i_p %	Year	Actual i %	Predicted i_p %
1948/49	32.52	18.52	1958/59	3.75	0.08
1949/50	19.00	22.69	1959/60	8.07	7.49
1950/51	20.16	19.24	1960/61	20.96	11.82
1951/52	5.16	15.32	1961/62	18.25	16.22
1952/53	−0.12	7.20	1962/63	15.74	16.40
1953/54	6.02	−0.68	1963/64	17.31	19.69
1954/55	6.49	6.07	1964/65	13.61	13.55
1955/56	−0.33	1.35	1965/66	−1.21	10.77
1956/57	−12.45	0.01	1966/67	10.33	4.95
1957/58	−0.25	−4.74	1967/68	16.83	16.88

In fact the turning points, especially the downturns, in expenditure on fixed capital formation are strongly influenced by official decisions, working directly on the public capital programme and indirectly, through monetary policy, on private investment. Accordingly it seems desirable to include in the investment equation some variable to take account of policy changes. Although it might be possible to construct a direct "policy" series, based on the capital budgets for each year and the credit advice given to the associated banks by the Central Bank, it was felt that such a series would have serious drawbacks. Instead it was decided to attempt to isolate the circumstances in which a policy restrictive to investment expenditure might be expected to operate.

Historically, it appears that policy is strongly affected by the level of the external reserves and by the size of the current account deficit on the balance of payments. Taking the December figures for each year, the percentage changes in the level of external reserves (external assets in the earlier years) were calculated. In the case of the balance of payments, the surplus or deficit in each year was expressed as a percentage of that year's total imports of goods or services. Not surprisingly, attempts to use these series directly in the equations were not successful, as changes in the reserves or the external deficit have a major impact on policy only in certain circumstances. Therefore the series were used to construct a dummy variable which seeks to isolate these years when a policy response might be expected.

Years containing both a decline in reserves of more than 10 per cent and a deficit equivalent to more than 10 per cent of imports were ascribed a value of 2. When the fall in reserves and the ratio of the deficit to imports were both more than 5 per cent (but not both more than 10 per cent), or when one series was above 10 per cent but the other below 5 per cent, a value of one was given. For other years a zero value was entered. This procedure gives a dummy variable having a value of 2 for 1951 and 1955, a value of 1 for 1948, 1950, 1956 and 1965, and a value of 0 for the remaining years between 1948 and 1968. In order to allow for the inevitable delays in the authorities receiving information and taking decisions, and in policy measures actually affecting the level of investment, the dummy variable Z has been included in the equations only in lagged forms.

Including the dummy variable, lagged by both one and two years, in both the original formulation and the revised formulations investigated in §4.4, the following results are obtained.

				R^2
(5.1)	i_p	=	$3.86 + \underset{(1.629)}{0.667d} + \underset{(3.538)}{0.474i_{-1}} - \underset{(2.428)}{5.047Z_{-1}} - \underset{(2.102)}{4.489Z_{-2}} + k$	.777
(5.2)	i_p	=	$6.124 + \underset{(2.020)}{1.012d} - \underset{(1.885)}{4.872Z_{-1}} - \underset{(1.524)}{4.217Z_{-2}} - \underset{(1.909)}{0.385(i_{-1} - 1.53d_{-1})} + k$	.634
(5.3)	i_p	=	$8.293 + \underset{(1.486)}{0.721d} - \underset{(2.709)}{7.039Z_{-1}} - \underset{(0.960)}{2.605Z_{-2}} - \underset{(1.486)}{0.495(i_{-1} - 1.55y_{-1})} + k$	.679

The fit of the investment equations is considerably improved by the inclusion of the lagged dummy variable. Clearly the formulation 5.1 containing also the simple lagged investment term is better than either of the alternatives. Comparing this with equation 3.3, it can be seen that the coefficient of d has been reduced from 1.023 to 0.667, a result more in keeping with commonsense expectations that investment would tend to be inelastic in the short run with respect to changes in final demand.

The usefulness of this equation for predictive purposes depends of course on the maintenance of the policy reaction to successive deficits in the balance of payments on current account accompanied by falling external assets. Any change in the official view as to what constitutes an "acceptable" level of reserves, with perhaps growth being desired irrespective of the level of reserves for a longer period than that implied in the equation, or even a quicker reaction to apparent trends could undermine the formulation of the equation.

§4.6 *Amending the Model: Price Changes*

An approach to the problem of differential price changes between the components of d was adopted by introducing a further variable into the equations for c, g, x, and i in current values. This term involves, for each equation, the difference between the percentage change in the implied price index (p_c, p_g, p_x or p_i) of the variable to be estimated, and the percentage change in the implied price index for gross national product. The expectation is that this term will pick up any differences that arise between these four variables merely on account of price changes—which the original model was unable to handle.

The effect of the introduction of this new variable on each equation can be seen below.

			R^2
(6.1)	c_p	$= 0.852+0.698d-0.153(c_{-1}-y_{-1})+0.038(p_y-p_c)+k$ $(6.556)\quad(1.331)\qquad(0.155)$	.749
(6.2)	g_p	$= -0.689+1.109d+0.079(g_{-1}-y_{-1})-0.592(p_y-p_g)+k$ $(4.589)\quad(0.510)\qquad(2.144)$	.730
(6.3)	x_p	$= -0.435+1.143d-0.037(x_{-1}-m_{-1})+0.095(p_y-p_x)+k$ $(4.113)\quad(0.380)\qquad(0.362)$	.523
(6.4)	i_p	$=3.876+0.664d+0.468i_{-1}-4.936Z_{-1}-4.709Z_{-2}+0.107(p_y-p_i)+k$ $(1.564)\quad(3.265)\quad(2.165)\quad(1.767)\quad(0.147)$	.777

Of the four equations the only one altered substantially is the government expenditure equation, whose R^2 increases from 0.652 to 0.730. Because of the improvement in fit, and because the price term itself emerges as significant at the 5 per cent level, equation 6.2 seems to be a preferable alternative to equation 3.2.

§4.7 *Testing the Model*

In the light of the foregoing discussion it appears that the best combination of equations for use as a consistency check in forecasting current price expenditure in National Accounts terms is as follows:

(3.1) $c_p = 0.87+0.696d-0.157(c_{-1}-y_{-1})+k$

(6.2) $g_p = -0.69+1.109d+0.079(g_{-1}-y_{-1})-0.592(p_y-p_g)+k$

(5.1) $i_p = 3.86+0.667d+0.474i_{-1}-5.047Z_{-1}-4.489Z_{-2}+k$

(3.4) $x_p = -0.16+1.118d-0.03(x_{-1}-m_{-1})+k$

(3.5) $m_p = -4.39+1.886d-0.885(p_y-p_m)-0.409(m_{-1}-d_{-1})+k$

(3.6) $y_p = 1.36+0.714d+0.350(p_y-p_m)-0.341(y_{-1}-d_{-1})+k$

As a test of the model the results have been calculated from 1969 and are compared below with the preliminary National Accounts estimates, as given in the June 1970 *Quarterly Economic Commentary.*

1969 *Actual* (preliminary estimate)		*Calculated*	
c =	11.9	c_p =	9.8
g =	12.4	g_p =	14.5
i =	23.8	i_p =	20.3
x =	9.8	x_p =	14.4
m =	17.5	m_p =	17.2
y =	11.6	y_p =	11.9
	d = 13.0		

The results predicted by the model are in reasonably close accordance with the preliminary National Accounts for 1969. The actual consumption increase is rather higher than the calculated, while the actual export figure is a good deal lower than the calculated. This shortfall can be explained in part by the stagnation in cattle and beef exports, and partly, though to a much lesser extent, by the maintenance men's dispute of early 1969.

The model naturally is not very successful at handling the type of factor which causes these divergences. While d implicitly contains these factors the equations do not explicitly isolate and distribute them in any meaningful way. For instance, for the given value of d=13.0 derived on the basis of assumptions regarding x and c, the model "overpredicts" x and "underpredicts" c. The equations distribute d according to the "normal" pattern of the past.

§4.8 *Constant Price Consistency Model*

Rather than working in current price terms, the equations were re-estimated with the variables expressed in constant 1958 prices (Table A.6 and B.6 in *National Income and Expenditure* 1968). It is hoped that this may eliminate to some extent the effect of different price changes on the various components of the National Accounts. The results are given below. The notation is as in the current price model, except that each term here applies to the percentage change at constant prices.

		R^2
(8.1)	$c_p = -0.34 + \underset{(5.597)}{0.736d} - \underset{(1.145)}{0.173(c_{-1} - y_{-1})} + k$	.695
(8.2)	$g_p = 0.72 + \underset{(1.903)}{0.514d} + \underset{(0.206)}{0.048(g_{-1} - y_{-1})} + k$	.178
(8.3)	$i_p = 4.07 + \underset{(0.993)}{0.742d} + \underset{(3.080)}{0.489i_{-1}} - \underset{(1.902)}{5.838Z_{-1}} - \underset{(1.109)}{2.908Z_{-2}} + k$	.724
(8.4)	$x_p = 1.412 + \underset{(2.551)}{0.940d} + \underset{(0.453)}{0.052(x_{-1} - m_{-1})} + k$	.304
(8.5)	$m_p = -1.804 + \underset{(4.277)}{2.030d} - \underset{(1.142)}{0.240(m_{-1} - d_{-1})} + k$	.539
(8.6)	$y_p = 0.559 + \underset{(5.522)}{0.642d} - \underset{(1.239)}{0.228(y_{-1} - d_{-1})} + k$	.657

In terms of goodness of fit the results are lower than using current price terms. The government expenditure equation performs very badly. This is in part explained by the conceptual difficulty, common to all National Accounts items but more particularly so to Government Expenditure, of arriving at a constant price value. The remaining equations seem satisfactory enough given that the data is in constant price terms. Some experiments were carried out on the import equation giving:

		R^2
(8.7)	$m'_p = 4.655 + \underset{(1.602)}{0.842d} - \underset{(0.817)}{0.139(m_{-1} - d_{-1})} - \underset{(3.297)}{7.395Z_{-1}} + k$	.725

where Z is the policy proxy dummy described in 4.5. The inclusion of Z_{-1} improves the fit of the equation, but commonsense dictates caution where the coefficient of a rather arbitrary dummy variable is so large in relation to the variation in the dependent variable.

An exercise similar to that carried out for the current value equations has been undertaken comparing the model predictions with the preliminary estimates for 1969.

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Actual (preliminary estimate)		*Calculated*	
c	= 4.4	c_p	= 3.7
g	= 3.6	g_p	= 3.5
i	= 13.9	i_p	= 14.5
x	= 5.8	x_p	= 6.2
m	= 11.9	m_p	= 8.2
		(m'_p	= 8.3)
y	= 4.0	y_p	= 4.2

$d = 6.0$

As in the case of the current price estimates, the predictions in constant price terms agree reasonably well with the preliminary estimates except in this case for imports of goods and services.

CONSUMPTION AND IMPORT FUNCTIONS

In the *Irish Economy in* 1966 (ERI Paper No. 33) the staff of the Economic Research Institute presented a consumption function and an import function, which can be regarded together as a semi-recursive model or separately as two independent single-equation models.

§4.9 *The Consumption Function*

Personal Consumer Expenditure is considered as a function of average weekly wage earnings in transportable goods industries, agricultural prices, other final supply (defined below), and a lagged adjustment variable, gross national product less personal expenditure. The period over which the model is estimated is 1948/49–1963/64.

The dependent variable is the year to year percentage change in personal expenditure on consumers goods and services at current prices, c.

The first independent variable (e) is the percentage change in average weekly wage earnings in transportable goods industries. This is obtained from the annual index for 1948–1950, and thereafter by taking the average of the four quarterly figures published in the Quarterly Industrial Inquiry (CSO).

The use of this index as a non-agricultural income variable is justified on the grounds that it moves fairly closely in line with employee remuneration. A comparison of percentage changes in the index with percentage changes in employee remuneration obtained in *National Income and Expenditure* (Table A.2 and B.2) 1968, from 1948/49 to 1967/68, bears out this relationship.

Year	Non-agricultural employee remuneration	Wages in transportable goods industries	Year	Non-agricultural employee remuneration	Wages in transportable goods industries
1948/49	7.5	4.2	1958/59	5.3	3.7
1949/50	9.2	3.6	1959/60	8.4	6.8
1950/51	9.5	9.5	1960/61	10.0	5.8
1951/52	−4.8	5.2	1961/62	11.2	9.9
1952/53	7.9	7.5	1962/63	8.1	3.7
1953/54	3.5	1.0	1963/64	16.3	12.3
1954/55	4.8	5.9	1964/65	6.5	3.0
1955/56	5.0	5.7	1965/66	8.6	9.0
1956/57	−0.1	2.6	1966/67	7.7	7.1
1957/58	3.8	5.7	1967/68	10.4	8.6

The coefficient of correlation between these two series of percentage changes is .7558 which is reasonably high—but any serious divergence between them is likely to be reflected in the estimates of c.

The second independent variable is the percentage change in the agricultural price index p_a (1953=100), taken as an indicator of agricultural income.

The relationship between agricultural income and the agricultural price index is not quite as strong as in the case of non-agricultural income and the index of average weekly wage earnings.

Year	Agricultural income	Agricultural price index	Year	Agricultural income	Agricultural price index
1948/49	7.9	0.4	1958/59	8.8	−0.1
1949/50	−5.4	4.4	1959/60	2.2	−2.7
1950/51	6.9	10.0	1960/61	4.7	0.4
1951/52	16.2	3.4	1961/62	3.2	1.7
1952/53	8.4	6.8	1962/63	−1.5	0.5
1953/54	−6.1	−1.3	1963/64	16.7	10.7
1954/55	9.9	4.5	1964/65	0.8	4.1
1955/56	−9.3	−9.3	1965/66	−4.3	−1.5
1956/57	10.1	6.7	1966/67	7.6	2.1
1957/58	−8.3	2.7	1967/68	13.1	10.2

The coefficient of correlation in this case is 0.6397. Other variables are percentage changes in other final supply s_o, where $S_o = G + I + X + B$, which "allows for autonomous changes in exports, investment and possibly government expenditure to exert an influence on gross national product and thus simultaneously on personal consumption"; and finally, as in the consistency model, a lagged term $(y_{-1} - c_{-1})$ is introduced as an adjustment variable.

The regression equation presented is:

$$(9.1) \qquad c = 0.699 + 0.6685e + 0.2367p_a + 0.1505(y_{-1} - c_{-1}) \qquad R^2 = .805$$
$$(4.421) \quad (2.618) \quad (1.356)$$

As s_o was not significant it was dropped from the analysis.

Taking the period 1948/49–1967/68 the regressions were re-run giving the following two equations:

$$(9.2) \qquad c = 0.844 + 0.5666e + 0.2068p_a + 0.0849s_o + 0.1747(y_{-1} - c_{-1}) \qquad R^2 = .802$$
$$(4.225) \quad (2.032) \quad (1.033) \quad (1.641)$$

and

$$(9.3) \qquad c = 1.109 + 0.6041e + 0.2750p_a + 0.1548(y_{-1} - c_{-1}) \qquad R^2 = .788$$
$$(4.699) \quad (3.545) \quad (1.476)$$

The coefficient of s_o is again not significant and dropping it from the equation makes little difference to the fit. However, given the nature of the model as a whole, maintaining s_o seems to be sensible in itself. The accordance between the re-estimated equation and the original is satisfactory. Experiments were carried out including numbers employed in transportable goods industries as an additional variable. These failed to improve the fit of the equation, possibly because of a high degree of collinearity between changes in earnings and employment. Alternative functions based on National Accounts income categories were calculated, but were less successful than the consumption functions shown.

§4.10 *The Import Function*

The year to year percentage change in imports of goods and services (m) is considered as a function of the year to year percentage change in total final supply (s_t) where $S_t = C + I + G + X + B$, an adjustment variable consisting of the difference between percentage changes in y and m lagged one year $(y_{-1} - m_{-1})$, and a dummy variable v for the period 1954/55 to 1957/58. The period of the regression was 1953/54 to 1963/64—the earlier years being dropped due to the "violent fluctuations to which imports were subject in the early post-war years." There were thus 11 observations for the regression equation calculated, and shown below.

$$(10.1) \qquad m = -1.551 + 1.3785s_t + 0.1833(y_{-1} - m_{-1}) + 4.127v_t \qquad R^2 = .943$$
$$(7.945) \quad (1.577) \quad (3.419)$$

Using the period 1953/54 to 1967/68, i.e. 15 observations, the equation becomes:

$$(10.2) \qquad m = -3.258 + 1.6078s_t + 0.2045(y_{-1} - m_{-1}) + 3.4034v_t \qquad R^2 = .929$$
$$(10.064) \quad (1.652) \quad (2.568)$$

In the import function total final supply s_t is clearly crucial; R^2 using s_t alone is 0.86. Taking the shorter period and using total final supply rather than final demand gives better results in terms of R^2 than obtained in the consistency model (equation 3.5), but the improvement is at the expense of an explanation of the earlier period's movements. However the use of s_t which includes B rather than d which excludes it, along with a comparison between the equations, seems to imply that the state of stocks is highly relevant for imports. As a check on this the equation of the consistency model for imports has been recalculated using the years 1953/54 to 1967/68. The equation is:

$$(10.3)\quad m = -3.532 + \underset{(4.765)}{1.749d} - \underset{(0.415)}{0.360(m_{-1} - d_{-1})} - \underset{(0.748)}{0.298(p_y - p_m)} \qquad R^2 = .695$$

and using d as the sole independent variable the R^2 is .678. This result does seem to agree with the hypothesis that changes in the level of stocks are important when considering the level of imports.

4.11 *Testing the Consumption and Import Functions*

Taking the consumption function and the import function together we have a model which enables us to test the implications of alternative assumptions regarding the course of the economy. For the consumption function (equation 9.2) the impact of different levels of earnings, agricultural prices, exports, Government expenditure and investment (the latter three appearing as components of other final supply) on consumer expenditure can be assessed. In the semi-recursive form of the model, this calculated value for consumption together with the assumed value for s_o used in the consumption function, gives a value for s_t which forms the current independent variable for the import function (equation 10.2).

If the equations are regarded as independent of each other, then a value of s_t based on any set of assumptions, and not necessarily compatible with the calculated value of c, can be used for the import function.

Tests have been carried out for 1969 compared with the preliminary National Accounts estimates, giving the following results.

1969

	Actual (preliminary estimate)	*Calculated* *Recursive*	*Calculated* *Independent*
c =	11.9	9.7	9.7
m =	17.5	14.6	15.5

RECURSIVE MODEL

§4.12 *Leser's Recursive Model*

In Appendix I of the *Irish Economy in* 1967 (ESRI Paper No. 39) C. E. V. Leser introduced a simple recursive model of the Irish economy. The notation used is as in previous models with Yd (personal disposable income), Ba (value of physical changes in agricultural stocks), Bn (=B−Ba), Z (a dummy variable=1 for 1954/55 and 1957/58; =2 for 1955/56; and=0 elsewhere) and Z′ ($=Z+Z_{-1}$); brought in as additional variables. The analysis is carried out in first differences as opposed to the percentage changes of the previous models, and the period of cover is 1953/54 to 1964/65. The equations presented by Leser are given below.

(12.1) $$\triangle M = 1.200+0.8238\triangle I+0.5574(\triangle X+\triangle Ba)+11.416Z$$
$R^2 = 0.948$
$S = 5.30$

(12.2) $$\triangle Y-\triangle G = 1.623+0.5491\triangle I+0.9214(\triangle X+\triangle Ba)+1.7976\, e$$
$R^2 = 0.944$
$S = 7.09$

(12.3) $$\triangle Yd = -3.980+0.9762(\triangle Y-\triangle G)$$
$R^2 = 0.935$
$S = 6.91$

(12.4) $$\triangle C = -32.520+0.6469\triangle Yd+508.81\left(\frac{Yd-C}{Yd}\right)_{-1}+4.325Z'$$
$R^2 = 0.937$
$S = 5.84$

This model has already been tested by Geary (*Economic and Social Review,* Vol. 1, No. 1) for the years 1947–1953 and gives rather disappointing results. It also behaved rather poorly when tested as a predictor for 1967 and 1968. However, the earlier years were excluded from the data used to derive the forecasting equation —presumably on account of the violent changes evident in the earlier period.

The equations have been re-estimated using data to 1968 and the results are as follows.

(12.5) $$\triangle M = -4.187+0.5731\triangle I+0.9639(\triangle X+\triangle Ba)+2.7317Z$$
$R^2 = .849$
$S = 13.0$

(12.6) $$\triangle Y-\triangle G = 9.977+0.8110\triangle I+0.7141(\triangle X+\triangle Ba)+0.9726e$$
$R^2 = .9295$
$S = 9.0$

(12.7) $$\triangle Yd = -4.196+0.9484(\triangle Y-\triangle G)$$
$R^2 = .9171$
$S = 8.9$

(12.8) $$\triangle C = -22.5377+0.5873\ Yd+394.98\left(\frac{Yd-C}{Yd}\right)_{-1}+5.341Z'$$
$R^2 = .9252$
$S = 7.038$

As can be seen from the re-run equations there has been some considerable shift in the coefficients of the first two equations as a result of the addition of three years observations. Geary has noted that "the data, at the △ level, fluctuate from year to year in quite fantastic degree" and this in part accounts for the very great change in the coefficients. The evidence of the re-run equations appears to contradict Leser's conclusion: "The first two equations show that fixed investment has a larger effect upon imports than have exports, whilst the opposite applies to domestic production."

The equations have been re-estimated using data from 1958/59 to 1967/68. While this only provides 10 observations in all the period seems more homogeneous and the equations thus estimated may provide more useful forecasting tools. The shorter period also eliminates the use of the dummy variables for the import function and the consumption function.

$$\Delta M = -9.302 + \underset{(0.862)}{0.362}\,\Delta I + \underset{(4.985)}{1.175}(\Delta X + \Delta Ba) \quad (12.9)$$

$R^2 = .902$
S = 10.5

$$\Delta Y - \Delta G = 16.187 + \underset{(2.175)}{0.934}\,\Delta I + \underset{(1.866)}{0.516}(\Delta X + \Delta Ba) + \underset{(1.370)}{0.937}e \quad (12.10)$$

$R^2 = .883$
S = 10.4

$$\Delta Yd = -10.169 + \underset{(7.892)}{1.033}(\Delta Y - \Delta G) \quad (12.11)$$

$R^2 = .886$
S = 9.8

$$\Delta C = -28.116 + \underset{(12.258)}{0.633}\,\Delta Yd + \underset{(6.152)}{425.397}\left(\frac{Yd - C}{Yd}\right)_{-1} \quad (12.12)$$

$R^2 = .971$
S = 4.1

The results of taking the shorter period are mixed. The coefficient of the investment term in the import equation has become insignificant—yet on commonsense grounds this term should be included as imports of producers capital goods makes up a not inconsiderable proportion of gross domestic fixed capital formation.

Once more the alteration of the period over which the equations have been calculated has led to a considerable change in many of the coefficients.

In view of this apparent instability of the coefficients, the poor forecasting (and rear-casting) results of the original model, and the difficulty of using the model for current forecasting purposes (due to the fact that some of the necessary data are not available until the publication of the full National Accounts for a year) it seems that this model is best regarded as an interesting experimental exercise, rather than as an actual forecasting tool.

RETAIL SALES MODEL

§4.13 *Baker's "Retail Sales" Model*

Finally, the retail sales model of T. J. Baker appearing as Appendix II of "The Irish Economy in 1967" is considered. This, in contrast with the earlier models is a quarterly partial model, seeking to explain changes in retail sales in terms of demand factors: average weekly wage earnings in transportable goods industries; the agricultural price index; and consumer credit. The variables used are:

C: Index of Retail Sales

E: Index of Weekly wage earnings in transportable goods industries 1953=100

P: Agricultural Price Index 1953=100

H: Hire Purchase debt outstanding plus personal and professional bank advances (£m)

Z: Dummy variable for the introduction of turnover and wholesale taxes.

C,E,P are seasonally corrected quarterly data. No seasonal correction was necessary for H. The whole analysis was conducted in terms of % changes. The model was based on data from the second quarter of 1961 to the fourth quarter of 1966, providing 23 observations on the variables.

The following equations were presented.

$$(13.1)\qquad c = -0.26 + \underset{(1.752)}{0.184e} + \underset{(2.206)}{0.225e_{-1}} + \underset{(0.836)}{0.117p_{-1}} + \underset{(3.647)}{0.434h} + \underset{(4.428)}{2.40Z} \qquad R^2 = .808$$

and

$$(13.2)\qquad c = -0.29 + \underset{(3.229)}{0.419e_{-\frac{1}{2}}} + \underset{(4.608)}{0.470h} + \underset{(5.551)}{2.62Z} \qquad R^2 = .796$$

where $e_{-\frac{1}{2}}$ is obtained by considering the percentage change in the two-quarter moving total of E.

The model has been updated by the addition of data up to the end of 1969, increasing the number of observations from 23 to 35. Updating this model is made difficult by the fact that the series personal and professional bank advances has been discontinued since the end of 1967. Instead, recourse was had to the sum of two series: professional, scientific and miscellaneous services; and personal advances. While this sum does not measure the same thing it was felt that percentage changes will get over most of this difficulty—except of course at the point where the break occurs. Similarly, the changes in definition of hire purchase debt and instalment credit outstanding at the beginning of 1967 is not thought to have seriously affected the quarter to quarter percentage changes.

Since the average weekly wage earnings figure for the first quarter of 1969 does not reflect the effect of maintenance men's dispute on total earnings and hence by inference on retail sales, a dummy variable Z′ has been used for the first and second quarters of 1969. Regressions were calculated for various combinations of the variables with the best result as follows.

$$(13.3)\qquad C = -0.19 + \underset{(2.518)}{0.270e} + \underset{(2.309)}{0.233e_{-1}} + \underset{(1.426)}{0.149p_{-1}} + \underset{(2.948)}{0.274h} + \underset{(4.364)}{2.434Z} + \underset{(4.298)}{3.286Z'} \qquad R^2 = .750$$

Compared with equation 13.1, the inclusion of the additional observations has had the effect of considerably reducing the coefficient of the credit variable. This is in many ways a welcome development, as the original value of the credit coefficient was too high to be entirely credible, and must have either reflected some degree of colinearity or a tendency for the credit variable to act as a proxy for some factor not specified in the formulation.

As the updated model is based on data to the end of 1969, there are insufficient data available outside the period of observation for meaningful testing of the equation as a forecasting tool.

§4.14 *Conclusion*

None of the models described here has ever been applied in a mechanistic way in arriving at the Institute's forecasts of National Accounts. Nevertheless, most of them in their original forms have proved useful tools, when considered in conjunction with other methods, in the forecasting process. It is hoped that the versions presented here, updated and in some cases modified, will continue to make a modest but useful contribution to improving the quality of macro-economic short-term forecasting, both within the Institute and elsewhere.