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*Examining the benefits of load shedding strategies using
a rolling-horizon stochastic mixed complementarity
equilibrium model*

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Abstract: As a result of government policies increasing the amount of electricity generated from fluctuating renewable sources in many countries, the requirement for flexibility in the corresponding electricity systems increases. On the demand side, load shedding is one demand response mechanism contributing to an increased flexibility. Traditionally, load shedding was based on rather static or rotational strategies, whereby the system operator chooses the consumers for load shedding. However, ongoing technological developments provide the basis for smarter and more efficient load shedding strategies. We therefore examine the costs and strategies associated with such mechanisms by modelling an electricity market with different types of generators and consumers. Some consumers provide flexibility through load shedding only while others additionally have the ability to generate their own electricity. Focusing on the impacts of how and to whom consumers with own generation ability can supply electricity, the presence of market power and generator uncertainty we propose a rolling horizon stochastic mixed complementarity equilibrium model, where the individual optimisation problems of each player are solved simultaneously and in equilibrium. We find that a non-static strategy reduces consumer costs while allowing consumers to provide own generation to the whole market results in minimal benefits. The presence of market power was found to increase costs to consumers. We also consider the optimal foresight horizon that market players should consider. This novel study finds that the optimal foresight horizon is mainly driven by the daily load structure and, to a lower degree, by the uncertainty of the generators' availability.

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1. Introduction

Many governments have adopted policies for expanding the use of renewable energy sources (RES) aimed at reducing greenhouse gas emissions. As a result of an increasing use particularly of fluctuating RES, such as wind and solar energy, the volatility of the system residual load will increase strongly leading to growing flexibility requirements [1]. In order to meet these requirements, electricity markets need to become more flexible. While traditionally, flexibility has been mainly provided by the supply side, demand side flexibility has gained increasing interest over the last decade and is expected to become increasingly important in the future [2, 3, 4]. Kirby and Hirst [5] as well as Chen et al. [6], for instance, describe the system benefits (mainly efficiency gains and cost reductions) of an increased demand side flexibility. In this context, Palensky and Dietrich [2] distinguish between four categories of demand side management: energy efficiency, time of use, demand response and spinning reserve. Aimed at exploring market-based solutions to meet short-term flexibility requirements, we focus on the demand response category in this paper. Within the demand response category, Albadi and El-Saadany [7] distinguish between load reduction/shedding, load shifting and customer owned distributed self-generation, whereas Bayer [8] distinguishes between reduction/shedding, shifting and increase of load. While the majority of research concentrates on load shifting, we shall focus on the examination of benefits of load shedding strategies, i.e. on the temporary short-term reduction of load in situations where the demand for electricity exceeds the supply capacity or where there is inadequate transmission infrastructure to deliver sufficient electricity to the areas and consumers where it is needed.

Traditionally, load shedding involved strategies where the system operator chooses the consumers that must shed their load - mostly following a rather static or a rotational scheme. Under a static scheme, the system operator can shed load of specific consumers according to predefined conditions (e.g., sheddable capacity and corresponding price) laid down in a contract or according to predefined priorities [9] of consumers. Under a rotational scheme, the system operator can shed load in a specific part of the electricity network at a time, where the affected areas and consumers will change over time in order to ensure a fair burden sharing. While being a common event in many developing countries, load shedding, particularly the rotational scheme is rather a measure of last resort in developed countries today, used by the system operator to avoid a total blackout of the power system.

However, the increasing digitisation driven by ongoing developments in information and communication technology (ICT) enables the transformation of electricity distribution grids

towards active distribution grids [10, 11, 12] and provides the basis for smarter and more efficient (non-static) load shedding strategies. For instance, shedding load of a particular consumer would not need to result in a complete blackout for this consumer but could simply imply a partial load reduction (“brownout”). Such interruptible and curtailable load programmes have also been reported and explored by others focussing on electricity [7, 10, 13, 14, 15] as well as gas transportation [16]. In essence, while implying curtailments for some consumers, such approaches, are aimed at avoiding blackouts and therefore at increasing energy security on a system level. The European Energy Security Strategy [17] and European Directive on Security of Network and Information Systems [18] both acknowledge the need for increasing energy system security and underline the relevance of such approaches, while at the same time highlighting the need for addressing these challenges in a competitive market environment.

Our focus in this paper is to examine the potential costs and benefits of different strategies for load shedding as one set of instruments within the field of demand response. For this purpose, we assume a competitive electricity market with multiple generators and different types of consumers which can be distinguished according to their load shedding ability and costs. We also assume that some consumers provide flexibility to the market through load shedding only while others additionally have the ability to generate their own electricity by auxiliary power generation units (APUs). Moreover, we consider uncertain generator availability. In such an environment, we are particularly interested in exploring the following research questions:

1. What are the benefits of allowing consumers with own generation to provide generation to the whole market?
2. How do ‘smart’ (non-static) load shedding strategies compare with static and rotational load shedding schemes and how do they differ in terms of costs to consumers?
3. How does the presence/absence of market power affect costs to consumers?
4. Is there an optimal foresight horizon for consumers with generation ability to plan their auxiliary generation and what are the drivers determining this optimality?
5. What are the benefits of a stochastic planning approach in the light of the uncertainties?

Demand response has been studied intensely in literature (see e.g. reviews by [7, 19, 20, 21, 22, 23]). Load shedding, in particular, has been investigated using heuristic techniques [24] as well as linear or nonlinear programming techniques [25]. Wang and Billinton [26], for instance, consider time-dependent, linear load shedding cost functions of different consumer types in an optimal load shedding approach. However, in order to explore the research questions set out

above, most existing approaches are limited with respect to at least one of the following two characteristics:

- The load shedding cost functions are assumed to be linear.
- Load shedding is optimised from a central planning perspective using a single optimisation problem.

In relation to the first limitation, we wish to note that the costs associated with load shedding should not be assumed to increase linearly when the amount of load shedding is increased. Low amounts of lost load, for instance, may only lead to low-cost effects (e.g., reduced illumination) whereas higher amounts of lost load may induce much higher losses across different consumer types [11]. In relation to the second limitation, a central planning optimisation does not take into account individual optimisation targets of different players. Hence, methods are needed that allow for the simultaneous consideration of multiple, individual optimisation problems (such as complementarity problems) and for the incorporation of consumer-specific, nonlinear load shedding cost functions. Moreover, with a view to our research questions, the methods should be able to consider market power, electricity generation by consumers and stochastic supply. Chen et al. [6], for instance, use a game-theoretic equilibrium model with a quadratic load shedding cost function. The model by De Jonghe et al. [3] is very similar. However, both don't take into account market power, APU generation or stochastic supply.

We therefore propose using a game-theoretic equilibrium model, namely a mixed complementarity problem (MCP) with quadratic load shedding cost functions, to analyse interactions of different players in a competitive electricity market. MCPs allow the optimisation problems of multiple individual players to be solved simultaneously and in equilibrium by combining the Karush-Kuhn-Tucker (KKT) conditions for optimality of each of the players and connecting them via market clearing conditions. In addition, MCPs allow both primal variables (eg., power generation) and dual variables (eg., prices) to be constrained together [27] while also allowing players with constrained optimisation problems to be modelled as either price-takers or price-makers, hence, incorporating market power into such models [28]. MCPs have been used to model various types of energy markets [29, 30, 31, 32, 33].

We apply the proposed MCP in the context of a case study based on demand side data for Ireland. The players that we consider in the case study, include different types of generators and consumers. The generators produce electricity to maximise their profits and may be price-takers or price-makers as described above. The consumers choose how much of their load to

shed in order to meet their demand at minimum cost and may differ in terms of their electricity demand profiles, their load shedding potential and cost functions and their ability to generate their own electricity. We consider consumers with the ability to generate electricity as *active* load shedding consumers and consumers without this ability as *passive* load shedding consumers. As load shedding is of greatest significance in times when demand is high and supply is low, we initially assume that RES generation is not available during the entire time period and, in addition, that one of the conventional generators is unavailable for generation and the time when it will return online is the model’s main source of uncertainty.

In order to take this uncertainty into account, we use a rolling horizon stochastic MCP in this work, which involves solving multiple MCPs along multiple sequences paths where the results from the previous problem are used to update the parameters for the subsequent problem, i.e. for each problem solved, only a subset of all time steps are considered. The alternative approach to solving optimisation/equilibrium models would be the perfect foresight approach which involves solving the problem once over all time steps. However, rolling horizon models have been used to solve many optimisation/equilibrium problems in energy markets [34, 35, 36] as they have been shown to model such markets more realistically than perfect foresight approaches and in a computationally efficient manner [29, 35].

The contribution of our paper can be summarised as follows: We examine the benefits of load shedding strategies using a rolling horizon stochastic MCP with quadratic load shedding cost functions to allow the optimisation problems of multiple individual players to be solved simultaneously. Beyond the existing work by Chen et al. [6] and De Jonghe et al. [3], we consider market power, own generation by consumers and stochastic generator availability. Note that the proposed model is a short-term model, i.e. we do not consider aspects related to investment planning in this paper. One aspect of rolling horizon models that has not been investigated in the literature thus far is: What level of look-ahead foresight should the players in the model consider (i.e., how should the timesteps be subdivided)? This work examines this novel question in the context of the market described.

The remainder of this paper is structured as follows: In section 2, we introduce the mathematical model. In section 3, we present the data of our case study followed by the results. In section 4, we discuss and interpret the results before concluding the paper in section 5. In Appendix A, we provide the Karush-Kuhn-Tucker conditions for each player type that we consider in our model. In Appendix B, we provide additional data related to the case study.

2. Model formulation

In this section we describe the formulation of the rolling-horizon stochastic mixed complementarity approach. The model involves solving multiple rolling-horizon sequences of stochastic Mixed Complementarity Problems (MCPs). Each stochastic MCP models an electricity market with $|I|$ conventional generators, $|J|$ passive load shedding consumers and $|K|$ active load shedding consumers. The conventional generators produce electricity in order to maximise their profits (subject to capacity constraints) and may be modelled as either price-takers or price-makers. Thus, the model can incorporate market power. The two types of consumers choose how much of their load to shed, subject to constraints, in order to meet their demand at minimum cost. In addition, active load shedding consumers also have the ability to generate their own electricity, known as auxiliary generation, to help meet their demand (again, subject to capacity and storage constraints).

Each of these three types of players has separate optimisation problems that are connected through market clearing conditions. The stochastic MCP is made up of these market clearing conditions along with the Karush-Kuhn-Tucker (KKT) conditions for optimality from each of the players. Thus, the MCP solves the optimisation problem of each player simultaneously and in equilibrium.

There are $|R|$ rolls in each sequence path l (described in detail in Section 2.2) with $|L|$ paths in total. At each roll, the MCP is solved. Hence, the MCP is solved $|L| \times |R|$ times in total. Each MCP has H timesteps. For roll $r \in R, \forall l \in L$, the set of hourly timesteps for the MCP is $T = \{r, r+1, \dots, H+r-1\}$. Hence, the set of timesteps changes for each roll along a path. Once the MCP for a given roll is solved, reference demand and storage parameters are updated for the next MCP to be solved in the sequence (see Section 2.2.1). When the last MCP of a given rolling horizon sequence is solved (i.e., the MCP for roll $r = |R|$), demand and storage parameters for auxiliary generation are reset to their original levels and the model moves forward to the MCP for the first roll of the next sequence path.

For the MCP at roll r , each player has first-stage decisions that represent ‘here and now’ decisions or actual decisions for timestep $t = r$. These decisions are scenario-independent. Furthermore, each player also has second-stage decisions that represent ‘wait and see’ or hypothetical decisions for all other hourly timesteps (i.e., $r < t < H + r - 1$). These decisions are scenario-dependent. One, and only one, of the conventional generators is assumed to be unreliable, i.e., this generator is available/unavailable for generation at different timesteps. Each MCP is solved over $|S|$ scenarios with each scenario s representing different outage lengths for

the unreliable generator (see Section 2.2). Once the unreliable generator returns online it is assumed it does not go offline again. We believe this assumption is reasonable, as the model is solved over relatively short timescale (no more than 48 hours). In addition, to include scenarios where the unreliable generator could return online and then offline again would require an excess amount of scenarios, which would make the model computationally intractable. All other generators are assumed to be fully available for generation for each hourly timestep and scenario.

Tables 1 - 4 describe the sets, variables and parameters used in the model. The following conventions are used: lower-case Roman letters indicate indices or variables, upper-case Roman letters represent parameters (i.e., data, functions), while Greek letters indicate endogenous or exogenous prices. The variables in parentheses alongside each constraint in this section are the Lagrange multipliers associated with those constraints.

Table 1: Sets for MCP solved at roll r .

$r \in R$	Rolls. For each roll an MCP is solved.
$l \in L$	Path of rolling horizon sequences.
$i \in I$	Conventional generators.
$j \in J$	Passive load shedding consumers.
$k \in K$	Active load shedding consumers.
$t \in T(r) = \{r, \dots, r + H - 1\}$	Hourly timesteps for roll r where H is the time horizon.

Table 2: Primal variables for MCP solved at roll r . All units are MW.

$g_{i,t,s}$	Electricity generated by generator i in timestep t and scenario s .
$g_{i,t=r}^{\text{FS}}$	Electricity generated by generator i for scenario-independent first timestep ($t = r$).
$\Delta g_{j,t,s}^{\text{P}}$	Load shed by passive load shedding consumer j in timestep t and scenario s .
$\Delta g_{k,t=r}^{\text{P,FS}}$	Load shed by passive load shedding consumer j for scenario-independent first timestep ($t = r$).
$\Delta g_{k,t,s}^{\text{A}}$	Load shed by active load shedding consumer k in timestep t and scenario s .
$\Delta g_{j,t=r}^{\text{A,FS}}$	Load shed by active load shedding consumer k for scenario-independent first timestep ($t = r$).
$g_{k,t,s}^{\text{APU}}$	Electricity generated by active load shedding consumer k in timestep t and scenario s .
$g_{k,t=r}^{\text{APU,FS}}$	Electricity generated by active load shedding consumer k for scenario-independent first timestep ($t = r$).

Table 3: Dual variables for MCP solved at roll r .

$\gamma_{t,s}$	System electricity price for timestep t and scenario s (€/MW h).
$\gamma_{t=r}$	System electricity price for scenario-independent first timestep ($t = r$) (€/MW h).
$\lambda_{G\#}$	Lagrange multipliers associated with the constraints in generators' problems (unit depends on constraint).
$\lambda_{P\#}$	Lagrange multipliers associated with the constraints in passive load shedding consumers' problems (unit depends on constraint).
$\lambda_{A\#}$	Lagrange multipliers associated with the constraints in active load shedding consumers' problems (unit depends on constraint).

Table 4: Parameters for MCP solved at roll r .

PR_s	Probability associated with scenario s for MCP solved at roll r .
PR_l	Probability associated with path l .
H	Time horizon, i.e., total number of hourly timesteps for each MCP.
F_i	Marginal cost for generator i (€/MW h).
G_i^{\max}	Generator i 's maximum capacity for each timestep (MW).
M_i^G	Binary parameter indicating whether generator i is a price maker ($M_i^G = 1$) or price taker ($M_i^G = 0$).
$D_{j,t}^P$	Reference demand for passive load shedding consumer j at timestep t (MW).
$\Delta g_j^{P,\max}$	Maximum amount of electricity passive load shedding consumer j can shed at timestep t (MW).
$C_{j,t}^P(\cdot)$	Passive load shedding consumer j 's marginal cost function for time t (€/MW h).
$E_{j,t}^P$	Intercept for passive load shedding consumer j 's marginal cost function for time t (€/MW h).
$B_{j,t}^P$	Slope for passive load shedding consumer j 's marginal cost function for time t .
$D_{k,t}^A$	Reference demand for active load shedding consumer k at timestep t (MW).
$\Delta g_k^{A,\max}$	Maximum amount of electricity active load shedding consumer k can shed at timestep t (MW).
$C_{k,t}^A(\cdot)$	Active load shedding consumer k 's marginal cost function for time t (€/MW h).
$E_{k,t}^A$	Intercept for active load shedding consumer k 's marginal cost function for time t (€/MW h).
$B_{k,t}^A$	Slope for active load shedding consumer k 's marginal cost function for time t .
F_k^{APU}	Marginal cost of auxiliary generation for active load shedding consumer k (€/MW h).
$g_k^{\text{APU},\max}$	Maximum amount of electricity active load shedding consumer k can generate at each timestep (MW).
V_k	Maximum capacity of electrical energy active load shedding consumer k can store ^a (MW h).

^a In reality, while the energy in storage would be some sort of fuel (e.g., diesel or gas), V_k represents the total amount of electrical energy the stored fuel can generate.

2.1. Stochastic mixed complementarity problem for roll r

We now describe in detail the optimisation problems of the generators (equations (1a)-(3), Section 2.1.1), passive load shedding consumers (equations (4a)-(5), Section 2.1.2) and the active load shedding consumers (equations (6a)-(7), Section 2.1.3) in addition to the market clearing conditions (equations (8a)-(8b), Section 2.1.4) that connect them. The corresponding KKT conditions for the different optimisation problems (equations (A.1)-(A.24)) can be found in Appendix A.

2.1.1. Generator i 's problem

Generator i maximises its expected profits (revenues less cost) by choosing the amount of electricity to generate. For timestep $t = r$, it makes scenario-independent first-stage decisions ($g_{i,t=r}^{\text{FS}}$), representing actual decisions. For timesteps $t > r$ it makes second-stage hypothetical decisions, for each scenario s ($g_{i,t,s}$). The marginal price generators receive is $\gamma_{t=r}$ for timestep $t = r$ and $\gamma_{t,s}$ for timesteps $t > r$, $\forall s \in S$. The marginal cost of generation for generator i is F_i . Generator i 's problem at roll r is

$$\max_{g_{i,t=r}^{\text{FS}}, g_{i,t,s}} (\gamma_{t=r} - F_i)g_{i,t=r}^{\text{FS}} + \sum_{s \in S} PR_s \sum_{t>r}^{r+H-1} (\gamma_{t,s} - F_i)g_{i,t,s}, \quad (1a)$$

subject to

$$g_{i,t=r}^{\text{FS}} \leq G_i^{\text{max}}, (\lambda_{G1_{i,t=r}}^{\text{FS}}) \quad (1b)$$

$$g_{i,t,s} \leq G_i^{\text{max}}, \forall s, t > r, (\lambda_{G1_{i,t,s}}). \quad (1c)$$

Constraints (1b) and (1c) ensure that the amount of electricity generator i produces in each hourly timestep and scenario is capped. Generator i 's primal decision variables are also constrained to be non-negative.

If generator i is assumed to be a price taker then its decision variables cannot affect the prices $\gamma_{t=r}$ and $\gamma_{t,s}$. Hence, these price variables are assumed exogenous to generator i 's problem whilst still being variables of the overall MCP. If generator i is assumed to be a price-maker, then its decision variables can affect price. As a result we derive the following relationship between

prices and generation for price-makers:

$$\begin{aligned}
\gamma_{t=r} = & -\frac{1}{2} \left(\frac{(\sum_{j \in J} B_{j,t=r}^P)(\sum_{k \in K} B_{k,t=r}^A)}{(\sum_{j \in J} B_{j,t=r}^P) + (\sum_{k \in K} B_{k,t=r}^A)} \right) \left(\sum_{i \in I} g_{i,t=r}^{\text{FS}} + \sum_{k \in K} g_{k,t=r}^{\text{APU,FS}} \right. \\
& - \sum_{j \in J} \left(D_{j,t=r}^P + \frac{1}{2 \sum_{j \in J} B_{j,t=r}^P} (E_{j,t=r}^P + \lambda_{P1_{j,t=r}}^{\text{FS}} - \lambda_{P2_{j,t=r}}^{\text{FS}}) \right) \\
& \left. - \sum_{k \in K} \left(D_{k,t=r}^A + \frac{1}{2 \sum_{k \in K} B_{k,t=r}^A} (E_{k,t=r}^A + \lambda_{A1_{k,t=r}}^{\text{FS}} + \lambda_{A2_{k,t=r}}^{\text{FS}} - \lambda_{A5_{k,t=r}}^{\text{FS}}) \right) \right), \tag{2}
\end{aligned}$$

and

$$\begin{aligned}
\gamma_{t,s} = & -\frac{1}{2} \left(\frac{(\sum_{j \in J} B_{j,t}^P)(\sum_{k \in K} B_{k,t}^A)}{(\sum_{j \in J} B_{j,t}^P) + (\sum_{k \in K} B_{k,t}^A)} \right) \left(\sum_{i \in I} g_{i,t,s} + \sum_{k \in K} g_{k,t,s}^{\text{APU}} \right. \\
& - \sum_{j \in J} \left(D_{j,t}^P + \frac{PR_s}{2 \sum_{j \in J} B_{j,t}^P} (\lambda_{P1_{j,t,s}} - \lambda_{P2_{j,t,s}} + \frac{E_{j,t}^P}{PR_s}) \right) \\
& \left. - \sum_{k \in K} \left(D_{k,t}^A + \frac{PR_s}{2 \sum_{k \in K} B_{k,t}^A} (\lambda_{A1_{k,t,s}} + \lambda_{A2_{k,t,s}} - \lambda_{A5_{k,t,s}} + \frac{E_{k,t}^A}{PR_s}) \right) \right), \quad \forall s, t > r. \tag{3}
\end{aligned}$$

These relationships are then substituted into the objective function (1a). Equation (2) is determined by combining the market clearing condition (8a below), with the KKT conditions that determine how passive and active load shedding change with the system price, equations (A.6) and (A.12) respectively. Similarly, equation (3) is determined by combining equations (8b), (A.7) and (A.13).

If generator i is a price-taker, then their problem is linear and hence convex. If generator i 's is a price-maker, then their problem is strictly convex, assuming $B_{j,t}^P > 0, \forall j \in J, t \in T$ and $B_{k,t}^A > 0, \forall k \in K, t \in T$. Note: M_i^G is a binary parameter that is used in the KKT conditions to indicate whether generator i is a price maker ($M_i^G = 1$) or price taker ($M_i^G = 0$).

2.1.2. Passive load shedding consumer j 's problem

Passive load shedding consumer j seeks to minimise the expected cost of meeting their demand plus the expected cost of any load shedding. Their demand consists of a reference demand less any load shedding. Reference demand ($D_{j,t}^P$) is the demand consumer group j would have at time t if they did not shed any of their load. For timestep $t = r$, passive load shedding consumers make scenario-independent first-stage decisions ($\Delta g_{j,t=r}^{\text{P,FS}}$), representing actual decisions. For timesteps $t > r$ they make second-stage hypothetical decisions, for each

scenario s ($\Delta g_{j,t,s}^P$). Passive load shedding consumer j 's problem at roll r is:

$$\begin{aligned} \min_{\Delta g_{j,t=r}^{P,FS}, \Delta g_{j,t,s}^P} \quad & \gamma_{t=r}(D_{j,t=r}^P - \Delta g_{j,t=r}^{P,FS}) + \Delta g_{j,t=r}^{P,FS} C_{j,t=r}(\Delta g_{j,t=r}^{P,FS}) + \\ & \sum_{s \in S} PR_s \sum_{t>r}^{r+H-1} (\gamma_{t,s}(D_{j,t}^P - \Delta g_{j,t,s}^P) + \Delta g_{j,t,s}^P C_{j,t}^P(\Delta g_{j,t,s}^P)), \end{aligned} \quad (4a)$$

subject to

$$\Delta g_{j,t=r}^{P,FS} \leq \Delta g_j^{P,\max}, \quad (\lambda_{P1j,t=r}^{FS}), \quad (4b)$$

$$\Delta g_{j,t,s}^P \leq \Delta g_j^{P,\max}, \quad \forall s, t > r, \quad (\lambda_{P1j,t,s}), \quad (4c)$$

$$\Delta g_{j,t=r}^{P,FS} \geq 0, \quad (\lambda_{P2j,t=r}^{FS}), \quad (4d)$$

$$\Delta g_{j,t,s}^P \geq 0 \quad \forall s, t > r, \quad (\lambda_{P2j,t,s}). \quad (4e)$$

Equations (4b) and (4c) constrain the amount of their load passive consumer's can shed for each hourly timestep and scenario while constraints (4d) and (4e) ensure that load-shedding are non-negative. We assume the marginal cost of load shedding for passive consumers j at time $t, \forall s \in S$ is linear:

$$C_{j,t}^P(x) = E_{j,t}^P + B_{j,t}^P x, \quad (5)$$

which means their total cost of load shedding is quadratic. Assuming $B_{j,t}^P > 0, \forall t \in T$, consumer j 's problem is strictly convex.

2.1.3. Active load shedding consumer k 's problem

Active load shedding consumer k seeks to minimise the expected cost of meeting their demand plus the expected cost of any load shedding or auxiliary generation. Their demand consists of a reference demand less any load shedding. Reference demand ($D_{k,t}^A$) is the demand consumer group k would have at time t if they did not shed any of their load or produce any auxiliary generation. For timestep $t = r$, active load shedding consumers make scenario-independent first-stage decisions on the amount of their load to shed ($\Delta g_{k,t=r}^{A,FS}$) and on the level of auxiliary generation ($g_{k,t=r}^{APU,FS}$), both representing actual decisions. For timesteps $t > r$ they make second-stage hypothetical decisions, for each scenario s ($\Delta g_{k,t,s}^A$ and $g_{k,t,s}^{APU}$). Active load shedding

consumer k 's problem at roll r is:

$$\begin{aligned}
& \min_{\substack{\Delta g_{k,t=r}^{\text{A,FS}}, \\ g_{k,t=r}^{\text{APU,FS}}, \\ \Delta g_{k,t,s}^{\text{A}}, \\ g_{k,t,s}^{\text{APU}}}} \gamma_{t=r} (D_{k,t=r}^{\text{A}} - \Delta g_{k,t=r}^{\text{A,FS}} - g_{k,t=r}^{\text{APU,FS}}) + \Delta g_{k,t=r}^{\text{A,FS}} C_{k,t=r}^{\text{A}} (\Delta g_{k,t=r}^{\text{A,FS}}) + g_{k,t=r}^{\text{APU,FS}} F_k^{\text{APU}} \\
& + \sum_{s \in S} PR_s \sum_{t>r}^{r+H-1} (\gamma_{t,s} (D_{k,t}^{\text{A}} - \Delta g_{k,t,s}^{\text{A}} - g_{k,t,s}^{\text{APU}}) + \Delta g_{k,t,s}^{\text{A}} C_{k,t} (\Delta g_{k,t,s}^{\text{A}}) + g_{k,t,s}^{\text{APU}} F_k^{\text{APU}}), \tag{6a}
\end{aligned}$$

subject to

$$\Delta g_{k,t=r}^{\text{A,FS}} \leq \Delta g_k^{\text{A,max}}, \quad (\lambda_{A1k,t=r}^{\text{FS}}), \tag{6b}$$

$$\Delta g_{k,t,s}^{\text{A}} \leq \Delta g_k^{\text{A,max}}, \quad \forall s, t > r, \quad (\lambda_{A1k,t,s}), \tag{6c}$$

$$\Delta g_{k,t=r}^{\text{A,FS}} + g_{k,t=r}^{\text{APU,FS}} \leq D_{k,t=r}^{\text{A}}, \quad (\lambda_{A2k,t=r}^{\text{FS}}), \tag{6d}$$

$$\Delta g_{k,t,s}^{\text{A}} + g_{k,t,s}^{\text{APU}} \leq D_{k,t}^{\text{A}}, \quad \forall s, t > r, \quad (\lambda_{A2k,t,s}), \tag{6e}$$

$$g_{k,t=r}^{\text{APU,FS}} \leq g_k^{\text{APU,max}}, \quad (\lambda_{A3k,t=r}^{\text{FS}}), \tag{6f}$$

$$g_{k,t,s}^{\text{APU}} \leq g_k^{\text{APU,max}} \quad \forall s, t > r, \quad (\lambda_{A3k,t,s}), \tag{6g}$$

$$g_{k,t=r}^{\text{APU,FS}} + \sum_{t>r}^{r+H-1} g_{k,t,s}^{\text{APU}} \leq V_k, \quad \forall s, \quad (\lambda_{A4k,s}), \tag{6h}$$

$$\Delta g_{k,t=r}^{\text{A,FS}} \geq 0, \quad (\lambda_{A5k,t=r}^{\text{FS}}), \tag{6i}$$

$$\Delta g_{k,t,s}^{\text{A}} \geq 0, \quad \forall s, t > r, \quad (\lambda_{A5k,t,s}). \tag{6j}$$

Equations (6b) and (6c) constrain the amount of the load active consumer's can shed for each hourly timestep and scenario while equations (6d) and (6e) ensure their auxiliary generation is capped at their demand, i.e., they cannot provide auxiliary generation to meet demand other than their own. Equations (6f) and (6g) constrains the amount of auxiliary generation they can produce at each timestep and scenario while equation (6h) constrains the total amount of auxiliary generation they can produce for scenario s over the entire time horizon of roll r . Note: the variable $g_{k,t=r}^{\text{APU,FS}}$ appears in every one of the $|S|$ instances of constraint (6h) reflecting how this variable represents an actual decision that must hold for each possible future scenario. The parameter V_k represents the maximum capacity of electrical energy that active load shedding consumer k can store (e.g., through diesel tanks) and is updated after the MCP at each roll r is solved; see Section 2.2.1. Constraints (6i) and (6j) ensure active load shedding consumer k 's load shedding decisions are non-negative.

Active load shedding consumer k 's marginal cost for auxiliary generation is F_k^{APU} while we assume their marginal cost for load shedding at time t is linear:

$$C_{k,t}(x) = E_{k,t}^{\text{A}} + B_{k,t}^{\text{A}}x, \quad (7)$$

which means their total cost of load shedding is quadratic. Assuming $B_{k,t}^{\text{A}} > 0, \forall t \in T$, consumer k 's problem is strictly convex.

2.1.4. Market clearing condition

The $|I| + |J| + |K|$ optimisation problems are connected via the following market clearing conditions:

$$\begin{aligned} \sum_{i \in I} g_{i,t=r}^{\text{FS}} + \sum_{k \in K} g_{k,t=r}^{\text{APU,FS}} &= \sum_{j \in J} (D_{j,t=r}^{\text{P}} - \Delta g_{j,t=r}^{\text{P,FS}}) + \sum_{k \in K} (D_{k,t=r}^{\text{A}} - \Delta g_{k,t=r}^{\text{A,FS}}), (\gamma_{t=r}), \quad (8a) \\ \sum_{i \in I} g_{i,t,s} + \sum_k g_{k,t,s}^{\text{APU}} &= \sum_{j \in J} (D_{j,t}^{\text{P}} - \Delta g_{j,t,s}^{\text{P}}) + \sum_{k \in K} (D_{k,t}^{\text{A}} - \Delta g_{k,t,s}^{\text{A}}), \forall s, t > r, (\gamma_{t,s}) \quad (8b) \end{aligned}$$

which state that the total amount of electricity produced from conventional generation and auxiliary generation must equal the sum of the passive and active load shedding demand (reference demand less load shedding). The prices $\gamma_{t=r}$ and $\gamma_{t,s}$ are the free Lagrange multiplier associated with conditions (8a) and (8b) respectively.

2.1.5. The complete MCP for roll r

The KKT conditions for each of the three types of players are presented in Appendix A.1 - Appendix A.3 respectively. As each of the optimisation problems are convex, these conditions are both necessary and sufficient for optimality for each type of player [27]. The MCP for roll r consists of conditions (A.1) - (A.24) in addition to the market clearing conditions (8).

2.2. Solving the model

As mentioned previously, the overall model involves solving the stochastic MCP for $|R|$ rolls along each possible sequence path l . Figure 1 describes these paths. Each path p represents different sequences of rolls with each node presenting a different instance of when the MCP is solved. Circular nodes in Figure 1 model a situation where the unreliable generator is initially offline and the other players are unsure of when it will return. Consequently, the MCPs solved at these rolls contain multiple scenarios, representing the uncertain information the players have when making their decisions. Diamond shaped nodes reflect a situation where the unreliable generator has returned online all players assume it will not go on outage again. As a result,

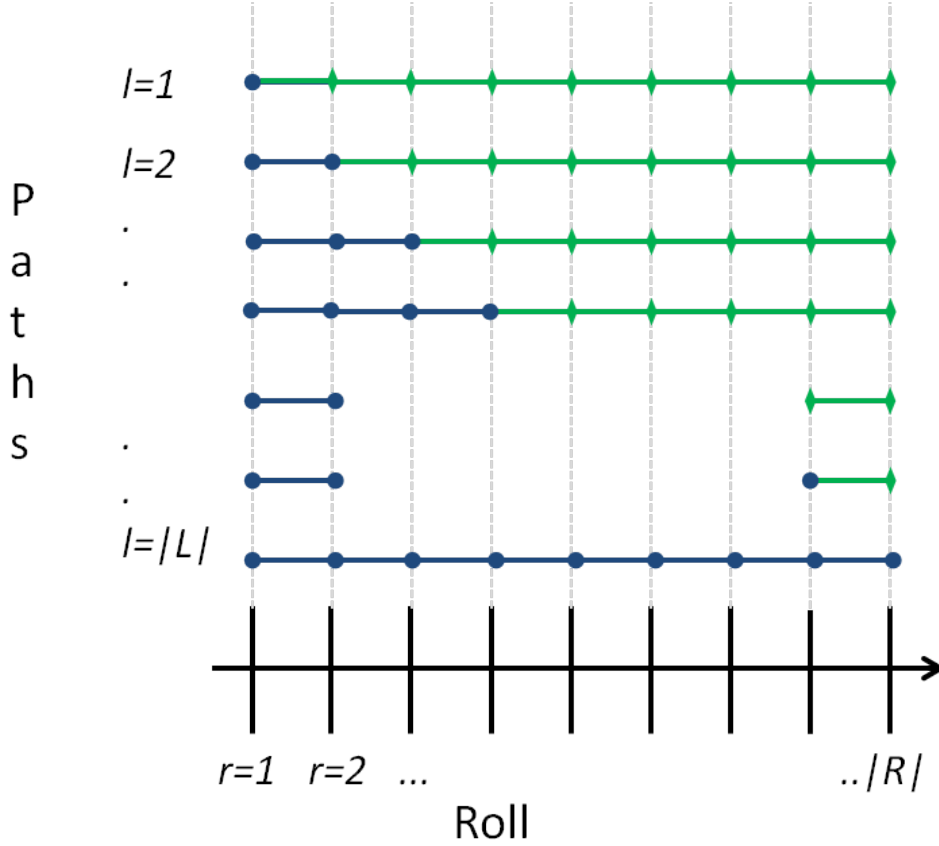


Figure 1: Schematic of rolling horizon sequence paths. For circular nodes, the MCP is solved with multiple scenarios (see Figure 2) while for diamond shaped nodes, the MCP is solved with only one scenario.

MCPs solved at these rolls are deterministic, i.e., $S = \{1\}$ and $PR_{s=1} = 1$. Each path l represents different time points when the unreliable returns online. For path l the unreliable generator is assumed initially offline (with uncertainty about its return) for the MCPs solved at rolls $r \leq l$ while, for rolls $r > l$, the unreliable generator is assumed to have returned online for all timesteps and scenarios.

Figure 2 describes the $|S|$ scenarios for the MCPs associated with circular nodes in Figure 1. It reflects the uncertain information the other players have about the return of the unreliable generator when making decisions at these rolls. Circular nodes in Figure 2 represent timesteps when the unreliable generator is offline while diamond shaped nodes represent timesteps where this generator is online. For scenario s , the unreliable generator is assumed offline for $t-r+1 \leq s$ and online for $t-r+1 > s$. Consequently, these scenarios have the same probabilities as the paths, i.e., $PR_s = PR_l$, ($\{\forall s \in S, \forall l \in L \mid s = l\}$). For the first hourly timestep ($t = r$) for

each roll described with circular nodes in Figure 2, the unreliable generator is assumed offline in every scenario. Hence the decisions made at this timestep are scenario-independent for each player.

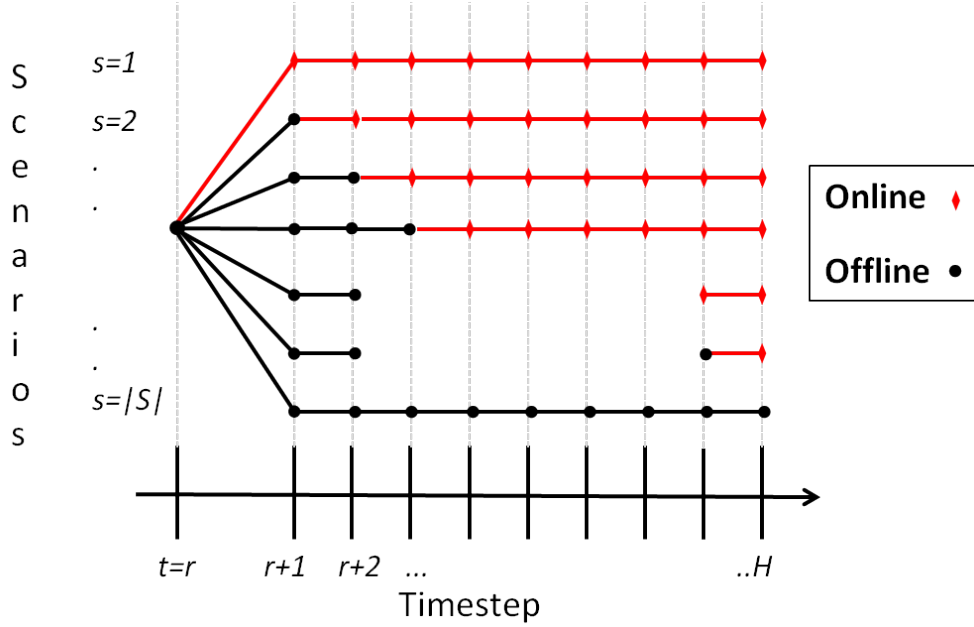


Figure 2: Schematic of scenarios associated with nodes the MCP is solved with the unreliable generator is initially assumed offline and its return is uncertain.

2.2.1. Update rules

When a MCP is solved, the model moves forward to the next roll on the path and the following parameters are updated before the next roll:

$$D_{j,t,s}^P \rightarrow D_{j,t+1,s}^P \quad \forall j, t, s, \quad (9a)$$

$$D_{k,t,s}^A \rightarrow D_{k,t+1,s}^A \quad \forall k, t, s, \quad (9b)$$

$$E_{j,t,s}^P \rightarrow E_{j,t+1,s}^P \quad \forall j, t, s, \quad (9c)$$

$$E_{k,t,s}^A \rightarrow E_{k,t+1,s}^A \quad \forall k, t, s, \quad (9d)$$

$$B_{j,t,s}^P \rightarrow B_{j,t+1,s}^P \quad \forall j, t, s, \quad (9e)$$

$$B_{k,t,s}^A \rightarrow B_{k,t+1,s}^A \quad \forall k, t, s, \quad (9f)$$

$$V_k \rightarrow V_k - g_{k,t=r}^{\text{APU,FS}} \quad \forall k. \quad (9g)$$

Equations (9a) and (9b) reflect what happens in real-world electricity markets: when a new time period begins (i.e., when the model moves to a new roll), those in the market have updated information regarding demand. Similarly, Equations (9c) - (9f) reflect how the cost of load shedding for the consumers changes when made load shedding decisions are being made for a new time period. As $g_{k,t=r}^{\text{APU,FS}}$ represents the actual amount of auxiliary generation (in contrast to the hypothetical $g_{k,t,s}^{\text{APU}}$ decisions), equation (9g) reflects how the amount of electrical energy in storage is reduced after it is used for auxiliary generation.

After the last roll of a sequence path is solved, these parameters are reset to their original values (described in Section 3.1) for roll $r = 1$ of the next sequence path.

3. Case Study and Results

In this section, we present the case study assumptions (section 3.1), followed by the results for these assumptions (section 3.2). The assumptions concerning the load shedding costs constitute a crucial part of the input data for the analysis in this paper. With respect to this aspect, our analysis builds on Leahy and Toll [37] who estimate the value of lost load (VOLL) for different types of consumers in Ireland (see section 3.1.1 below for further details on how their results are used for our analysis).

3.1. Model input assumptions for the case study

3.1.1. Generation and demand data

In this section, numerical examples are presented. We firstly describe the parameters for the main policy test case, the *Base Case*. This policy assumes that the regulator/system operator maintains a non-static load-shedding strategy which means that, at each hourly timestep, the ability of both consumers to shed their load is not constrained nor is their demand prioritised over the other player's demand. In addition, the *Base Case* policy assumes that active consumers have the ability to use their auxiliary generation to meet their demand. The *Base Case* is formulated with $|I| = 5$ generators, $|J| = 1$ passive load shedding consumers, $|K| = 1$ active load shedding consumers and $|L| = 48$ rolling horizon sequence paths. Each sequence path has $|R| = 48$ rolls. For the MCPs solved when the unreliable generator is initially assumed offline, there are $|S| = 48$ scenarios with the probability associated with these (PR_s) described in Figure 3a and Table B.7. These probabilities are normalised such that $\sum_s PR_s = 1$ and are initially taken from a log-normal distribution with values of 2.233 and 1 for the location and scale parameters respectively. These parameters are chosen such that each player assumes that

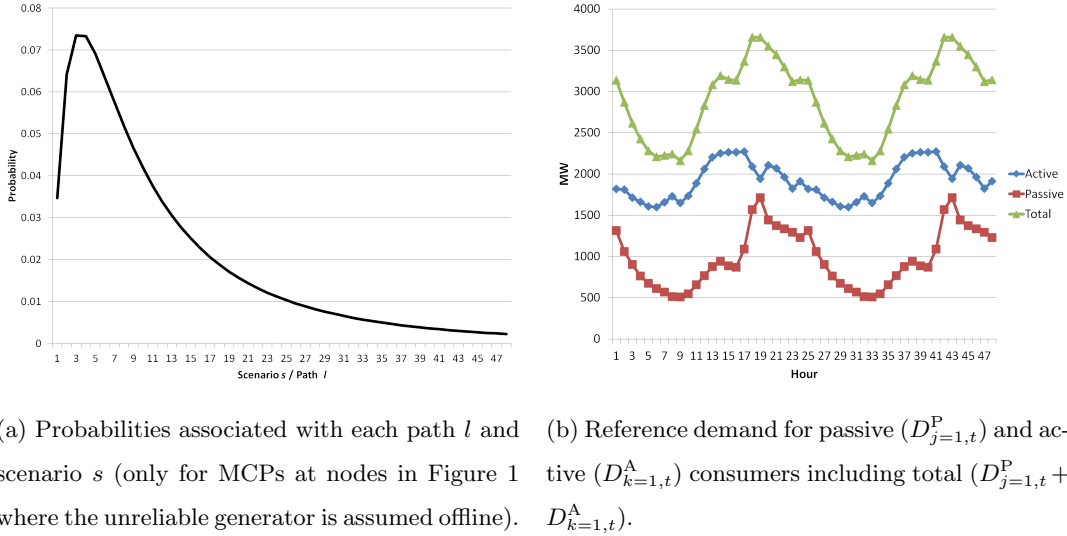


Figure 3: Input parameters.

the expected outage length of the unreliable generator is 12 hours. Note: these probabilities are also the same as those associated with the $|L| = 48$ rolling horizon sequence paths (PR_l). For MCPs solved once the unreliable generator has returned online there is only $|S| = 1$ scenario for all timesteps. For each MCP solved there are $H = 24$ hourly timesteps, i.e., each player has a look-ahead foresight of 24 hours. Hence, the set of hourly timesteps are $t = \{1, 2, \dots, 24\}$ for rolls $r = 1$, $t = \{2, 3, \dots, 25\}$ for rolls $r = 2$ and $t = \{48, 49, \dots, 72\}$ for rolls $r = |R| = 48$.

The five different thermal generators represent different technologies with marginal cost and maximum capacity values described in Table 5. The baseload generator ($i = 1$) is assumed to be a modern hard coal power plant with high efficiency. Midload generators are assumed to be combined cycle gas turbines (CCGTs) with high ($i = 2$) and moderate efficiency ($i = 3$) respectively. Peak generator $i = 4$ is assumed to be an open cycle gas turbine (OCGT) whereas peak generator $i = 5$ is assumed to be an oil fired gas turbine. The marginal costs in Table 5 were calculated using fuel and carbon prices of the corresponding futures markets for 2017 as obtained from the European Energy Exchange (www.eex.com). For this purpose, we used the average market results of the futures markets for 2017 as traded during the first six months of 2016. All generators are assumed to be price-takers except for generator $i = 5$, i.e., $M_i^G = 0$, for $i = 1, 2, 3, 4$ and $M_i^G = 1$, for $i = 5$. The unreliable generator, that is assumed initially offline for some rolls, is generator $i = 4$ while all other generators are assumed online and available for all rolls and timesteps.

Table 5: Values for generator marginal cost and maximum hourly capacity.

Generator	F_i	G_i^{\max}
$i = 1$	22	800
$i = 2$	30	100
$i = 3$	43	600
$i = 4$	50	600
$i = 5$	133	700

Table 6: Parameter values for passive and active load shedding consumer.

Parameter	Value
$\Delta g_{j=1}^{\text{P,max}}$	500
$E_{j=1,t}^{\text{P}}$	200 ($\forall t$)
$\Delta g_{k=1}^{\text{A,max}}$	500
$E_{k=1,t}^{\text{A}}$	150 ($\forall t$)
$F_{k=1}^{\text{APU}}$	176
$g_{k=1}^{\text{APU,max}}$	200
$V_{k=1}$	100

Table 6 displays the maximum amount of electricity both types of consumers can shed in each hourly timestep as well as the intercepts of their load shedding marginal cost functions, which we assume do not vary in time. In addition, Table 6 also displays the marginal cost, maximum hourly capacity and total storage capacity for the active load shedding consumer’s auxiliary generation. The total storage capacity is equal to five hours worth of generation. Figure 3b displays reference demand for both types of consumers (as well as their sum) while Table B.8 gives the values for these reference demands in addition to the slopes associated with both types of consumers’ marginal cost functions. The values for both the parameters in the marginal load shedding cost functions as well as the reference demands are obtained on the basis of Leahy and Toll [37]. While they distinguish between industrial, commercial and residential consumers, we focus on passive and active load shedding consumers. We assume residential consumers to be passive while we assume industrial, in addition to commercial consumers, to be active, i.e. we aggregate industrial and commercial consumers for illustrative purposes. The passive reference demand values $D_{j,t}^P$ in Table B.8 are therefore identical with the demand values in Leahy and Toll [37] while the active reference demand values $D_{k,t}^A$ represent the sum of the industrial and commercial demand values in Leahy and Toll [37].

The slopes $B_{j,t}^P$ and $B_{k,t}^A$ of the marginal load shedding cost functions are calculated on the basis of the time-dependent VOLL values, according to equations (10a) and (10b). As for the reference demand, the values of lost load for the passive consumers ($VOLL_{j,t}^P$) are identical to the residential consumer VOLLs in Leahy and Toll [37]. The values of lost load of the active consumers ($VOLL_{k,t}^A$) are calculated as the demand-weighted average of the industrial and commercial VOLLs. The calculation according to equations (10a) and (10b) is based on the idea that low amounts of lost or shed load lead to rather low-cost effects and that the marginal load shedding costs increase with increasing amounts of load shedding [11]. Equations (10a) and (10b) assume the marginal load shedding costs of the last MWh of demand that can be shed to be equal to the VOLL for each considered type of consumer. Since we are aware of the uncertainty related to this assumption, we shall consider different marginal load shedding cost function slopes aimed at exploring the sensitivity of this assumption (see section 4).

$$B_{j,t}^P = \frac{VOLL_{j,t}^P}{D_{j,t}^P} \quad (10a)$$

$$B_{k,t}^A = \frac{VOLL_{k,t}^A}{D_{k,t}^A} \quad (10b)$$

Note: the parameters $D_{j,t,s}^P, D_{k,t,s}^A, B_{j,t,s}^P, B_{k,t,s}^A$ and V_k update after each roll¹, thus the values described here for these parameters are the values for rolls $r = 1$ only with the values for all other rolls being obtained as described in Section 2.2.1. All other parameters remain the same for each path solved.

3.1.2. Considered load shedding policies

The above describes the values for the parameters used in the *Base Case* which represents a non-static load shedding policy. We now proceed and describe the other policies, in particular the difference between them and the *Base Case*:

1. APU-to-Market Case:

Constraints (6d) and (6e) ensure that the active load shedding consumer can only provide auxiliary generation to meet their own demand. In this case, these constraints are removed from the MCPs solved and thus, the active load shedding consumer can provide auxiliary generation to the full market. As with the *Base Case*, this test represents a non-static load shedding policy.

2. Static (Priority/Rotational) Policies:

(a) Priority (No passive load shedding) Case:

In this case, only the active load shedding consumer's demand is shed. As such, the passive load shedding consumer's demand is prioritised, i.e., $\Delta g_j^{P,\max} = 0$.

(b) Priority (No passive load shedding & no APU) Case:

This case is similar to the previous case except, in this case, the active load shedding consumer also cannot provide any auxiliary generation, i.e., $g_k^{\text{APU},\max} = 0$.

(c) Priority (No active load shedding & no APU) Case:

In this case, only the passive load shedding consumer's demand is shed. As such, the active load shedding consumer's demand is prioritised, i.e., $\Delta g_k^{A,\max} = 0$. In addition, the active load shedding consumer cannot provide any auxiliary generation, i.e., $g_k^{\text{APU},\max} = 0$.

(d) Rotational load shedding Case:

In this case, load shedding switches between the two types of consumers. For the first 24-hour period, only the passive load shedding consumer's demand can be shed while for the second 24-hour period, only the active load shedding consumer's demand can be shed.

¹As $E_{j,t,s}^P$ and $E_{k,t,s}^A$ are constant through time, there is no need to update these parameters after each roll.

(e) *Rotational load shedding & no APU Case:*

This case is similar to the previous case except, in this case, the active load shedding consumer also cannot provide any auxiliary generation, i.e., $g_k^{\text{APU,max}} = 0$, for all time periods.

3. *No Market Power Case:*

In the *Base Case*, generator $i = 5$ is assumed to be a price-maker. In this case it is assumed to be a price-taker along with all other generators. As with the *Base Case*, this test represents a non-static load shedding policy.

In addition to these policy test cases, we also consider test cases for different levels of look-ahead foresight for each of the players. In the *Base Case*, there are $H = 24$ hourly timesteps. With the aim of identifying the optimal level of foresight, we also consider similar cases where each player has $H = 2, 6, 12$ and 48 hours of look-ahead foresight.

3.2. Results

In the following, we present the results of our study for the input data described above. We start by presenting and comparing the results for different load shedding policies, followed by describing the effect of market power. Subsequently, we present our findings concerning the optimal level of foresight and the value of stochastic solution as well as the expected value of perfect information.

3.2.1. Load shedding policies

In this section the results from the different load shedding policies are compared. Figure 4 displays the Total Expected Consumer Costs (TECC) for the different test cases. This cost is calculated by taking the optimal decisions from the scenario-independent timesteps $t = r$ from each MCP solved:

$$\text{TECC} = \sum_l PR_l \text{TECC}_l, \quad (11)$$

where

$$\begin{aligned} \text{TECC}_l = \sum_r \left(\gamma_{t=r} (D_{j,t=r}^{\text{P}} + D_{k,t=r}^{\text{A}} - \Delta g_{j,t=r}^{\text{P,FS}} - \Delta g_{k,t=r}^{\text{A,FS}} - g_{k,t=r}^{\text{APU,FS}}) \right. \\ \left. + \Delta g_{j,t=r}^{\text{P,FS}} C_{j,t=r} (\Delta g_{j,t=r}^{\text{P,FS}}) + \Delta g_{k,t=r}^{\text{A,FS}} C_{k,t=r}^{\text{A}} (\Delta g_{k,t=r}^{\text{A,FS}}) + g_{k,t=r}^{\text{APU,FS}} F_k^{\text{APU}} \right). \end{aligned} \quad (12)$$

These decisions represent the actual decisions the players would make assuming scenario path l and hence this cost is calculated as a weighted sum of the costs associated with each path.

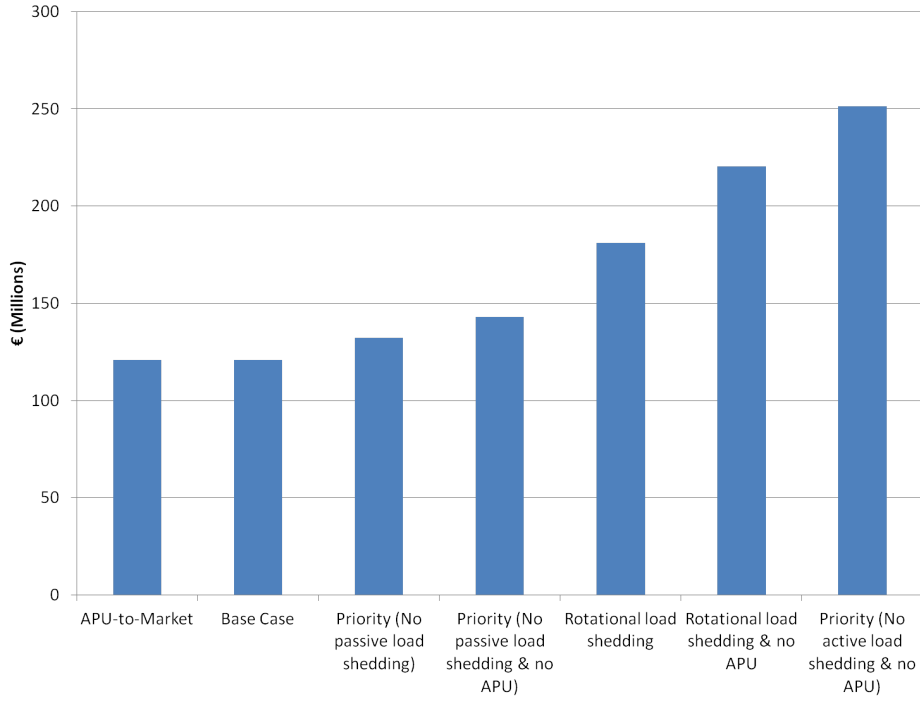


Figure 4: Total expected consumer costs associated with different load shedding policies.

Figure 4 shows that the *Base Case* and *APU-to-Market* case have the lowest TECC. In fact, for the data considered in this work, these two cases give the same optimal decisions values for each MCP solved. The difference between the two cases is that constraints (6d) and (6e) are excluded from the *APU-to-Market* case. However, in the *Base Case*, these two constraints are never binding. This is because APU generation is constrained (through both hourly and storage capacity constraints)² on the amount of electricity it can provide. Thus, APU generation is never large enough to be able to meet demand from active consumers, never mind demand from both types of consumers. In order to examine a case where APU generation is large enough to meet at least some of both types of demand, we also, as a sensitivity check, examined both of these cases with extremely low (and unrealistic) reference demand for active consumers. However, only a minor reduction in TECC was found by allowing auxiliary generation be available to the full market. Henceforth, the *Base Case* and *APU-to-Market* are considered equivalent.

Figure 4 also shows that, overall, the *Base Case* has the lowest TECC. For each of the other

²Given that the model is solved over a relatively short period of time (48) hours, we assume the active consumer is unable to refuel the storage tank associated with APU generation.

cases, load shedding and/or the APU are restricted in some way. Excluding the *Base Case*, the lowest TECC is seen when passive consumers' load is prioritised (i.e., there is load shedding for active consumers only) while the highest TECC is found when active consumers' load is prioritised. Consequently, when load shedding rotates between active and passive consumers, there are intermediate levels of TECC. This suggests that if a regulator had to prioritise between the two types of consumers then active consumers should be chosen for load shedding while rotational policies should be chosen ahead of prioritising active consumers only. Figure 4 also shows that removing auxiliary generation increases TECC. This can be seen when the *Priority (No passive load shedding)* case is compared with the *Priority (No passive load shedding & no APU)* case and similarly when the *Rotational load shedding* case is compared with the *Rotational load shedding & no APU* case.

In a similar manner to equations (11) and (12), expected total amount of load shedding and expected prices are also calculated; see Figures 5 and 6. These results match with those seen in Figure 4 with decreased potential for load shedding leading to lower amounts of load shedding, increased expected prices and hence increased TECC. However, when the APU is removed, both load shedding and TECC increase. This is because, in these cases, the extra load shedding is not used to reduce costs but rather to replace auxiliary generation. These results highlight the benefits of auxiliary generation to the market.

3.2.2. Effect of market power

Figure 7 displays the TECC and total expected amount of load shedding for the *Base Case* and the case where no market power is present. Unsurprisingly both TECC and load shedding are lower when there is no market power present. In the *Base Case* the price-maker (generator $i = 5$) holds back on the amount of electricity it can provide. It is able to do this at times of high demand and low supply, i.e., when all other generators are supplying electricity at maximum capacity and are hence unable to prevent the price-maker's strategic behaviour. Consequently, load shedding, prices and the price-maker's profits all increase.

Figure 8 presents the generator profits for the *Base Case* and the *No Market Power* case and shows that each of the generators substantially increase its profits when market power is present. This is despite only generator $i = 5$ being modelled as a price-maker. As the price-maker strategically chooses its production to increase the system price and hence its profit, all other generators benefit too. Ironically, generator $i = 5$ has the smallest percentage increase (469%) in profit between the *Base Case* and the *No Market Power* case. This is because this

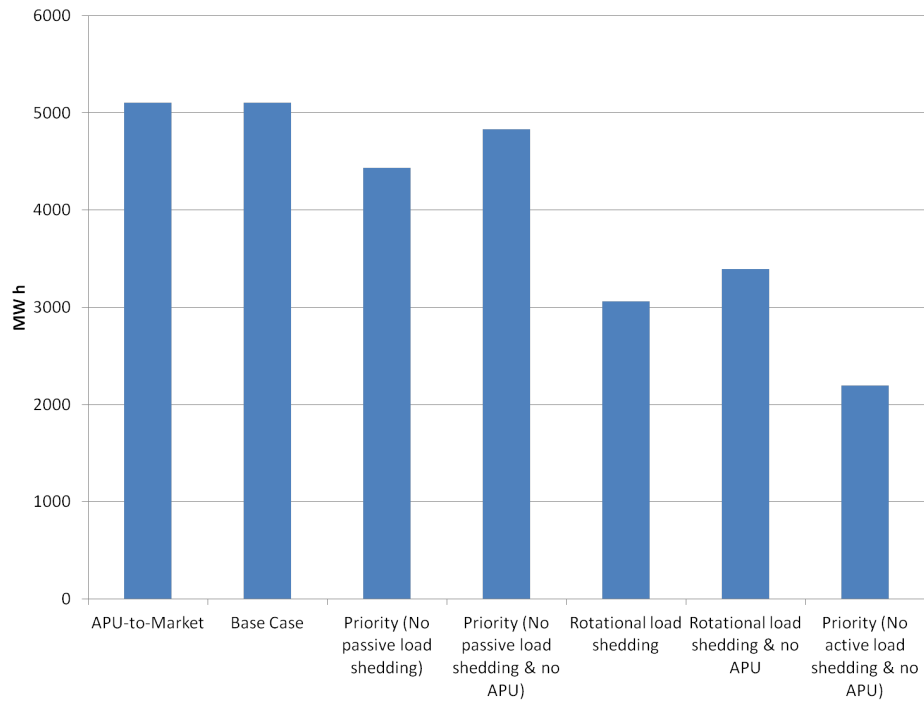


Figure 5: Total expected amount of load shed for different load shedding policies.

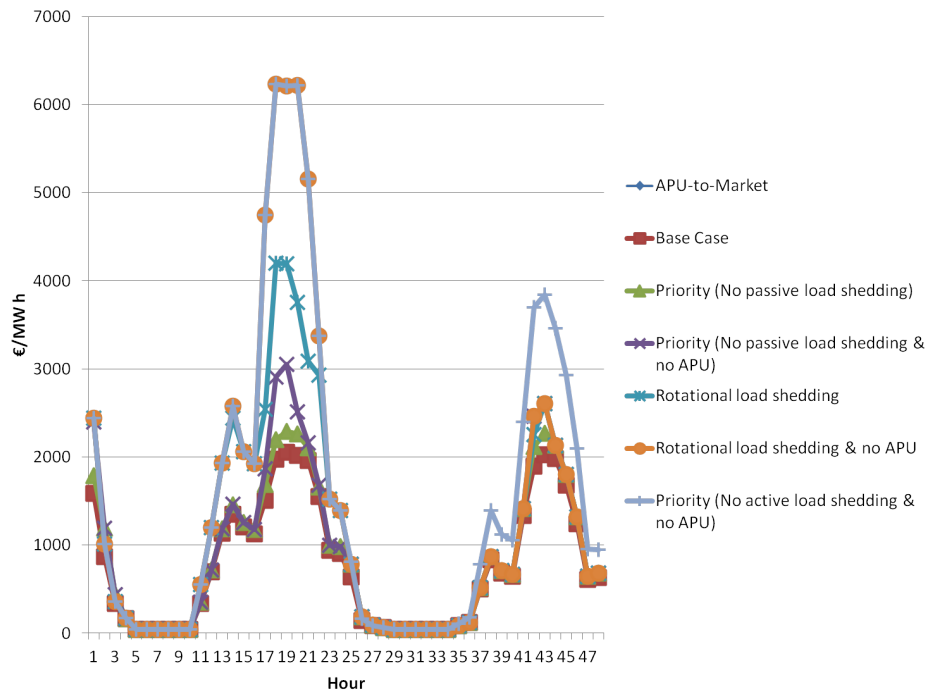


Figure 6: Expected price time series for different load shedding policies.

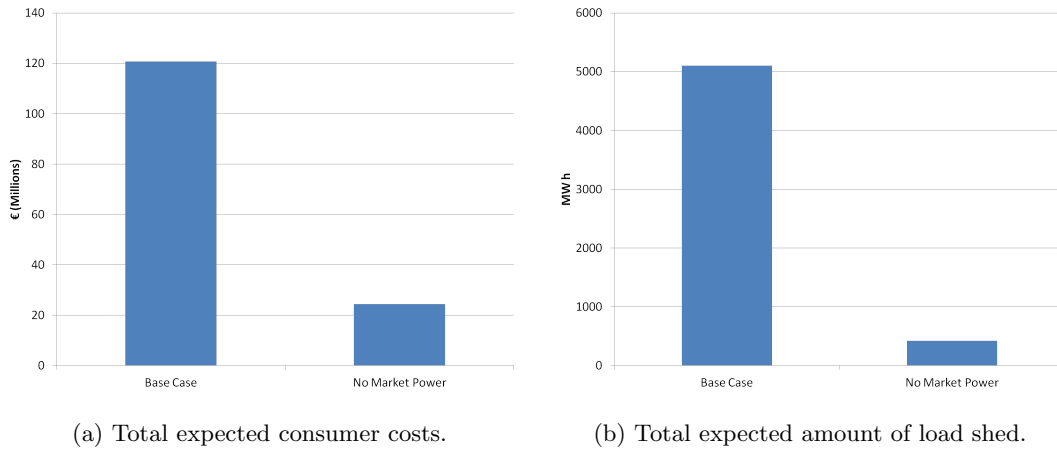


Figure 7: Results with and without market power.

generator is modelled as a peaking unit and is generating the least amount in comparison to the other units.

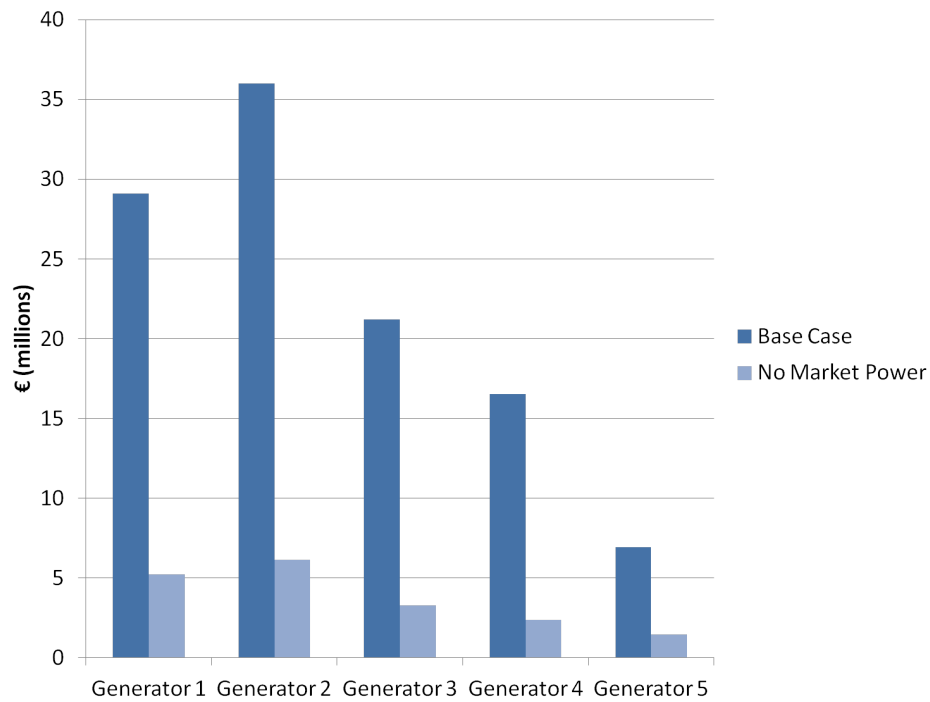


Figure 8: Expected generator profits with and without market power present.

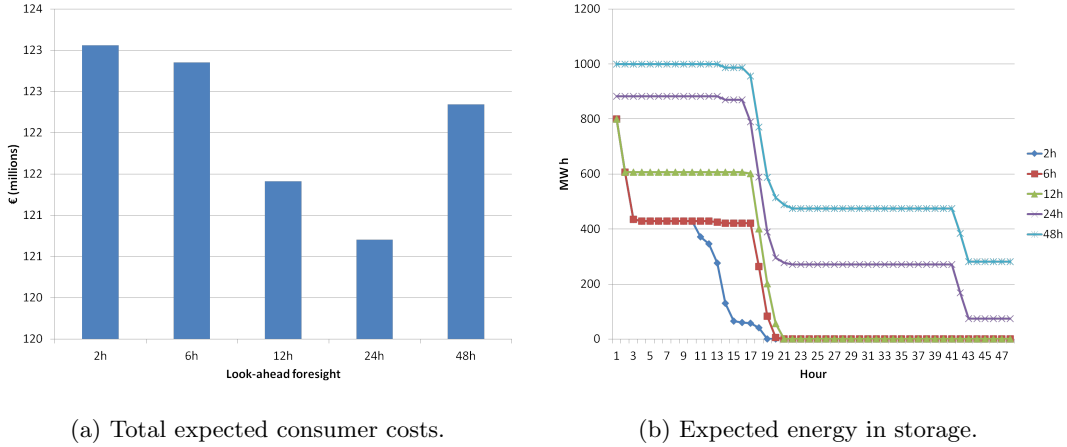


Figure 9: Effect of different levels of look-ahead foresight.

3.2.3. Optimal level of foresight

This section examines the cases where the players in the model have different levels of look-ahead foresight when making decisions at each MCP. Figure 9a displays the TECC when the model considers $H = 2, 6, 12, 24$ and 48 hours of foresight while Figure 9b gives the expected amount of energy in storage associated with the APU. Reference demand is initially high; see Figure 3b. Thus, when $H = 2$ and $H = 6$, the active consumers have limited foresight and thus use the energy they have in storage to meet this initially high level of demand without concern for the larger demand peak that occurs at later timesteps. Consequently, in these cases, there is reduced auxiliary generation to meet the peak demand of the first 24 hours and no auxiliary generation to meet the peak demand of the second 24 hours. This results in higher TECC.

When $H = 12$ the active consumers do not use the APU to meet the initially high demand and save it for the first peak in demand. However, with only 12 hours of foresight they have no concern for the peak in the second 24 hour period and thus empty the tank when the first peak occurs. This leads to reduced TECC compared with the cases when $H = 2$ and $H = 6$ but increased costs compared with the $H = 24$ case. When $H = 24$, the active consumers do not empty the tank when the peak in the first 24 hours occurs and save enough energy to meet some of the peak demand in the second 24 hours.

This trend suggests that the more foresight players have in the rolling-horizon model, the more informed decisions they can make. However, the $H = 48$ case shows increased TECC compared with the $H = 12$ and $H = 24$ cases. The active consumers in the $H = 48$ case are over conservative when using the APU to meet the daily demand peaks. This is because, in this

case, they become concerned with peaks in the third and fourth 24 hour periods despite the model being solved over 48 rolls, i.e., despite the players only having to make actual decisions for 48 hours. For instance, when making decisions at roll $r = 44$, the set of timesteps considered is $t = \{44, 45, \dots, 91\}$ and consequently the active consumers save energy excessively for future timesteps. Thus, in the rolling-horizon model considered, the $H = 48$ case contains too many timesteps.

To further examine the optimal level of look-ahead foresight, Figure 10 displays the TECC (normalised to the $H = 24$ results in each case) for different sensitivity checks, for each level of foresight:

1. *Original Case:*

Provides the same results as those presented in Figure 9a.

2. *6 Hours Expected Outage Length:*

In this case the log-normal distribution, that determines the PR_l and PR_s parameters, is adjusted such that the expected outage length is 6 hours instead of 12 with values of 1.519 and 0.75 for the location and scale parameters respectively.

3. *Two Peaks Daily:*

In this case the reference demand for both types of consumers is adjusted such that there are two peaks in each 24 hour period in contrast to the one peak seen in Figure 3b.

4. *Two Peaks Daily and 6 Hours Expected Outage Length:*

In this case, the two previous cases are combined.

In the *6 Hours Expected Outage Length* case, $H = 24$ hours of foresight still leads to the lowest TECC. While the expected outage is six hours, outage lengths greater than this still have significant probabilities associated with them and thus it remains prudent for active consumers to save energy for peak demands that do not occur in the first 6 or 12 hour periods. Similarly, for the *Two Peaks Daily* case, $H = 24$ hours of foresight also leads to the lowest TECC. While the active consumers in the $H = 6$ and $H = 12$ cases now save energy for the first peak in demand they still do not have enough for the two peaks in the second 24 hours and thus endure greater costs. When there is $H = 48$ hours of foresight, for each case, the active consumers are again overly conservative and concerned with the peaks in the third and fourth 24 hour periods and thus do not use the APU effectively to reduce costs.

When the *6 Hours Expected Outage Length* and *Two Peaks Daily* cases are combined, the optimal level of foresight changes to $H = 12$ hours. In this case, the active consumers with $H = 24$ hours of foresight unnecessarily become overly concerned with the peaks in future

timesteps. This is because the probability of the unreliable generator returning at these time steps is reduced. These results suggest that it is a combination of the demand profile plus expected outage length that determines the optimal level of foresight that players in rolling-horizons models should have.

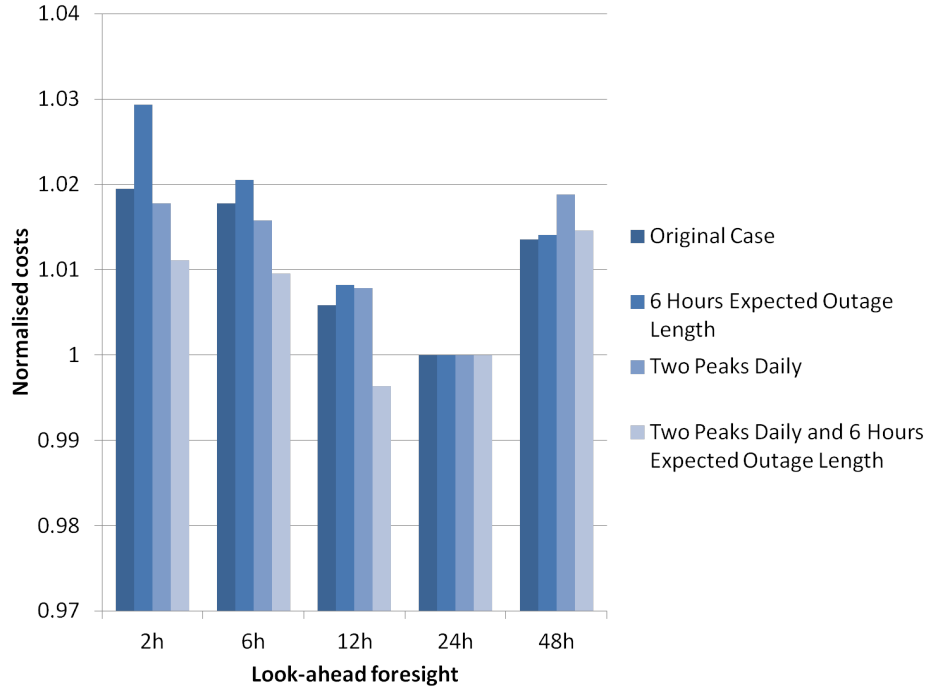


Figure 10: Normalised total expected consumer costs for different sensitivity checks.

3.2.4. Value of stochastic solution and expected value of perfect information

To justify the use of stochastics and rolling-horizons in the modelling approach employed, we also calculate well known metrics in stochastic programming: the Value of Stochastic Solution (VSS) and the Expected Value of Perfect Information (EVPI) [38]. The EVPI measures the increase in costs as a result of having uncertainty in the model, i.e., it measures the cost of not having perfect information. In this work, it is calculated using the following three steps:

1. For each MCP, taking the difference in expected consumer costs (sum of objective functions (4a) and (6a)) between the *Base Case* and when, for each MCP, the scenarios described in Figure 2 are replaced with a deterministic process whereby each player knows exactly when the unreliable generator will return.

2. For each path, averaging these values over the total number of stochastic MCPs in that path³ (circular shaped nodes in Figure 1)).
3. Calculating a weighted average over all paths, using the probabilities associated with each path.

The EVPI's value was found to be €5,018,824 which is 19% of the corresponding cost associated with the *Base Case*.

The VSS measures the cost of considering expected values instead of stochastic solutions. In this work, it is calculated in a similar way to the EVPI described above but, for each MCP, the scenarios described in Figure 2 are replaced with a deterministic process whereby each player assumes, with probability one, that the unreliable generator will return in 12 hours (the expected outage length), regardless of which path or scenario they are on. It's value was found to be €2,289,030 which is 9.7% of the corresponding cost associated with the *Base Case*.

4. Discussion

The five main findings of our research can be summarised as follows. While the first three are particularly relevant for system operators or regulators, the fourth provides insights particularly valuable for the players in the market - consumers as well as producers. The fifth finding is related to the methodology.

First, allowing consumers with auxiliary generation ability to supply their generation to the entire market only results in marginal benefits. Again, this finding needs to be interpreted in the context of our assumptions but even the sensitivity check with an unrealistically low reference demand for active consumers did not show a noteworthy benefit for this policy.

Second, a load shedding policy which does not discriminate or prioritise between different consumer groups leads to the lowest consumer costs: amongst the considered policies, smart load shedding policies (considered as *Base Case* in section 3) lead to the lowest costs to consumers, followed by shedding load of active consumers only and a rotational load shedding scheme (alternating between shedding active and passive consumers). Shedding load of passive consumers only leads to the highest costs to consumers. At the same time, the amount of load shedding (in MWh) is highest for smart load shedding and lowest for shedding passive consumers only. Since high amounts of load shedding represent a high demand side flexibility, this finding suggests that increasing demand side flexibility can strongly reduce consumer costs. Obviously, these results

³There is no EVPI or VSS associated with deterministic MCPs (diamond-shaped nodes in Figure 1)

are driven by our assumptions to a large extent, particularly concerning the load shedding cost functions for the two different consumer groups. Leahy & Tol [37], on the basis of whose results we derive the cost functions for our case study, find the VOLL to be highest for the residential sector. This assumption, together with the fact that we did not consider households with auxiliary generation ability, is important for interpreting the results. However, the generalised result that a load shedding policy which does not discriminate or prioritise between different consumer groups leads to the lowest consumer costs holds regardless of these assumptions. In addition, we ran the model with different slopes for the load shedding cost functions as a sensitivity check. While the total costs to consumers changed strongly with the slopes, the ranking of the policies remained unchanged. Finally, our results suggest that the importance of a deeper understanding of the load shedding costs and the value of lost load for different consumer groups will increase in future.

Third, preventing market power will significantly reduce consumer costs as well as the amount of load shedding. While this finding is not surprising, it is especially relevant for markets with dominant players on the generation side. Moreover, it is interesting to note that costs and load shedding between absence and presence of market power differ by a factor of six approximately. Costs and load shedding between the *Base Case* and *Priority (No active load shedding & no APU)* policy differ by a factor of approximately two only.

Fourth, concerning the optimal look-ahead foresight for the players in the market, we found that a very short look-ahead foresight might be disadvantageous particularly for active load shedding consumers since they might use up a limited amount of fuel in the storage of their auxiliary generation too quickly. From the perspective of active consumers, it is interesting to note that an unlimited increase of the look-ahead foresight is not necessarily beneficial either. This is because the optimal foresight is driven by a combination of the (daily) demand profile and the uncertainty concerning the length of a shortage period, which is the main source of uncertainty considered in our model.

Fifth, the calculated EVPI and VSS provide proof that it is advisable to use a rolling-horizon stochastic programming approach in this context. Incorporating stochastics into the model more realistically replicates the uncertain information those in the market have when making decisions. Additionally, and in contrast to a perfect foresight approach, the rolling-horizon model allows for the capture of the actual decisions players would make for each timestep, as well as the hypothetical decision they would consider making. Consequently, the results obtained replicate real-world markets more convincingly.

Critically reflecting our approach, we wish to acknowledge its short-term nature. This means that we do neither consider refueling strategies for the limited auxiliary generation storage nor do we consider interactions between load shedding and investment decisions. Moreover, we did not analyse the impact of own generation ability or decentralised storage availability for residential consumers within the scope of this paper. Concerning the data, while being based on actual values for Ireland, the case study has an illustrative character. In particular, there is uncertainty around the VOLL which, at the same time, drives the results. These limitations will be subject to future research activities.

5. Conclusions

In this paper, we examine the costs and strategies associated with load shedding as one demand response mechanism contributing to an increased demand side flexibility. For this purpose, we model an electricity market with different types of generators and consumers. More specifically, we propose a rolling horizon stochastic mixed complementarity equilibrium model.

Overall, we found that leveraging demand side flexibility can reduce costs to consumers. When comparing different policies, a smart load shedding policy, which does not discriminate between consumer groups, leads to the lowest costs. Furthermore, we found that allowing consumers with generation ability to supply their generation to the entire market only provides marginal benefits, whereas the prevention of market power is highly important in terms of ensuring low costs to consumers. Our study also included the analysis of the optimal foresight horizon that market players should consider, which has not been studied before in this context. We found that the optimal foresight horizon is driven by the daily load structure and, to a lower extent, by the uncertainty of the generators availability. In the light of these findings and bearing the smart meter rollout in mind, which is imminent particularly in many European countries, it is therefore now important that the specification of the technical requirements allows realisation of such policies - ideally without or with limited loss of comfort. This will ensure that full use of the smart infrastructure to be installed can be made in future.

Future research in this area should explore the impact of decentralised storage and own generation ability of residential consumers as well as interdependencies between the introduction of load shedding policies and investment decisions into generation capacity. Moreover, a deeper understanding of load shedding cost functions should be sought for different consumer groups.

Appendix A. Karush-Kuhn-Tucker conditions

This appendix presents the Karush-Kuhn-Tucker (KKT) conditions for optimality for each of the three types of players modelled in this work. These conditions, along with the market clearing conditions (8), make up the mixed complementarity problem solved at roll r . The ‘perb’ notation $0 \leq a \perp b \geq 0$ is equivalent to $a \geq 0$, $b \geq 0$ and $a \cdot b = 0$.

Appendix A.1. Generators’ KKT conditions

The generators’ KKT conditions are

$$0 \leq g_{i,t=r}^{\text{FS}} \perp -\gamma_{t=r} - M_i^G g_{i,t=r}^{\text{FS}} \frac{\partial \gamma_{t=r}}{g_{i,t=r}^{\text{FS}}} + F_i + \lambda_{G1_{i,t=r}}^{\text{FS}} \geq 0, \quad \forall i, \quad (\text{A.1})$$

$$0 \leq g_{i,t,s} \perp PR_s \left(-\gamma_{t,s} - M_i^G g_{i,t,s} \frac{\partial \gamma_{t,s}}{g_{i,t,s}} + F_i \right) + \lambda_{G1_{i,t,s}} \geq 0, \quad \forall i, s, t > r, \quad (\text{A.2})$$

$$0 \leq \lambda_{G1_{i,t=r}}^{\text{FS}} \perp -g_{i,t=r}^{\text{FS}} + G_i^{\text{max}} \geq 0, \quad \forall i, \quad (\text{A.3})$$

$$0 \leq \lambda_{G1_{i,t,s}} \perp -g_{i,t,s} + G_i^{\text{max}} \geq 0, \quad \forall i, s, t > r, \quad (\text{A.4})$$

where M_i^G is a binary parameter indicating whether generator i is a price maker ($M_i^G = 1$) or price taker ($M_i^G = 0$) and

$$\frac{\partial \gamma_{t=r}}{\partial g_{i,t=r}^{\text{FS}}} = \frac{\partial \gamma_{t,s}}{\partial g_{i,t,s}} = -\frac{1}{2} \left(\frac{(\sum_{j \in J} B_{j,t}^{\text{P}})(\sum_{k \in K} B_{k,t}^{\text{A}})}{(\sum_{j \in J} B_{j,t}^{\text{P}}) + (\sum_{k \in K} B_{k,t}^{\text{A}})} \right), \quad \forall i, s, t. \quad (\text{A.5})$$

Appendix A.2. Passive load shedding consumers’ KKT conditions

The passive load shedding consumers’ KKT conditions are

$$-\gamma_{t=r} + E_{j,t=r}^{\text{P}} + 2B_{j,t=r}^{\text{P}} \Delta g_{j,t=r}^{\text{P,FS}} + \lambda_{P1_{j,t=r}}^{\text{FS}} - \lambda_{P2_{j,t=r}}^{\text{FS}} = 0, \quad \forall j, \quad (\text{A.6})$$

$$PR_s \left(-\gamma_{t,s} + E_{j,t}^{\text{P}} + 2B_{j,t}^{\text{P}} \Delta g_{j,t,s}^{\text{P}} \right) + \lambda_{P1_{j,t,s}} - \lambda_{P2_{j,t,s}} = 0, \quad \forall j, s, t > r, \quad (\text{A.7})$$

$$0 \leq \lambda_{P1_{j,t=r}}^{\text{FS}} \perp -\Delta g_{j,t=r}^{\text{P,FS}} + \Delta g_j^{\text{P,max}} \geq 0, \quad \forall j, \quad (\text{A.8})$$

$$0 \leq \lambda_{P1_{j,t,s}} \perp -\Delta g_{j,t,s}^{\text{P}} + \Delta g_j^{\text{P,max}} \geq 0, \quad \forall j, s, t > r, \quad (\text{A.9})$$

$$0 \leq \lambda_{P2_{j,t=r}}^{\text{FS}} \perp \Delta g_{j,t=r}^{\text{P,FS}} \geq 0, \quad \forall j, \quad (\text{A.10})$$

$$0 \leq \lambda_{P2_{j,t,s}} \perp \Delta g_{j,t,s}^{\text{P}} \geq 0, \quad \forall j, s, t > r. \quad (\text{A.11})$$

Appendix A.3. Active load shedding consumers' KKT conditions

The active load shedding consumers' KKT conditions are

$$-\gamma_{t=r} + E_{k,t=r}^A + 2B_{k,t=r}^A \Delta g_{k,t=r}^{A,FS} + \lambda_{A1k,t=r}^{FS} + \lambda_{A2k,t=r}^{FS} - \lambda_{A5k,t=r}^{FS} = 0, \quad \forall k, \quad (\text{A.12})$$

$$PR_s(-\gamma_{t,s} + E_k^A + 2B_k^A \Delta g_{k,t,s}^A) + \lambda_{A1k,t,s} + \lambda_{A2k,t,s} - \lambda_{A5k,t,s} = 0, \quad \forall k, s, t > r, \quad (\text{A.13})$$

$$0 \leq g_{k,t=r}^{APU,FS} \perp -\gamma_{t=r} + F_k^{APU} + \lambda_{A2k,t=r}^{FS} + \lambda_{A3k,t=r} + \sum_{s \in S} \lambda_{A4k,s} \geq 0, \quad \forall k, \quad (\text{A.14})$$

$$0 \leq g_{k,t,s}^{APU} \perp -\gamma_{t,s} + F_k^{APU} + \lambda_{A1k,t,s} + \lambda_{A3k,t,s} + \lambda_{A4k,s} \geq 0, \quad \forall k, s, t > r, \quad (\text{A.15})$$

$$0 \leq \lambda_{A1k,t=r}^{FS} \perp -\Delta g_{k,t=r}^{A,FS} + \Delta g_k^{A,max} \geq 0, \quad \forall k, \quad (\text{A.16})$$

$$0 \leq \lambda_{A1k,t,s} \perp -\Delta g_{k,t,s}^A + \Delta g_k^{A,max} \geq 0, \quad \forall k, s, t > r, \quad (\text{A.17})$$

$$0 \leq \lambda_{A2k,t=r}^{FS} \perp -\Delta g_{k,t=r}^A - g_{k,t=r}^{APU,FS} + D_{k,t=r}^A \geq 0, \quad \forall k, \quad (\text{A.18})$$

$$0 \leq \lambda_{A2k,t,s} \perp -\Delta g_{k,t,s}^A - g_{k,t,s}^{APU} + D_{k,t}^A \geq 0, \quad \forall k, s, t > r, \quad (\text{A.19})$$

$$0 \leq \lambda_{A3k,t=r}^{FS} \perp -g_{k,t=r}^{APU,FS} + g_k^{APU,max} \geq 0, \quad \forall k, \quad (\text{A.20})$$

$$0 \leq \lambda_{A3k,t,s} \perp -g_{k,t,s}^{APU} + g_k^{APU,max} \geq 0, \quad \forall k, s, t > r, \quad (\text{A.21})$$

$$0 \leq \lambda_{A4k,s} \perp -g_{k,t=r}^{APU,FS} - \sum_{t>r}^{r+H-1} g_{k,t,s}^{APU} + V_k \geq 0, \quad \forall k, s, \quad (\text{A.22})$$

$$0 \leq \lambda_{A5k,t=r}^{FS} \perp \Delta g_{k,t=r}^{A,FS} \geq 0, \quad \forall k, \quad (\text{A.23})$$

$$0 \leq \lambda_{A5k,t,s} \perp \Delta g_{k,t,s}^A \geq 0, \quad \forall k, s, t > r. \quad (\text{A.24})$$

Appendix B. Supplementary data

Table B.7: Probabilities associated with each path l and scenario s (only for nodes where unreliable generator is offline).

s/l	1	2	3	4	5	6	7	8	9	10	11	12
PR_s/PR_l	0.035	0.064	0.074	0.073	0.069	0.063	0.058	0.052	0.047	0.042	0.038	0.034
s/l	13	14	15	16	17	18	19	20	21	22	23	24
PR_s/PR_l	0.031	0.028	0.025	0.023	0.021	0.019	0.017	0.016	0.014	0.013	0.012	0.011
s/l	25	26	27	28	29	30	31	32	33	34	35	36
PR_s/PR_l	0.010	0.010	0.009	0.008	0.008	0.007	0.007	0.006	0.006	0.005	0.005	0.005
s/l	37	38	39	40	41	42	43	44	45	46	47	48
PR_s/PR_l	0.004	0.004	0.004	0.004	0.003	0.003	0.003	0.003	0.003	0.003	0.002	0.002

Table B.8: Values for reference demands and slopes of marginal load shedding cost functins.

t	1	2	3	4	5	6	7	8	9	10	11	12
$D_{j=1,t}^P$	1316	1062	902	766	674	613	568	513	512	546	659	770
$D_{k=1,t}^A$	1821	1809	1714	1661	1607	1598	1657	1730	1651	1736	1886	2061
$B_{j=1,t}^P$	4.7	2.4	1.2	1.7	1.2	2.2	15.0	68.2	102.0	85.7	58.2	40.4
$B_{k=1,t}^A$	4.5	4.6	4.8	5.0	5.1	5.2	5.0	4.8	5.0	4.8	4.4	4.0
t	13	14	15	16	17	18	19	20	21	22	23	24
$D_{j=1,t}^P$	2204	2250	2262	2265	2272	2091	1944	2108	2070	1963	1825	1911
$D_{k=1,t}^A$	879	941	885	872	1092	1569	1714	1441	1375	1336	1294	1230
$B_{j=1,t}^P$	32.6	28.4	31.6	33.5	23.6	13.8	14.2	20.7	23.4	25.6	25.0	16.1
$B_{k=1,t}^A$	3.7	3.7	3.7	3.6	3.6	4.0	4.3	3.9	4.0	4.2	4.5	4.3
t	25	26	27	28	29	30	31	32	33	34	35	36
$D_{j=1,t}^P$	1821	1809	1714	1661	1607	1598	1657	1730	1651	1736	1886	2061
$D_{k=1,t}^A$	1316	1062	902	766	674	613	568	513	512	546	659	770
$B_{j=1,t}^P$	4.7	2.4	1.2	1.7	1.2	2.2	15.0	68.2	102.0	85.7	58.2	40.4
$B_{k=1,t}^A$	4.5	4.6	4.8	5.0	5.1	5.2	5.0	4.8	5.0	4.8	4.4	4.0
t	37	38	39	40	41	42	43	44	45	46	47	48
$D_{j=1,t}^P$	2204	2250	2262	2265	2272	2091	1944	2108	2070	1963	1825	1911
$D_{k=1,t}^A$	879	941	885	872	1092	1569	1714	1441	1375	1336	1294	1230
$B_{j=1,t}^P$	32.6	28.4	31.6	33.5	23.6	13.8	14.2	20.7	23.4	25.6	25.0	16.1
$B_{k=1,t}^A$	3.7	3.7	3.7	3.6	3.6	4.0	4.3	3.9	4.0	4.2	4.5	4.3

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