Inducing truthful revelation of generator reliability.

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Abstract

Liberalised electricity markets often include a capacity remuneration mechanism to allow generation firms recover their fixed costs. Various de-rating factors and/or penalties have been incorporated into such mechanisms in order to reward the unit based on the contribution they make to system security, which in turn depends on the unit's reliability. However, this reliability is known to the firm but not to the regulator. We adopt a mechanism design approach for capacity payments based on a declaration by the firm of their reliability. The mechanism scales payments and penalties according to this declared reliability such that the firm's profit-maximising strategy is to truthfully reveal its reliability. A stochastic Mixed Complementarity Problem (MCP) is used to model the interactions between the firms, and we apply this methodology to a test system using Irish electricity market data. Truth-telling is induced, increasing the efficiency of capacity payments while eliminating the requirement for the regulator to allocate resources to discovering reliability.

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Nomenclature

f	Index for generation firms
F	Total number of firms
t	Index for generation technologies
T	Total number of technologies
s	Index for scenarios representing units available/unavailable
$B_{f,t,s}$	Binary parameter indicating whether firm f's unit with technology t is available in scenario s
$R_{f,t}$	Parameter giving the initial reliability of each technology owned by each firm
$refurb_{f,t}$	Variable giving the refurbishment decisions of each firm for each technology
$\hat{R}_{f,t}$	Variable giving the declared reliability for each technology owned by each firm
$\overline{R}_{f,t}$	Parameter giving the maximum reliability for each technology owned by each firm
pr_s	Probability of scenario s
p	Index for demand period
$gen_{f,t,p,s}$	Variable giving the generation for technology t owned by firm f in demand period p for scenario s
$CAP_{f,t}$	Parameter giving the initial capacity of technology t held by firm f
$RCOST_t$	Parameter giving the refurbishment cost of technology t
$\gamma_{p,s}$	Variable giving the energy price generators receive in period p and scenario s
$\gamma_{p,s}$	Lagrange variable giving the energy price in period p and scenario s
$H(.)$	Capacity payments function
α	Variable giving scaling factor for capacity payments function
τ_t	Variable giving the penalty per MW a unit with technology t pays when unit if it is unavailable
$MC_{f,t}$	Parameter giving the marginal cost of technology t for firm f
Π_f	Variable giving profit of each firm f
Z_p	Parameter giving the intercept of the demand curve at period p
W	Parameter giving the slope of the demand curve
POT	Parameter giving the total amount of money for capacity payments
$\Gamma^0_{p,s}$	Maximum price consumers are willing pay in period p and scenario s

1   Introduction

Electricity markets have undergone a process of liberalisation in recent years. The absence of an active demand-side in electricity generation markets, along with the shared nature of the network, means that generators face a 'missing money' problem in relation to recovering their fixed costs (Stoft, 2002). Thus separate capacity remuneration mechanisms have been proposed as a means of compensating generators for the cost of holding capacity, separate from providing energy (Cramton and Ockenfels, 2012; Cramton and Stoft, 2008; Botterud and Doorman, 2008).

In compensating generators for providing capacity, over and above the energy they provide, capacity remuneration must differentiate between the rated capacity of the unit and the capacity they can be expected to reliably provide. No generation unit is completely reliable, in that there is always a non-zero probability that the unit will be on either scheduled or forced outage when required. In the case of variable renewable generators, such as wind or photovoltaic generators, their availability is obviously weather-dependent, as well as being subject to forced or scheduled outage for maintenance. An efficient capacity remuneration mechanism will take this inherent unreliability into account.

Several capacity remuneration mechanisms include different ways of 'de-rating' the unit's capacity in order to account for unreliability. For example, the Electricity Market Reforms (EMR) in Great Britain have included a new capacity market incorporating a de-rating factor. The de-rating factor of thermal generators is determined administratively for each technology, based on their historic availability during the last seven winters (National Grid, 2015b). In the electricity market of Pennysylvania-New Jersey-Maryland (PJM), conventional generators are de-rated according to their forced outage rates, while wind is de-rated to 13% of its installed capacity and solar generation is de-rated to 38% of installed capacity (Bowring et al., 2013). The Single Electricity Market (SEM) of the island of Ireland is currently undergoing a redesign and will include a new capacity remuneration mechanism. The regulators envisage that generation capacity offered in the capacity market shall be de-rated, and explicitly state that the method of de-rating capacity should be "...reflective of its [the unit's] ability to deliver capacity at times of system stress." (CER and NIAUR, 2015).

These examples are not exhaustive and do not cover all possible capacity mechanisms or de-rating methodologies. However, the principle that installed capacity should be de-rated when considering appropriate revenues from capacity remuneration mechanisms has been established in general. The methods currently used in, for example, Great Britain and PJM to determine the de-rating can be viewed as somewhat *ad hoc*. They are based on historical data, which may have been determined partly as a result of strategic withholding of capacity by firms. They are calculated on a technology-basis, rather than on the basis of the individual units and/or the firms that own them. For this reason, a firm that owns a unit with higher reliability than the average unit of that particular technology (i.e. a lower than average forced outage rate) has a diminished incentive to ensure the continuing reliability of that unit, while a firm with a unit of less than average reliability (i.e. a higher than average forced outage rate) has a diminished incentive to refurbish the unit and improve its forced outage rate. Finally the interaction between units is not taken into account (*ex ante*, at least).

The issue of the appropriate de-rating factor to apply can be viewed as a problem of imperfect information. The firm possesses the best information regarding the reliability of its particular unit. Instead of estimating de-rating factors for entire technologies, the regulator may instead wish to induce the firm to reveal its (best estimate of its) true reliability. A regulator that simply scales capacity payments according to a firm's declared reliability will not overcome this informational asymmetry, as firms will declare the highest possible reliability in order to claim the highest possible capacity payment. However, if a penalty is incurred by generators during periods of unavailability, it is possible to scale both the capacity payment and the penalty by the firm's declared level of reliability. If these payment and penalty factors are well-chosen, the firm's profit-maximising strategy will be that of a truthful declaration of its reliability.

In this paper, we use mechanism design as the means by which a regulator (the principal) can construct a payment structure which will, in turn, induce generators to truthfully declare their units' reliability. Mechanism design problems that address informational asymmetries are characterized by one player (the principal) who conditions their actions on some information that is privately known by the other players (agents). The principal incentivises the agents to report their information truthfully. Mechanism design was first introduced by Hurwicz (1960, 1972) and was further developed by Maskin (1999) and Myerson (1979, 1982, 1986). It has been applied in many areas of economics such as monopoly pricing, optimal taxation, contract theory, principle agent theory, and auction theory (Zou et al., 2015).

In recent years, mechanism design has also been applied to electricity markets. For instance, in Zou et al. (2015), a market operator employs mechanism design to induce generators to reveal their true marginal costs of generation in a pool-based electricity market. Similarly, in Silva et al. (2001), mechanism design is used to incentivize generators to reveal their true marginal cost in an economic dispatch model. In Hobbs et al. (2000), a Vickery-Clarke-Groves auction is proposed and mechanism design is utilised to incentivise bidders to bid truthfully. Their framework is applied to both power and gas pipeline auctions. Building on Hobbs et al. (2000), Zou (2009) designs a double-sided auction mechanism for an electricity market that controls market power and enhances total social welfare. Each of these papers uses mechanism design to incentivise generators to reveal either their true bids or marginal costs. However, to the best knowledge of these authors, mechanism design has not been used to model electricity capacity markets, and in particular to induce truthful revaluation of generator reliability/forced outage rates.

This paper does not address the related question of the optimal level of reliability for the system, or how to incentivise generators to provide this optimal level. Rather we concentrate instead on addressing the informational asymmetry between the regulator/market operator and the firms regarding the reliability of each unit, and therefore the appropriate level of capacity compensation for each firm. We illustrate the effects of the mechanism by applying the methodology to a test system using Irish electricity market data, but the method is readily applicable to any system. We demonstrate how the profit-maximising strategy of firms is indeed to declare its (best estimate of its) true reliability, relieving the regulator of responsibility in this regard. We also examine the impact of refurbishment of existing units, allowing firms to improve the reliability of their generation portfolio, as well as investing in new generation or retiring old units.

We model the interactions between firms as a stochastic Mixed Complementarity Problem (MCP). MCPs allow the profit maximisation problems of multiple competing firms to be solved simultaneously and in equilibrium by combining the Karush-Khun-Tucker (KKT) conditions for optimality of each of the players and connecting them via market clearing conditions. The stochasticity arises from the inherent unreliability of the units, each of which has a probability of being unavailable (i.e. on forced outage) during any given period. Given that refurbishing a unit will render a unit less likely to be on forced outage, the probabilities of the scenarios in the stochastic problem are dependant on the refurbishment decisions made by the firms, and thus depend on the equilibrium solution to the problem itself.

The remainder of this paper is structured as follows: Firstly, in Section 2, we introduce the methodology employed while, in Section 3, we discuss the data used in our case study. In Section 4, we present the results of the paper, firstly, when refurbishment of units is not possible and secondly, when it is. Finally, in Sections 5 and 6 we discuss the findings and conclude the paper, respectively.

2 Methodology

In order to design the mechanism, we formulate the problem as a game with F generation firms and a regulator. The principal agent is the regulator and all other agents are the generation firms. The firms' objectives are to maximise expected profits, which they earn in both energy and capacity markets. The

regulator seeks information from the firms, namely the reliability of each firm's generation units. While we assume that this is the sole objective of the regulator, other metrics that the regulator may wish to optimise, such as consumer cost and surplus, are also analysed in Section 4.

Capacity remuneration mechanisms are catagorised by the EU Agency for Cooperation of Energy Regulators (ACER) as price-based or quantity-based (ACER, 2013). The capacity payment considered here is a price-based mechanism, whereby a fixed sum of money to remunerate generators for capacity provision is determined administratively. This total fixed sum is then divided between all generators. Thus the regulator determines the price of capacity, and allows the quantity to emerge through the market.

In order to determine the magnitude of our capacity payment, we consider the intercept of the demand curve in the period of highest demand, i.e. the peak demand. This is the total quantity of generation required. This total quantity is multiplied by the Weighted Annualised Capital Cost (WACC) of the cheapest conventional generator to determine the total capacity payment amount. Thus the payment per MW is equal to the payment a marginal generator would require in order to break even on its fixed costs (as they will break even on its variable costs through the energy market). This methodology is a simplified version of that which is used at present in the Single Electricity Market (SEM) of Ireland (CER and NIAUR, 2006).

This price-based mechanism differs to the quantity-based capacity remuneration mechanisms that exist in many markets, including most markets in the USA. However, including a derating factor in a quantity-based mechanism is far more challenging. This is partly because a quantity-based mechanism requires the regulator to determine the total amount of investment required, but this in turn will depend on the reliability of the resulting generation fleet. More importantly, however, quantity-based mechanisms typically use an auction mechanism to determine the capacity price. However, in order to know what price to bid in such an auction, the firms will need to know *ex ante* the scaling factors and penalty factors that will apply, given the market-clearing price. These however will only be determined *ex post*, once the auction has cleared.

Although quantity-based mechanisms are more popular, some recent work has suggested that quantity-based mechanisms are more vulnerable to the exploitation of market power (Khalfallah, 2011) and less vulnerable to overinvestment than previously thought (Lynch and Devine, 2017). We believe that if price-based mechanisms can also be shown to induce truthful revelation of reliability, as is the aim of this paper, there are strong arguments for considering price-based mechanisms in capacity remuneration mechanisms. We leave the consideration of the incorporation of a similar mechanism into quantity-based capacity payment mechanisms for further work.

Having determined the magnitude of the capacity payment, it would typically be divided between generators on the basis of their installed capacity, perhaps derated by some administratively-determined factor. Instead, we design a mechanism that divides the total capacity payment among generators on the basis of the firms' declared reliability as well as their installed capacities. In order to induce truth-telling by the generators, the capacity revenues received by each firm under the mechanism must be a function both of the reliability declared by the generator, and of the actual level of reliability. We therefore include a penalty term whereby the generator is penalised for periods of unavailability. This penalty will obviously apply depending on the firms actual, rather than their declared, reliability.

Such non-delivery penalties are a standard feature of many capacity payment mechanisms, including the new mechanism in Great Britain.

Subsections 2.1, 2.2 and 2.3 outline the formulation and equilibrium solutions of the model, and subsection 2.4 demonstrates that the model as formulated satisfies the conditions of incentive compatibility and the revelation principle that are standard features of mechanism design problems.

2.1 Firm f's problem

Firm f maximises the revenue they receive from the capacity and energy markets less the cost of refurbishment and less any penalty it must pay in scenarios where its units are unavailable. We consider stylised generation technologies: baseload, mid-merit and peaking. The firm's problem is as follows:

$$
\max_{\substack{gen_{f,t,p}, \\ \hat{R}_{f,t}, \\ refurb_{f,t}}} \Pi_f = \max_{\substack{gen_{f,t,p}, \\ \hat{R}_{f,t}, \\ refurb_{f,t}}} \sum_t CAP_{f,t} H(\alpha, \hat{R}_{f,t})
$$
$$
- refurb_{f,t} RCOST_t CAP_{f,t}
$$
$$
- \sum_s \left(pr_s (1 - B_{f,t,s}) CAP_{f,t} \tau_t \hat{R}_{f,t} \right)
$$
$$
+ \sum_{p,s} \left(pr_s B_{f,t,s} gen_{f,t,p,s} (\gamma_p - MC_{f,t}) \right)
$$

(1)

subject to:

$$
gen_{f,t,p,s} \leq CAP_{f,t}, \ \forall t, p, s, \ (\lambda^1_{f,t,p,s}) \tag{2}
$$
$$
\hat{R}_{f,t} \leq \overline{R_{f,t}}, \ \forall \ t, \ (\lambda^2_{f,t}) \tag{3}
$$
$$
R_{f,t} + refurb_{f,t} \leq \overline{R_{f,t}}, \ \forall \ t, \ (\lambda^3_{f,t}) \tag{4}
$$

where t represents the different energy technologies, p represents different time periods and s different scenarios. The decision variables for firm f are $\hat{R}_{f,t}$, $gen_{f,t,p,s}$ and $refurb_{f,t}$ representing declared reliability, generation and refurbishment decisions respectively. The parameter $B_{f,t,s}$ is a binary indicator, describing whether firm f with technology t is available ($B_{f,t,s} = 1$) or unavailable ($B_{f,t,s} = 0$) in scenario s. The probability of the technology being available (i.e., the probability that $B_{f,t,s} = 1$) is determined by two factors. The first is the initial reliability ($R_{f,t}$) of the unit, which is a parameter of the model. The second factor is any refurbishment decisions made by the firm, which by definition make the unit more reliable by increasing the probability that $B_{f,t,s} = 1$. Thus, the probability of firm f's technology t being available is given by the total reliability of the technology after any refurbishment decisions, i.e., $R_{f,t} + refurb_{f,t}$. Each scenario s represents a different combination of all units being online/offline. In

total, there are $2^{F \times T}$ scenarios and the probability associated with scenario s is calculated as follows:

$$pr_s = \prod_{f,t}(R_{f,t} + refurb_{f,t})^{B_{f,t,s}}(1 - R_{f,t} - refurb_{f,t})^{1-B_{f,t,s}} \ \forall \ s. \tag{5}$$

Equation (5) follows from the definitions of $R_{f,t}$, $refurb_{f,t}$ and $B_{f,t,s}$. The first half of the expression is active only for those technologies that are available ($B_{f,t,s} = 1$) in scenario s, and so their contribution to the probability of scenario s is given by the reliability of the technologies ($R_{f,t} + refurb_{f,t}$). The second half of the expression is active only for those technologies that are on forced outage ($B_{f,t,s} = 0$), and so their contribution to the probability of scenario s is given by one minus the reliability of the technologies ($1 - R_{f,t} - refurb_{f,t}$).

The parameter $RCOST_t$ is the cost of refurbishment per technology while $CAP_{f,t}$ and $MC_{f,t}$ are the initial endowment of generation capacity and the marginal cost of production of each technology, respectively. The energy price at each period for scenario s is $\gamma_{p,s}$. It is exogenous to firm f's problem and is determined via the market clearing conditions. Each firm f is also assumed to be a price-taker.

The revenue per MW of installed capacity is $H(\alpha, \hat{R}_{f,t})$ which is a function of the declared reliability of each unit and a scaling factor α. This scaling factor is exogenous to firm f's problem but is a variable of the overall problem determined via the market clearing conditions. The objective of this paper is to determine the function $H(.)$, set by the regulator, in order to ensure that the firms' profit-maximising strategy is to declare their units' reliability truthfully; see Section 2.4. The penalty per MW of installed capacity that firms pay when their units are unavailable is τ_t. This penalty varies per technology and is scaled such that the penalty increases when firms increase their levels of declared reliability. This penalty is also set by the regulator to ensure firms declare their units' reliability truthfully; see Section 2.4. Thus the mechanism designed by the regulator is comprised of the function $H(.)$ and the parameters α and τ. Using the principles of mechanism design, our model solves for $H(.)$, α and τ in order to produce an equilibrium which renders the firm's incentives compatible with those of the regulator, i.e. truthful declaration of reliability.

Constraint (2) ensures that generation for a given unit and time period cannot exceed the amount of installed capacity while constraints (3) and (4) provide upper bounds for the declared and actual reliability of each unit. The variables in brackets alongside constraints (2) - (4) are the Lagrange multipliers associated with those constraints. All primal (decision) variables of this problem are also constrained to be non-negative.

2.2 Market clearing conditions

The Market Clearing Conditions that combine each of the firms' problems are

$$\sum_{f,t} B_{f,t,s} gen_{f,t,p,s} = Z_p + W\gamma_{p,s}, \ \forall p, s, \ (\gamma_{p,s}), \tag{6}$$

$$POT = \sum_{f,t} CAP_{f,t} H(\alpha, \hat{R}_{f,t}), \ (\alpha). \tag{7}$$

7

Equation (6) represents an inverse demand curve where Z_p is the demand intercept for period p and W is a parameter representing the slope of the function. Equation (7) specifies that the capacity pot, which is set administratively and so is exogenous to the problem, should be divided between generators based on their installed capacity and declared reliability. The variables $\gamma_{p,s}$ and α are the free Lagrange multipliers associated with these constraints.

2.3 Nash-equilibria

The overall model is a mixed complementarity problem given by the Karush-Khun-Tucker (KKT) conditions for each firm, along with the market clearing conditions. Each of the firms' problems is non-convex due to the bi-linear terms associated with the $refurb_{f,t}$ variable in the probability function in the objective functions. As a result the KKT conditions are necessary but not sufficient for optimality. However the stationary KKT conditions associated with the $refurb_{f,t}$ variable can be examined in isolation[1]:

$$0 \leq refurb_{f,t} \perp RCOST_t CAP_{f,t} + \lambda_{f,t}^3$$
$$- \sum_s \frac{\partial pr_s}{\partial refurb_{f,t}} \left((1 - B_{f,t,s}) CAP_{f,t} \tau_t \hat{R_{f,t}} - B_{f,t,s} gen_{f,t,p,s} (\gamma_{p,s} - MC_{f,t}) \right) \geq 0, \tag{8}$$

where

$$\frac{\partial pr_s}{\partial refurb_{f,t}} = (-1)^{1-B_{f,t,s}} \prod_{\substack{(\hat{f},\hat{t}) \\ (\hat{f},\hat{t}) \neq (f,t)}} (R_{\hat{f},\hat{t}} + refurb_{\hat{f},\hat{t}})^{B_{\hat{f},\hat{t},s}} (1 - R_{\hat{f},\hat{t}} - refurb_{\hat{f},\hat{t}})^{1-B_{\hat{f},\hat{t},s}}, \tag{9}$$

where $\hat{f}$ and $\hat{t}$ are dummy indices representing each firm and technology respectively except firm f with technology t. If

$$\sum_s \frac{\partial pr_s}{\partial refurb_{f,t}} \left(B_{f,t,s} gen_{f,t,p,s} (\gamma_{p,s} - MC_{f,t}) \right) > RCOST_t CAP_{f,t} + \sum_s \frac{\partial pr_s}{\partial refurb_{f,t}} \left((1 - B_{f,t,s}) CAP_{f,t} \tau_t \hat{R_{f,t}} \right), \tag{10}$$

then condition (8) is only satisfied if $\lambda_{f,t}^3 > 0$, which implies that $refurb_{f,t} = \overline{R_{f,t}} - R_{f,t}$ (as $\lambda_{f,t}^3$ is the Lagarange multiplier associated with constraint (4)). In other words, if the marginal benefit of refurbishing technology t is greater than the marginal cost, it is only optimal for firms to refurbish to the maximum extent possible. Conversely, if

$$\sum_s \frac{\partial pr_s}{\partial refurb_{f,t}} \left(B_{f,t,s} gen_{f,t,p,s} (\gamma_{p,s} - MC_{f,t}) \right) < RCOST_t CAP_{f,t} + \sum_s \frac{\partial pr_s}{\partial refurb_{f,t}} \left((1 - B_{f,t,s}) CAP_{f,t} \tau_t \hat{R_{f,t}} \right), \tag{11}$$

then condition (8) is only satisfied if $refurb_{f,t} = 0$, i.e, if the marginal benefit of refurbishing is less than the marginal cost, then the optimal decision is not to refurbish at all. Finally if

$$\sum_s \frac{\partial pr_s}{\partial refurb_{f,t}} \left(B_{f,t,s} gen_{f,t,p,s} (\gamma_{p,s} - MC_{f,t}) \right) = RCOST_t CAP_{f,t} + \sum_s \frac{\partial pr_s}{\partial refurb_{f,t}} \left((1 - B_{f,t,s}) CAP_{f,t} \tau_t \hat{R_{f,t}} \right), \tag{12}$$

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[1] The 'perb" notation $0 \leq a \perp b \geq 0$ is equivalent to $a \geq 0$, $b \geq 0$ and $a.b = 0$.

then the marginal cost and benefit of refurbishing are the same and firm f is indifferent regarding the amount of refurbishment they undertake for technology t.

As a result of the conditions above, we can simplify the problem without loss of generality by assuming the firms' refurbishment decisions are binary parameters, whereby firms either refurbish their units to the maximum reliability ($refurb_{f,t} = \overline{R_{f,t}} - R_{f,t}$) or they do not refurbish their unit at all ($refurb_{f,t} = 0$). When seeking an equilibrium solution to the problem, we solve the MCP for each combination of refurbishing/not refurbishing for each of the units that can refurbish. For example, for the studies described in Section 4, there are six units with reliability less than one. Hence, we solve the MCP $2^6 = 64$ times, each time with a different combination of units refurbishing or not. We then choose the combination that gives a unique Nash-equilibrium for each of the firms by inspection.

When the $refurb_{f,t}$ variable is considered as a parameter, each of the firms' problem's becomes convex (assuming the function $H(.)$ is chosen appropriately; see Section 2.4) and hence the KKT conditions are both necessary and sufficient for optimality for each of the individual optimisation problems (Gabriel et al., 2012). The firms' KKT conditions are outlined in Appendix A. Because the KKT conditions are solved simultaneously and in equilibrium, the solutions obtained lead to a Nash-equilibrium. Furthermore, assuming different marginal costs for each generating unit, leads to unique optimal generation levels for each firm. A proof of this result can be found in Appendix B. Hence, assuming unique optimal values for $\hat{R}_{f,t}$ (see Section 2.4), unique Nash-equilibrium solutions are obtained.

2.4 Mechanism design

To solve the problem we also need to specify the form of the payment function $H(\alpha, \hat{R}_{f,t})$ which the regulator sets. There are two key conditions associated with mechanism design (RSAS, 2007):

1. Incentive compatibility: a mechanism is incentive compatible if it is the dominant strategy of each participant to report their private information truthfully. For this problem, the firm's private information is the reliability of its generation units. Hence, incentive compatibility is satisfied if

$$\Pi_f(\hat{R}^*_{f,t}) \geq \Pi_f(\hat{R}_{f,t}) \quad \forall \, \hat{R}^*_{f,t}, \hat{R}_{f,t} \in [0, \overline{R_{f,t}} - R_{f,t}] \,\, \forall f, \tag{13}$$

where $\hat{R}^*_{f,t}$ represents the firms' optimal declaration of their units' reliability.

2. Revelation principle: If a game has a Nash equilibrium then there exists a direct revelation mechanism where agents reveal their private information truthfully[2]. If a incentive compatible direct mechanism does not exist, then it is not optimal for players to truthfully declare their private information and the revelation principle is not satisfied.

For the market analysed in this paper, a Nash equilibrium exists if the KKT conditions for each of the firms is satisfied, assuming their individual optimisation problems are convex. To ensure this Nash equilibrium also occurs with the firms exhibiting truthful revelation of their reliability, we examine the

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[2]A direct mechanism is one where each agent is asked to report their individual preferences.

stationary KKT condition associated with the $\hat{R}_{f,t}$ variable:

$$0 \le \hat{R}_{f,t} \perp -CAP_{f,t}\frac{\partial H(\alpha, \hat{R}_{f,t})}{\partial \hat{R}_{f,t}} + \sum_s \left(pr_s(1 - B_{f,t,s})CAP_{f,t}\tau_t\right) + \lambda_{f,t}^2 \ge 0. \tag{14}$$

Assuming that the firms' true reliability is neither at its maximum nor zero[3], setting

$$\frac{\partial H(\alpha, \hat{R}_{f,t})}{\partial \hat{R}_{f,t}} = \sum_s pr_s(1 - B_{f,t,s})\tau_t, \tag{15}$$

with $\hat{R}_{f,t} = R_{f,t} + refurb_{f,t}$ ensures condition (14) is satisfied if and only if firms declare their true reliability. As condition (14) is only necessary for optimality, we also require

$$\frac{\partial^2 H(\alpha, \hat{R}_{f,t})}{\partial \hat{R}_{f,t}^2} < 0, \tag{16}$$

at $\hat{R}_{f,t} = R_{f,t} + refurb_{f,t}$, for truth-telling to occur. As there is only one value for the reliability of firms' generating units, the optimal solutions for $\hat{R}_{f,t}$ are unique. Any strictly concave function $H(\alpha, \hat{R}_{f,t})$ can satisfy equations (15) and (16). While there are many functions that satisfy these conditions, for simplicity we choose

$$H(\alpha, \hat{R}_{f,t}) = \alpha(1 - e^{-\hat{R}_{f,t}}). \tag{17}$$

Substituting this payment function into condition (15) gives

$$\tau_t = \frac{\alpha e^{-R_{f,t}}}{\sum_s pr_s(1 - B_{f,t,s})}. \tag{18}$$

Thus, if the regulator sets the penalty according to (18), each firm will truthfully declare their units' reliability. Equation (17) ensures that the most reliable units receive the highest proportion of the capacity pot which, consequently, encourages refurbishment of units. If $H(\alpha, \hat{R}_{f,t})$ is chosen to be linear, then condition (14) would only be satisfied if and only if firms declared their true reliability to be either zero or its maximum, regardless of their initial level of reliability[4]. Other possible functions for H include $H(\alpha, \hat{R}_{f,t}) = \alpha\sqrt{\hat{R}_{f,t}}$ and $H(\alpha, \hat{R}_{f,t}) = \alpha\ln(\hat{R}_{f,t})$. The numerical tests in Section 4 were also conducted with these alternative functions and the results were found to be qualitatively unaffected. Because of the strictly concave structure of these functions, the most reliable units still received the highest proportion of the pot while truth-telling also still occurred.

3 Input data

To illustrate the methodology, we solve the model for a market with three generation technologies, four generation firms and 8760 time periods. We construct the case study using data for 2010 from the Irish electricity market, as it was publicly available. However, the model and its results are transferable to any electricity market. The three generation technologies considered are denoted as baseload, midmerit and

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[3]If a firm's reliability is at its maximum, then it will always declare its reliability truthfully to ensure its profits are maximised. If a firm's generation unit has zero reliability, then that unit does not exist.

[4]Similarly, if the model penalty structure did not contain any penalty, then condition (14) would only be satisfied if firms declared themselves to be fully reliable, regardless of their initial level of reliability.

peaking capacity. We consider pulverised coal to be roughly representative of baseload units, combined cycle gas plants as representing midmerit units and open cycle gas turbines as the peaking technology for this study.

In reality, the Irish electricity market roughly contains 58 thermal generating units (EIRGRID, 2016). However, a regulator in practice will not want to choose a reliability penalty for each unit in isolation as this would require categorising each unit as its own 'category' with no competitors, and could give rise to market power concerns. Thus, we assume the regulator will group units into a relatively small number of categories of technologies. For our purposes we choose three categories, however a higher number is possible. In Great Britain, for example, nine categories of thermal units are chosen for the purposes of determining de-rating factors. The capacities of the units modelled are displayed in Table 1.

We model four generation technology firms, which is roughly representative of the number of firms that operate in the Irish generation market. A higher number can be chosen when implementing this method in an alternative market. Firm one is an integrated firm, with investments in all three generation technologies. Firm two has baseload capacity only, firm three has midmerit capacity only and firm four has peaking capacity only. We assume the integrated firm has 44.4% share of generation capacity as this represents the share of the ESB (CER and NIAUR, 2016). The ESB is a legacy monopolist in the now liberalised Irish electricity market. The proportions of the three generating technologies were obtained from Shortt et al. (2013) while the total generation capacity (6826 MW) is equal to total generation requirement for Irish electricity market in 2010 (CER and NIAUR, 2009) and is also roughly equal to the intercept of the demand curve for the peak time period.

	Firm 1	Firm 2	Firm 3	Firm 4
Baseload	970	1214	0	0
Mid merit	697	0	873	0
Peaking	1364	0	0	1708

Table 1: Capacities ($CAP_{f,t}$) of each firm (MW)

We use the marginal costs of production from Shortt et al. (2013); see Table 2. Due to economies of scale, we assume that the marginal costs for the integrated firm are one unit cheaper. Sensitivities were conducted using different marginal costs and they did not significantly affect the final results. As real-life refurbishment costs are not publicly available, we assume the cost of refurbishing one MW of capacity from a reliability of zero to that of one is assumed to be one tenth of the investment cost used in Shortt et al. (2013) and Hirth (2013); see Table 3. This assumption is sufficient for our model but can of course be updated with information relevant to the system in question should this methodology be employed by a regulator. In addition, we provide a detailed sensitivity analysis on these refurbishment costs in Section 4.3.

	Firm 1	Firm 2	Firm 3	Firm 4
Baseload	64	65		
Mid merit	39		40	
Peaking	82			83

Table 2: Marginal cost ($MC_{f,t}$) of each generating unit (€/MW)

	Baseload	Midmerit	Peaking
$RCOST_t$	10,000	6,500	4,500

Table 3: Refurbishment cost for each technology (€/MW)

The slope of the demand curve is calculated from the elasticities estimated from historical Irish data in Di Cosmo and Hyland (2013) and by rearranging the following equation

$$\text{elasticity} = \frac{d\hat{Q}}{d\hat{P}} \times \frac{\hat{P}}{\hat{Q}}, \tag{19}$$

where $\hat{Q}$ and $\hat{P}$ are mean quantities and prices, respectively, from Di Cosmo and Hyland (2013) while $\frac{d\hat{Q}}{d\hat{P}}$ represents the slope of the demand curve. We assume that residential demand is met by supply companies that contract forward for generation, and so do not consider the impact of residential demand in the analysis. Agricultural demand is also met mainly through retail markets, and furthermore is a very small percentage of total demand (2%). We therefore calculate only the slope of the curves for industrial and commercial electricity demand, at -0.20 and -0.05 respectively, and take the weighted average according to their shares in final electricity demand to represent the slope of the demand curve for electricity, $W = -0.14$.

The 8760 time periods represent one year's worth of hourly timesteps. Demand intercept values (Z_p) can be found in the online supplementary appendix and were obtained by solving equation (6) using the slope described above and actual price and net load (demand less renewables) data for the Irish electricity market from 2010. This data was obtained from Eirgrid and SONI, the electricity market operators in the Republic of Ireland and Northern Ireland respectively[5].

The initial levels of reliability of installed capacity are considered fixed for each technology and firm. These reliability levels can be thought of as the forced outage rates of the units and are based on forced outage rates of units found on the Irish system as per the regulators' validated model for studying the Irish system (CER and NIAUR, 2013). The forced outage rate takes a value between zero and one, where zero indicates no reliability (i.e. the unit will be continually on forced outage) and one indicates guaranteed reliability (the unit will always be available when required). These initial levels are given in Table 4. Midmerit units have slightly lower levels of reliability than baseload units as they are cycled more frequently (Troy et al., 2010), adding to wear and tear on the units, and lower reliability than peaking units, as midmerit plants are online more frequently. Peaking units are used least often and so have lower wear and tear and higher reliability. Additionally, we also found that using alternative reliability/de-rating data from Great Britain (National Grid, 2015a) does not have a significant impact on the results.

	Baseload	Midmerit	Peaking
Reliability	0.965	0.955	0.985

Table 4: Initial levels of reliability ($R_{f,t}$) for each technology and firm

Each of the 64 MCPs solved contains roughly 1.4×10^7 equations and variables. As there are no inter-temporal constraints, each time period, for each MCP, is solved separately. In total the model was

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solved with a Central Processing Unit (CPU) time of approximately 9.3 hours. Using the path solver[6] in GAMS[7], this was performed using a 3.3GHz i5-4590 quad-core processor with 8GB of RAM and with a convergence tolerance of 10^{-6}, the default for the PATH solver. This solver has being shown by Ferris et al. (1999) to converge quadratically.

4 Results

In order to examine the impact of the mechanism, we solve the model both when the mechanism is and is not included in the capacity payment structure. When the mechanism is included, we solve the model as outlined in subsections 2.1 - 2.3. We then compare these results to a capacity payment that is merely divided amongst generators on the basis of their installed capacity rather than their installed capacity and their declared reliability, i.e. a price-based capacity payment that does not incorporate our mechanism, including the α and τ terms.

We run each of these models twice, once when refurbishment of units is allowed, as outlined in subsections 2.1 - 2.3, and once with the variable $refurb_{f,t}$ omitted. This allows us to isolate the incentives to invest in refurbishment, as well as the effects of so doing.

4.1 Without refurbishment

We first present the results from the two models when refurbishment is not considered. When the mechanism is included, the declared reliability for each technology and firm are given in table 5[8]. The firms revealed their true reliability for each technology, indicating that the mechanism had the desired effect of inducing truthfulness.

	Firm 1	Firm 2	Firm 3	Firm 4
Baseload	0.965	0.965		
Midmerit	0.955		0.955	
Peak	0.985			0.985

Table 5: Declared reliability ($\hat{R}_{f,t}$) for each technology and firm without refurbishment.

Table 6 compares the capacity revenues that are earned per MW of installed capacity when the mechanism is or is not included in the capacity payment design. As the total capacity payment is set administratively, and as the firms do not play a role in bidding for capacity revenues, each technology earns the same revenue per MW installed, regardless of which firm owns the unit. When the mechanism is included, the units with highest reliability (peaking units) earn the highest revenues per MW installed, with midmerit units earning the least. When the mechanism is not included, the capacity payment per MW installed is equal for each technology type, and thus for each level of reliability. Thus capacity payments that do not take account of reliability will see a transfer of resources from relatively reliable units towards relatively unreliable ones.

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[6]www.http://gams.com/docs/pdf/path.pdf

[7]`www.gams.com`

[8]When the mechanism is not included, there is no declaration of reliability of the firms, as their capacity payments are independent of their declared reliability.

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As outlined in subsection 2.4, the value for τ_t that ensures truth-telling is given by equation (18). Figure 1 displays the relationship between τ_t and $R_{f,t}$ and shows that for units with low reliability, a relatively small penalty is needed in order to incentivise truth-telling. For units with high reliability, much larger penalties are needed in order to ensure firms declare their actual reliability. This is because these units are much less likely to be unavailable and without a large penalty they would be incentivised to overestimate their reliability in order to increase the proportion of the total capacity payment they receive.

	Capacity revenues with mechanism	Capacity revenues without mechanism
Baseload	4578	4583
Midmerit	4550	4583
Peak	4634	4583

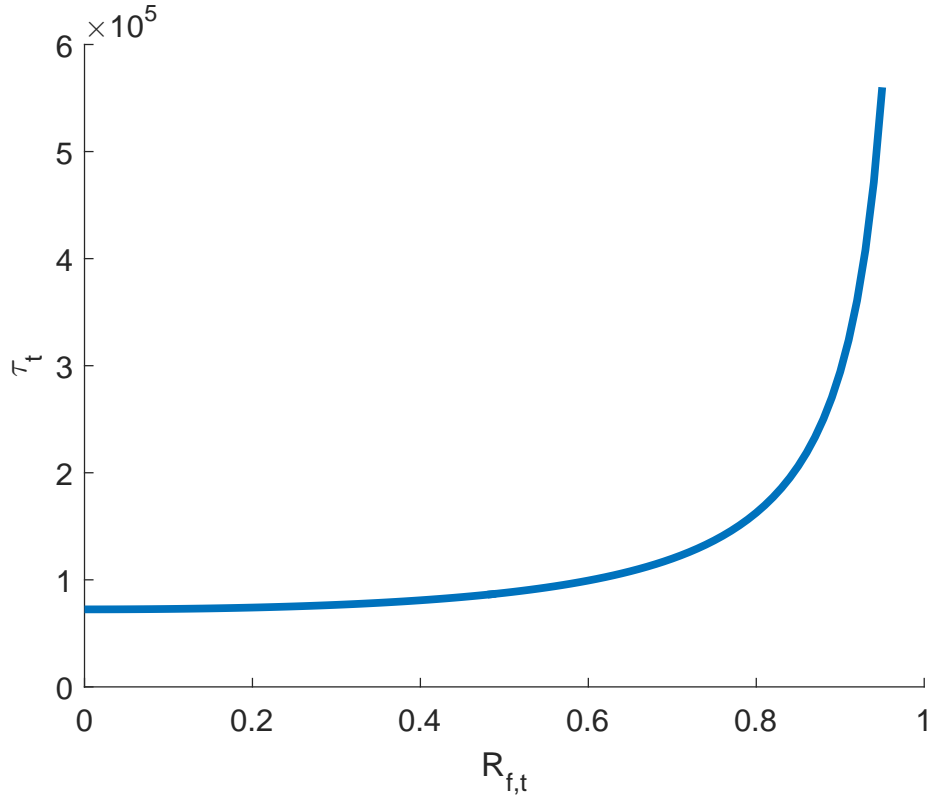Table 6: Capacity payment revenues for each generation technology (€/MW).

Figure 1: Values (€) for the penalty (τ_t) that ensures declared and actual reliability are the same for varying levels of reliability ($\alpha = 72402$).

Table 7 displays expected consumer costs from the energy market and expected consumer costs from the capacity market as well as their sum. These costs are calculated as follows:

$$\text{Expected consumer costs from energy market} = \sum_{f,t,p,s} pr_s \gamma_{p,s} gen_{f,t,p,s}, \qquad (20)$$

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Expected consumer costs from capacity market $= POT - \sum_{f,t,s} \left(pr_s(1 - B_{f,t,s})CAP_{f,t}\tau_t\hat{R}_{f,t} \right).$ (21)

Expected consumer costs from the capacity market are the value of the capacity pot less any penalties firms must pay when their generating units are offline. In addition, Table 7 also gives expected consumer surplus from the energy market (henceforth known as consumer surplus), which calculated is follows:

$$\text{Consumer surplus} = \frac{1}{2} \sum_{f,t,p,s} pr_s(\Gamma^0_{p,s} - \gamma_{p,s}) \times gen_{f,t,p,s},$$ (22)

where $\Gamma^0_{p,s}$ is the maximum price consumers are willing pay in period p and scenario s. This parameter is calculated by solving market clearing condition (6) with each value of $gen_{f,t,p,s}$ equal to zero. Table 7 shows that the mechanism reduces total expected consumer costs. As the reliability levels are the same, the probability associated with the different units being offline is unaffected by the mechanism. Consequently, equilibrium prices, generation levels and hence, consumer costs from the energy market and consumer surplus are all unaffected by the mechanism. In contrast, consumer costs from the capacity market are reduced as a result of the mechanism. This is because consumers recoup payments made by the generators through the penalty.

Table 8 shows that expected profits (calculated using objective function (1)) for each of the firms are reduced as a result of the mechanism. As before, this is a consequence of the penalty. Because the energy market is unaffected by mechanism in the case without refurbishment, expected profits from the energy market are the same with and without the mechanism.

	With mechanism	Without mechanism
Consumer costs from energy market	3,107	3,107
Consumer costs from capacity market	125	307
Total consumer costs	3,232	3,414
Consumer surplus	457,878	457,878

Table 7: Expected consumer costs and surplus (€ millions) with and without mechanism and without refurbishment

	With mechanism	Without mechanism
Firm 1	218,116	244,738
Firm 2	180,354	207,166
Firm 3	402,887	429,954
Firm 4	95,334	121,598

Table 8: Firms' expected profits (€ per MW installed) with and without mechanism and without refurbishment

4.2   With refurbishment

In this subsection we solve the model including the variable $refurb_{f,t}$, i.e., refurbishment of units is allowed. As above, we solve the model twice, including and omitting the mechanism (as described in

Section 2.1), and compare the outcomes. When the mechanism is included, each firm again declares their true reliability, and so the mechanism had the desired effect of truthfulness.

When refurbishment is allowed in the model, the extent to which firms take advantage of the opportunity to enhance the reliability of their units depends on whether they are incentivised to do so by their capacity payments. When the mechanism is included, all firms refurbish their units to the greatest extent possible, i.e., $R_{f,t} + refurb_{f,t} = 1 \, \forall f, t$. When the mechanism is omitted, the only incentive to refurbish a unit and thus increase its reliability stems from the opportunity to increase revenues in the energy market, by reducing the probability of being on forced outage during any given period. Under these circumstances, the integrated firm does not invest in refurbishment, but the other firms do; (see Table 9.

	Firm 1	Firm 2	Firm 3	Firm 4
Baseload	0.965	1		
Midmerit	0.955		1	
Peak	0.985			1

Table 9: Final reliability ($R_{f,t} + refurb_{f,t}$) without mechanism and with refurbishment.

This result is explained as follows. If the integrated firm refurbishes one of its three units, the probability associated with scenarios where that unit is unavailable deceases to zero. As a result, the firms' other two units are hindered from taking advantage of the high prices in these scenarios, and hence become less profitable. Consequently, the overall profit of the integrated firm is decreased if refurbishment takes place (for the inputs chosen here).

This result can be seen analytically in the $\frac{\partial pr_s}{\partial refurb_{f,t}}$ term in the stationary KKT condition associated with the variable $refurb_{f,t}$ (see equations (8) and (9)) which shows that the marginal benefit of refurbishing depends on the reliability of other units. Furthermore, when the mechanism is not included, there is consequently no penalty term, i.e., $\tau_t = 0 \, \forall t$ in equation (8). When the mechanism, and in particular the penalty, are included in capacity payments, the marginal cost of not refurbishing increases for the integrated firm (as the τ term in equation (8) becomes non-zero) and hence it is optimal for them to fully refurbish as shown in Table 9.

	With mechanism	Without mechanism
Consumer costs from energy market	2,438	2,569
Consumer costs from capacity market	307	307
Total consumer costs	2,745	2,876
Consumer surplus	458,277	458,140

Table 10: Expected consumer costs and surplus (€ millions) with and without mechanism and with refurbishment

Table 10 shows that the mechanism reduces expected total consumer costs. However, in contrast to Table 7, the reduction comes from the energy market and not the capacity market. In both cases, the expected consumer cost from the capacity market equals the value of the capacity pot. As each of the firms refurbish their units to 100% reliability with the mechanism in place, the probability associated with scenarios where one or more units is offline decreases to zero. Consequently, firms do not incur any
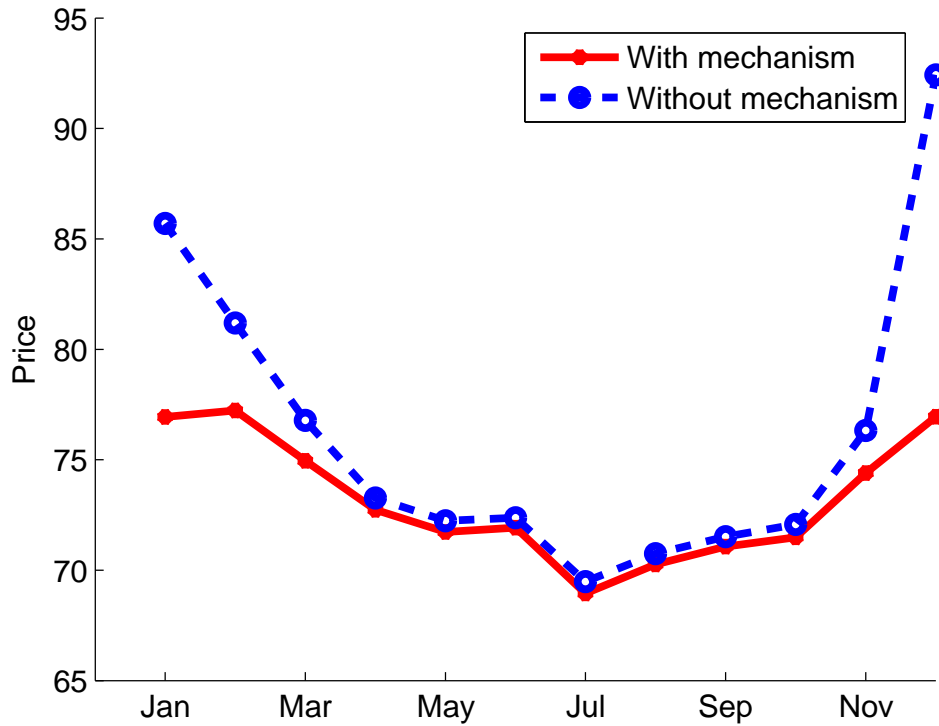
Figure 2: Monthly averaged expected prices (€/MW) with and without mechanism and with refurbishment

penalty.

Expected consumer costs from the energy market decrease with the mechanism in place. This is because of the decrease in equilibrium prices that consumer must pay; see Figure 2. Because Firm 1 does not refurbish its units when the mechanism is omitted, there are scenarios with probability greater than zero where some generating units are offline, which results in increased prices. The difference in prices is most prevalent in the high demand winter period from November to March. Table 10 also shows that consumer surplus is increased as a consequence of the decreased system prices. Similarly, Table 11 shows that the mechanism, and the resulting decreased prices, reduces the firms' expected profits. Again, as firms fully refurb their generating units when the mechanism is in place, the remuneration they receive from the capacity market is the same with and without the mechanism.

	With mechanism	Without mechanism
Firm 1	139,914	148,182
Firm 2	118,163	144,102
Firm 3	335,441	361,489
Firm 4	44,930	67,395

Table 11: Firms' expected profits (€ per MW installed) with and without mechanism and with refurbishment

4.3 Sensitivity on the costs of refurbishment

As mentioned in Section 3, real-life refurbishment costs are not publicly available. Consequently, we assumed refurbishment costs that are 10% of the cost of investing in a new unit. In this subsection, we perform a sensitivity analysis on this assumption. Table 12 displays the probability associated with the scenario where all generating units are online and fully available (henceforth known as the *Fully Available* scenario). This probability is proportional to the amount of refurbishment undertaken. If no refurbishment takes place, then the probability is equal to $\prod_{f,t} R_{f,t}$. If full refurbishment of all units takes place then the probability associated with the *Fully Available* scenario becomes equal to one.

Table 12 shows that the mechanism encourages refurbishment, regardless of the cost level, as the probability of the *Fully Available* scenario is always greater when the mechanism is in place. The only exception to this would be in a situation where refurbishment costs are extremely high and it is not optimal for any firm to refurbish any of the its units, regardless of the mechanism in place. Conversely, when the mechanism is omitted, Firm 1 never refurbishes its units, even when there is no refurbishment cost. The explanation for this is the same as that provided for the results in Section 4.2. When the refurbishment costs are 50% and 150% of investment costs it becomes too expensive for Firm 4 and Firm 2 to refurbish its units, respectively. In contrast, when the mechanism is in place all firms fully refurbish their units as long as refurbishment costs are less than 5.5 times investment costs. When refurbishment costs equal or exceed this unrealistic level, it becomes optimal for Firm 4 not to refurbish its unit and hence, the probability associated with the *Fully Available* scenario decreases.

Refurbishment costs as % of investment costs	With mechanism	Without mechanism
0%	1	0.908
10%	1	0.908
25%	1	0.908
50%	1	0.894
75%	1	0.894
100%	1	0.894
150%	1	0.863
550%	0.985	0.824

Table 12: Probability associated with all units being fully available ($\prod_{f,t} R_{f,t} + refurb_{f,t}$).

5 Discussion

The results indicate that it is possible to design a mechanism that divides capacity revenues among firms according to the reliability of their units, and that such a mechanism induces firms to truthfully reveal their reliability. This increases the information available to the regulator, incentivises refurbishment of units and increases the reliability of the units on the system.

For the case without refurbishment, equilibrium prices and levels of generation are the same

with and without the inclusion of the mechanism. This is because, for both designs, the reliability levels are fixed for each firm and incentives in the energy market are not affected by capacity payments. The total cost to consumers does, however, decrease when the mechanism is included, due to consumers recouping payments made by the generators through the penalty.

When refurbishment is allowed in the model, there are different levels of refurbishment depending on whether the mechanism is included. This can affect consumer outcomes through two channels. The first is that higher levels of reliability decrease the revenues recouped by consumers through the penalty mechanism. The second is that higher levels of reliability can lead to higher equilibrium levels of generation, which in turn lead to lower prices. For the input set chosen, the total reduction in consumer costs when the mechanism is included is 5.34%.

One obvious drawback that may occur to the reader is that our method assumes the regulator has prior knowledge of the reliability of the units, and uses this knowledge in determining the optimal levels of α and τ_t. However, this difficulty may be overcome by the regulator solving the model for many different levels of reliability, and thereby obtaining values for α and τ_t that will induce truth-telling for all initial levels of reliability. Given these values, the generators' profit-maximising strategies are those of truth-telling, regardless of their actual reliability. Sensitivity testing performed on this model induced truth-telling for any initial levels of reliability chosen, as the mechanism is designed to satisfy the requirement of incentive-compatibility.

In addition to the reduction in consumer costs, the mechanism may also lead to other regulatory savings not modelled here. This is because it likely poses a reduced regulatory burden compared to the current estimation and calculation of appropriate derating factors that regulators currently undertake. However a regulator may be reluctant to embrace a price-based capacity payment mechanism. Future work will examine the possibility of incorporating this mechanism into a quantity-based framework. For now, however, the inclusion of this truthtelling mechanism in a CRM should certainly be considered by regulatory authorities. Whether or not it is actually chosen will depend on the circumstances of the particular market, including costs, risks, over- or under-investment, the technology mix, the level of competition, the level of interconnection and policy aims such as renewable generation or security of supply concerns.

A regulator that calculates the appropriate amount of total remuneration may be reluctant to include a penalty term in the capacity remuneration mechanism, as that would lead to the total remuneration falling short of that required to induce the level of investment deemed necessary by the regulator. In this case, the regulator can recycle penalty revenues back to all firms by accounting for the penalty term in the market clearing condition, equation (7). This does not change equation (13), and so incentive compatibility is not affected and the firms will still truthfully reveal their reliability. However the total cost to consumers will be the same whether or not the mechanism is included. The only difference will be the new information gained and the increased refurbishment of units.

6 Conclusions

This paper designs a mechanism that incentivises electricity generation firms to truthfully reveal their level of reliability (or their forced outage rate) to the regulator while interactions between firms are modelled using a stochastic mixed complementarity problem. Such a mechanism has two advantages. The first is that regulators do not need to dedicate resources to determining appropriate derating factors to use for various generation units when awarding capacity revenues. The second is that there is now a transfer from relatively unreliable generators towards relatively reliable ones, providing an incentive to refurbish units and increase their reliability. This result was found to be robust to the refurbishment cost chosen. The increased system reliability translates into changes in the equilibrium quantities and prices of electricity that reduce consumer costs.

Further work will consider the impact of market entry and exit decisions and that of multiple time scales. A more sophisticated treatment of renewable generation may also be required if this method of declaration of reliability for variable generation such as wind and solar generation is to be included. The incorporation of risk aversion into the model may also lead to different outcomes depending on the capacity payment design chosen.

Acknowledgements

A KKT conditions for MCP

The Karush-Kuhn-Tucker optimality conditions for all firms are given below:

$$0 \leq \hat{R_{f,t}} \quad \perp \quad -CAP_{f,t}H(\alpha, \hat{R}_{f,t}) + \sum_s \left(pr_s(1 - B_{f,t,s})CAP_{f,t}\tau_t \right) + \lambda_{f,t}^2 \geq 0, \ \forall f, t, \quad (23)$$

$$0 \leq \lambda_{f,t,p,s}^1 \quad \perp \quad -gen_{f,t,p,s} + CAP_t \geq 0, \ \forall f, t, p, s, \quad (24)$$

$$0 \leq \lambda_{f,t}^2 \quad \perp \quad -\hat{R_{f,t}} + \overline{R_{f,t}} \geq 0, \ \forall f, t, \quad (25)$$

$$0 \leq \lambda_{f,t}^3 \quad \perp \quad -R_{f,t} - refurb_{f,t} + \overline{R_{f,t}} \geq 0, \ \forall f, t, \quad (26)$$

$$0 \leq gen_{f,t,p,s} \quad \perp \quad -pr_s B_{f,t,s}(\gamma_{p,s} - MC_{f,t}) + \lambda_{f,t,p,s}^1 \geq 0, \ \forall t, p, s. \quad (27)$$

Equations (23)-(27), along with market clearing conditions (6) and (7), represent the full mixed complementarity problem.

B Proof of uniqueness of generation levels

In this appendix, we prove, assuming different marginal costs for each generating unit, that the optimal generation decisions of each firm are unique.

Theorem 1. *If*

$$MC_{f,t} \neq MC_{\dot{f},\dot{t}}, \ \forall \ (f,t) \ \in (F \times T), \ \forall \ (\dot{f},\dot{t}) \ \in (F \times T), | \ (f,t) \neq (\dot{f},\dot{t}), \tag{28}$$

then $\exists! \ gen_{f,t,p,s} \ \forall \ f,t,p,s.$

Proof. **Proof by contradiction:**
Let the label 'A' represent one set of optimal solutions and the label 'B' represent another distinct set of optimal solutions.
Assume $\exists \ \hat{f} \in F, \ \hat{t} \in T, \ \hat{p} \in P$ and $\hat{s} \in S$ such that

$$gen_{\hat{f},\hat{t},\hat{p},\hat{s}}^{\mathrm{A}} \neq gen_{\hat{f},\hat{t},\hat{p},\hat{s}}^{\mathrm{B}}, \tag{29}$$

and, without loss of generality,

$$gen_{\hat{f},\hat{t},\hat{p},\hat{s}}^{\mathrm{A}} < gen_{\hat{f},\hat{t},\hat{p},\hat{s}}^{\mathrm{B}}. \tag{30}$$

Equation (30) implies that

$$gen_{\hat{f},\hat{t},\hat{p},\hat{s}}^{\mathrm{A}} \in [0, CAP_{\hat{f},\hat{t}}). \tag{31}$$

Therefore conditions (24) and (27) are satisfied if and only if

$$\gamma_{\hat{p},\hat{s}}^{\mathrm{A}} \leq MC_{\hat{f},\hat{t}}. \tag{32}$$

Similarly, Equation (30) also implies that

$$gen_{\hat{f},\hat{t},\hat{p},\hat{s}}^{\mathrm{B}} \in (0, CAP_{\hat{f},\hat{t}}]. \tag{33}$$

Therefore conditions (24) and (27) are satisfied if and only if

$$\gamma_{\hat{p},\hat{s}}^{\mathrm{B}} \geq MC_{\hat{f},\hat{t}}. \tag{34}$$

Combining equations (32) and (34) leads to

$$\gamma_{\hat{p},\hat{s}}^{\mathrm{B}} \geq MC_{\hat{f},\hat{t}} \geq \gamma_{\hat{p},\hat{s}}^{\mathrm{A}}. \tag{35}$$

If

$$gen_{\bar{f},\bar{t},\hat{p},\hat{s}}^{\mathrm{A}} \leq gen_{\bar{f},\bar{t},\hat{p},\hat{s}}^{\mathrm{B}} \ \forall \ \bar{f}, \bar{t} \in (F \times T) \setminus \{\hat{f}, \hat{t}\}, \tag{36}$$

then

$$\sum_{f,t} gen_{f,t,\hat{p},\hat{s}}^{\mathrm{A}} < \sum_{f,t} gen_{f,t,\hat{p},\hat{s}}^{\mathrm{B}}. \tag{37}$$

Using market clearing condition (6), this implies

$$\gamma^{\text{A}}_{\hat{p},\hat{s}} > \gamma^{\text{B}}_{\hat{p},\hat{s}}. \tag{38}$$

However, equation (38) contradicts equation (35).

Therefore $\exists \ \bar{f}, \bar{t} \in (F \times T) \setminus \{\hat{f}, \hat{t}\}$ such that

$$gen^{\text{A}}_{\bar{f},\bar{t},\hat{p},\hat{s}} > gen^{\text{B}}_{\bar{f},\bar{t},\hat{p},\hat{s}}. \tag{39}$$

Then following the same logic used in equations (30) - (35) leads to

$$\gamma^{\text{A}}_{\hat{p},\hat{s}} \geq MC_{\bar{f},\bar{t}} \geq \gamma^{\text{B}}_{\hat{p},\hat{s}}. \tag{40}$$

Assuming $MC_{\hat{f},\hat{t}} \neq MC_{\bar{f},\bar{t}}$, equation (40) contradicts equation (35). Consequently, we conclude that assuming

$$gen^{\text{A}}_{\hat{f},\hat{t},\hat{p},\hat{s}} \neq gen^{\text{B}}_{\hat{f},\hat{t},\hat{p},\hat{s}}, \tag{41}$$

leads to a contradiction and hence,

$$gen^{\text{A}}_{f,t,p,s} = gen^{\text{B}}_{f,t,p,s} \ \forall \ f \in F, t \in T, p \in P, s \in S, \tag{42}$$

i.e., the optimal generation decisions in the model proposed in this paper are unique. $\qquad \square$

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