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TECHNICAL DOCUMENTATION OF I3E MODEL

KELLY DE BRUIN AND AYKUT MERT YAKUT

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Kelly de Bruin
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THE AUTHORS

Kelly de Bruin is a Research Officer at the Economic and Social Research Institute (ESRI). Aykut Mert Yakut is a Postdoctoral Research Fellow at the Economic and Social Research Institute (ESRI).

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EXECUTIVE SUMMARY

This paper provides a technical description of the Ireland Environment, Energy and Economy (I3E) model. The I3E model is an intertemporal computable general equilibrium model with multiple firms, one representative household group, multiple commodities, government, enterprises, and rest of the world accounts. It describes the Irish economy in sectoral detail. This model includes a detailed description of energy inputs and concomitant greenhouse gas emissions and has been developed with the purpose of investigating the economic and environmental impacts of climate policies for Ireland.

1 Introduction

Climate change and the associated challenges are at the forefront of current policy debates. For example, all countries have committed to the Paris Climate agreement (with the exception of the US). In line with this, climate policy is becoming increasingly important domestically, as Ireland is obliged to decrease its emissions under the EU Commission's Climate and Energy Package. Ireland is required to deliver a 20% reduction in non-ETS greenhouse gas emissions by 2020 relative to 2005 levels, increasing to 30% in 2030. Designing appropriate energy policies is imperative to ensure a smooth and least-cost transition to a low-carbon economy. Furthermore, a better understanding of the implications of such a transition is necessary, especially on a sectoral level, to identify those sectors most hit by climate policies. In this sense data-based economic models can be very useful to advise policy, giving insights into the real-world effects of different policy measures. Such models can investigate the economic costs and sectoral implications of reaching the EU goals and which specific climate policies (e.g., carbon tax, energy tax or quotas) can be implemented to reach these goals, accounting for the economic behaviour of producers and consumers.

To analyse the impacts of energy-related policies on the Irish economy, a model with sectoral details and various energy inputs and technologies is needed. With this objective in mind, the Ireland Environment, Energy and Economy model (I3E model) is developed. This model focusses on the relationship between the economy, energy inputs and environmental impacts in the form of greenhouse gas (GHG) emissions. The I3E model is an intertemporal computable general equilibrium (CGE) model with multiple firms, one representative household group (RHG), multiple commodities, government, enterprises, and rest of the world accounts.

This paper serves as a reference document concerning the technical details of the I3E model. In what follows, the model structure and equations will be presented.

This document is structured as follows. In the next section, the sets, which represent the dimensions of the model, will be defined and clarified. Section 3 presents the equations dictating the behaviour of each agent in turn. Section 4 discusses the description of commodities in the model. In Section 5, the quantities are discussed and in Section 6, the prices. The energy part of the model is discussed in Section 7. Section 8 describes how economic growth is incorporated in I3E. Section 9 explains the savings–investment behaviour, and Section 10 gives the equilibrium conditions. The lists of activities and commodities can be found in Appendix A, the list of endogenous variables is presented in Appendix B, and finally, Appendix C provides the list of exogenous variables and parameters.

2 Sets

The variables and parameters used within the model are defined over various sets, which are collections of different variables or parameters with common characteristics. Each set has a unique definition and contains an elements that meet the defined criteria. The set logic is used to define equations over appropriate sets and allows us to exclude variables in the solution process. Each set and potential subsets are described in what follows.

- **Activities**

  All activities are assigned to the set a. This is the broadest set for domestic production activities. Along with several dimensions, activities are assigned to several subsets as follows

  - **Firm behaviour**

    An *Activity* determines its level of investment, i.e. investment by destination, by dividend maximisation as explained in Section 3.2.1. This procedure is based on Tobin's q, which is defined as the market value of a firm's capital stock divided by the replacement cost of it; in other words, the marginal value of capital. If a firm's total investment expenditure is high, the market value and the replacement cost of its capital stock are of a similar magnitude and Tobin's q approaches unity. If it is less than unity, the firm should not invest anymore. In the calibration process, some of the activities' Tobin's q values are calibrated as less than unity, and these activities are considered non-dividend maximisers. See Section 3.2.1 for details.

    Dividend maximising firms are assigned to the subset dm_a and non-dividend maximising firms to ndm_a.

  - **Production across commodities**

    Activities are allowed to produce multiple products, and the quantity of production is determined by revenue maximisation as explained in Section 3.2.3. However, some activities either produce a single commodity or their production of a single commodity exceeds 95% of their total production. For these activities, total output is a Leontief aggregate of the commodity (if the activity is the unique producer) or commodities (if production of a commodity exceeds the cut-off point). Otherwise, the activity's output is a constant elasticity of transformation (CET) aggregate of commodities that are produced by the activity in the base-year.

    Activities for which the output is defined by CET function are assigned to the subset $acet_a$ and activities where the output is defined by a Leontief function are assigned to the subset $aleo_a$.

  - **Energy demand**

    The sets $a1_a - a4_a$ are created to distinguish activities concerning the composition of their energy demands. The nested structures of production are explained in Section 3.2.2. The list of activities is provided in Appendix A.

- **Commodities**

  All commodities are assigned to the set c which includes the following subsets:

  - **Origin**

    The set cp_c contains commodities produced by domestic producers and sold in the domestic market. This set is defined over commodities for which the cells of activity rows and commodity columns on the Social Accounting Matrix (SAM) are positive. The set cpn_c covers *non-domestically produced commodities*

  - **Exports**

    The set ce_c is defined over export commodities, i.e. commodities for which the cells of commodity rows and the rest of the world columns on the SAM are positive. The set cen_c is the complementary set that covers *non-exported commodities*.

  - **Imports**

    The set cm_c contains import commodities, i.e. commodities for which the cells of the rest of the world rows and commodity columns on the SAM are positive. The set cmn_c covers non-imported commodities.

  - **Homogeneity**

    As explained in Section 3.2.3, activities are allowed to produce multiple products; however, there are a few commodities produced by a single activity, for instance, public administration. In this case, the commodity is homogeneous and assigned to the set ch_c. This set covers commodities for which the elasticity of substitution for the domestically produced commodity, $elas\_qxcs_c$, is zero. The set chn_c is the complementary set that covers *non-homogeneous commodities*, i.e. commodities that are produced by at least two distinct activities and $elas\_qxcs_c$ is positive.

  - **Composite goods**

    The model includes detailed nested structures of private composite consumption and production. The primary objective of creating such detailed nested structures is to reflect the compositions of households' and activities' demands for energy commodities as accurately as possible. In this sense, the way the nested structure is defined determines the substitutability between inputs to production and between goods for consumption. If goods are nested together, this represents higher substitutability between these goods compared to others. A Leontief relationship assumes no substitutability whereas a constant elasticity of substitution (CES) relationship assumes a substitution possibility. The values of the elasticity of substitution parameter, σ, for different elasticity relations are chosen in order to reflect the low and high substitution possibilities among the commodities based on expert judgement.

- **Factors of production**

  The set f contains factors of production. In this version, it has two elements, Labour and Capital.

- **Margins**
  The set m contains trade and transportation margins. Effectively, it has only one element since the national accounts give only the sum of trade and transportation margins and do not have distinguished data on each separately.

- **Households**
  The set hh is the broadest set for households. In this version, there is only one RHG, and the set has only one element. In the next versions, several subsets of the set hh can be created by considering, for instance, the area of residence (urban vs. rural), level of savings, etc.

3 Agents

This section will discuss the behaviour of each agent in the model in turn. The agents in the I3E model consist of households, activities, commodities, enterprises, and government,

3.1 Households

In this version, it is assumed that there is one RHG that solves the following intertemporal utility maximisation problem where the utility function is in the form of Constant Relative Risk Aversion (CRRA):

$$\begin{gathered} \max_{CC_t^{hh}} \sum_{t=1}^{\infty} \left(\frac{1+grw_t}{1+\rho^{hh}} \right)^t \frac{(CC_t^{hh})^{1-\theta^{hh}}}{1-\theta^{hh}} \\ s.t \\ SAV_t^{hh} + PCC_t^{hh}\, CC_t^{hh} \leq WINC_t^{hh} + CINC_t^{hh} + TR_t^{hh} + \overline{NFI}_t^{hh}\, ER_t + RCI_t^{hh} - WTAXS_t^{hh} \end{gathered} \tag{1}$$

where grw_t is the economic growth rate in the period t, ρ^{hh} is time preference rate, θ^{hh} is intertemporal elasticity of substitution, $WINC_t^{hh}$, $CINC_t^{hh}$, TR_t^{hh}, $\overline{NFI}_t^{hh}$, and RCI_t^{hh} are wage income, capital income (distributed dividends of enterprises), transfers from the government, net factor incomes from abroad (fixed in real terms), and revenue recycling income from the government (based on the total carbon tax collection), respectively, ER_t is exchange rate, $WTAXS_t^{hh}$ is wage income tax payments to the government, SAV_t^{hh} is savings, CC_t^{hh} is household-specific composite consumption and PCC_t^{hh} is its price. The RHS of the budget constraint represents household disposable income, INC_t^{hh}. The components of disposable income are as follows.[1]

$$WINC_t^{hh} = \sum_a W_t^{hh}\, WFDIST_{a,t}^{hh}\, LD_{a,t}^{hh} \tag{2}$$

$$WTAXS_t^{hh} = wtax^{hh}\, WINC_t^{hh} \tag{3}$$

[1] For the sake of space saving, explanations of variables and parameters will be given in the relevant subsections. You can see Section 3.2.1 and also Appendix B.

$$CINC_t^{hh} \; = \; shr\_cinc^{hh} \, DISDIV_t \tag{4}$$

In this set-up, household savings are determined endogenously by

$$SAV_t^{hh} \; = \; INC_t^{hh} \; - \; PCC_t^{hh} \, CC_t^{hh} \tag{5}$$

Household *hh* chooses the level of composite consumption to maximise her the present discounted value of intertemporal utility. The first-order condition (FOC) of this problem, equation (6), is the well-known consumption Euler equation and solves for the sequence of composite consumption where r_t is interest rate.

$$\frac{CC_{t+1}^{hh}}{CC_t^{hh}} \; = \; \left[(1+grw_t) \, \frac{1+r_t}{1+\rho^{hh}} \quad \frac{PCC_t^{hh}}{PCC_{t+1}^{hh}} \right]^{\frac{1}{\theta^{hh}}} \tag{6}$$

In the second stage, household *hh* disaggregates her composite consumption across commodities by maximising her intratemporal utility. The nested structure of household composite consumption is depicted by Figure 1.

Several composite commodities are included in the consumption nest; these are either constant elasticity of substitution (CES) or Leontief aggregates of directly observed commodities and/or other composite commodities. For the CES aggregates of the composite commodities, the values of elasticity of substitution parameters, σ in Figure 1, are chosen in a range of 1.2 and 2 in order to reflect the different substitution possibilities across different commodities that constitute the composite commodity.

Household composite consumption, CC, is assumed to be a CES aggregate of composite commodities of Transportation (TRP), Residential Energy (REN), Nourishment (NTR), Services (SER), and other commodities (OTC). As described above this reflects that different services goods are easier to substitute with each other than for example substituting services goods with nourishment goods. The logic here is that consumers are more likely to substitute food products with, e.g., agricultural products if prices of food products increase than to increase their consumption of services as food prices increase.

The composite commodity TRP is a Leontief aggregate of land, air, and water transportation commodities where land transportation (LND) is also a Leontief aggregate of public and private transportation commodities. The choice of a Leontief relationship here is warranted by the low level of substitutability between transport types; a consumer will not substitute their daily car commute with air or water transport due to increases in petrol prices. It should be noted that the original land transportation commodity (LTS with NACE Code 49) covers the public transportation demand of households. In I3E we assign a share of household demand energy commodities including gasoline, diesel, and electricity for private transportation purposes. We refer to private transportation energy use as the composite commodity LND, which is assumed to be a Leontief aggregate of that energy commodities.[2] The REN is disaggregated into lighting

2 According to the energy balances, the private consumption of liquid petroleum gas is devoted both to private transportation and to residential heating. Since the former is a quite tiny portion of the total demand, it is assumed to be zero and liquid petroleum gas is assumed to be a part of the residential heating demand.

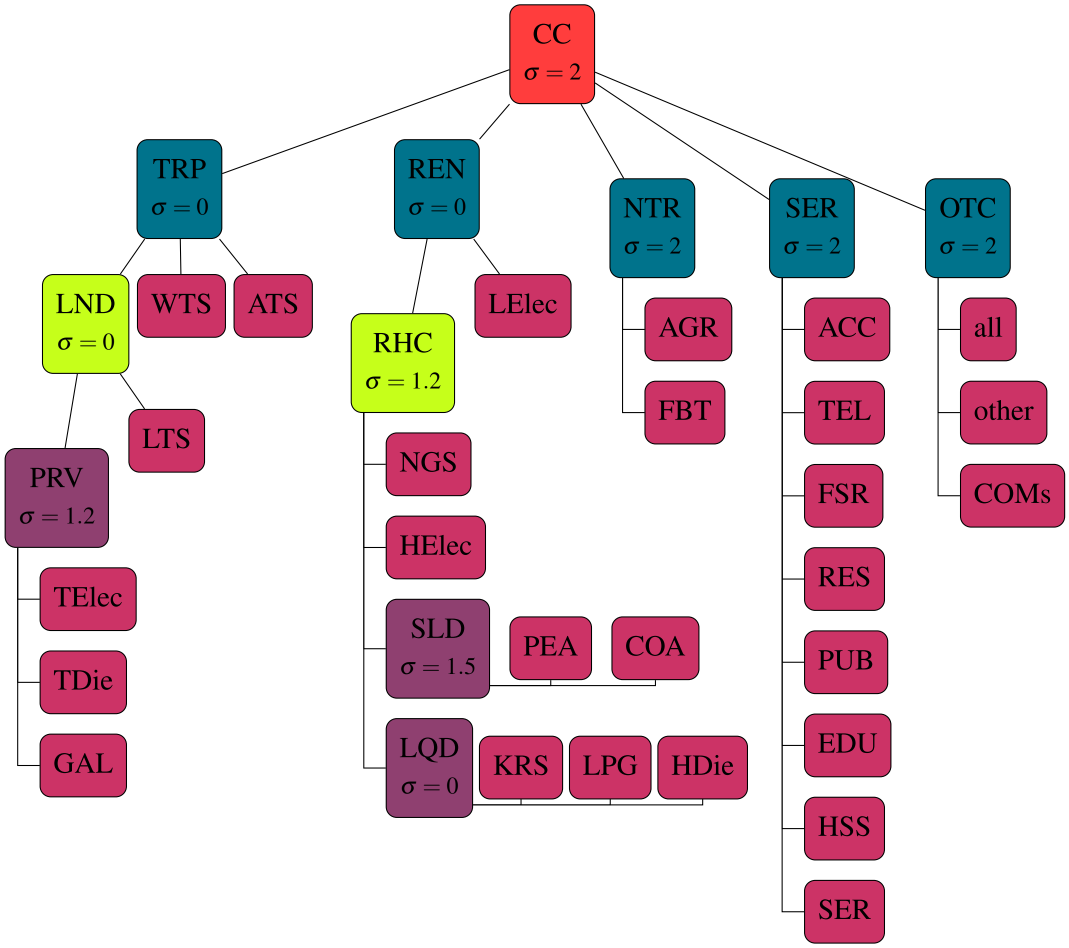

Figure 1: Nested Structure of Consumption

electricity and residential heating which is further disaggregated into natural gas supply, solid fuel, heating electricity, and liquid fuel. Moreover, solid (liquid) fuel is a CES (Leontief) aggregate of peat and coal (kerosene, liquid petroleum gas, and diesel for heating purposes). The total electricity consumption of households, the commodity *ELC*, is known from the SAM, and it is disaggregated into electricity demand by transportation, lighting, and heating purposes by using the data provided by SEAI (2013, Table 19). Similarly, total private consumption of diesel is disaggregated into diesel demand for transportation and heating by using the energy balances. The composite commodity *NTR* is a CES aggregate of the commodities agriculture and food, beverage, and tobacco while the composite commodity *SER* is a CES aggregate of several service commodities. The composite commodity *OTC* is a CES aggregate of all remaining commodities that are demanded by households.

3.2 Activities

3.2.1 Investment by Destination

The term "investment by destination" refers to investment expenditures of firms / sectors. In the model we distinguish between dividend maximisers and non-dividend maximisers; we discuss each in turn. In a fully intertemporal setting, the investment decision can be endogenised via a dividend maximisation problem where a firm maximises the present discounted value of its dividend stream, $DIV_{dm,t}$ (i.e. the present value of firm, $V_{dm,t}$) by choosing the level of physical investment, $PSI_{dm,t}$, and levels of factors of production (i.e. capital, $K_{dm,t}$, and labour, $LD^{hh}_{dm,t}$).

$$\max_{PSI_{dm,t}, K_{dm,t}, LD^{hh}_{dm,t}} V_{dm,t} = \sum_{t=1}^{\infty} \left(\frac{1+grw_t}{1+r_t} \right)^t DIV_{dm,t} \quad s.t \tag{7}$$

$$K_{dm,t+1}\,(1 + grw_t) = (1-\delta_{dm})\,K_{dm,t} + PSI_{dm,t} \tag{8}$$

where δ_{dm} is the depreciation rate. Activity-specific capital stock ($K_{dm,t}$) evolves according to capital accumulation function, (8). The Lagrange multiplier of this maximisation problem, $q_{dm,t}$, which is constrained by capital accumulation function is the well-known Tobin's q[3], i.e. the marginal value of capital:

$$DIV_{dm,t} = (1-corptax)\,WK_{dm,t}\,K_{dm,t} - INV_{dm,t} \tag{9}$$

$$INV_{dm,t} = PPSI_t\,PSI_{dm,t} + PVA_{dm,t}\,ADJ_{dm,t} \tag{10}$$

$$ADJ_{dm,t} = \phi_{dm} \frac{PSI^2_{dm,t}}{K_{dm,t}} \tag{11}$$

where $WK_{dm,t}$ is price of capital and $PPSI_t$ is price of investment. Sectoral dividend is equal to net-of-corporate tax sectoral profit minus total investment expenditures, $INV_{dm,t}$, which includes the cost of new investment equipments and the adjustment cost. Adjustment cost is an increasing and convex function of investment; for a given level of sectoral capital stock, the cost of installing new capital equipment will be greater. Adjustment cost, $ADJ_{dm,t}$, is measured by the price of the value added, $PVA_{dm,t}$, because it is assumed that installation of new capital requires the resources of the firm, which leads to interruption of production process and thus losses of output. Sectoral profit is equal to the sectoral value added minus the total labour cost.

$$WK_{dm,t}\,K_{dm,t} = PVA_{dm,t}\,VA_{dm,t} - \sum_{hh} \widetilde{W}^{hh}_t\,BX_t\,LD^{hh}_{dm,t}\,WFDIST^{hh}_{dm,t} \tag{12}$$

[3] Tobin's q is the ratio of the market value of existing capital to its replacement cost. See Hayashi (1982) for further discussion.

where $\widetilde{W}_t^{hh}$ is the effective wage rate, BX_t is the level of labour productivity, $LD_{dm,t}^{hh}$ is labour demand of activity dm, and $WFDIST_{dm,t}^{hh}$ is the ratio of factor payments to labour hh from activity dm. We need the last term since there is only one type of labour and its price that is determined by equilibrium conditions in the labour market is common to all activities. However, since productivity of each type of labour is different, payments to labour cannot be the same across sectors. The term $WFDIST_{dm,t}^{hh}$ captures the distance between average and sectoral payments to labour.

Given technological change, the wage per effective labour, $\widetilde{W}_t$, is constant while the wage per labour, W_t – the wage received by labour, grows at the rate of technological change. For the given level of labour supply, labour is paid more in every period due to increases in its productivity. Hence, the following relation holds.

$$W_t^{hh} \;=\; \widetilde{W}_t^{hh}\, BX_t$$

The real value added, $VA_{dm,t}$, is assumed to be a CES aggregate of sectoral capital stock and sectoral labour input:

$$VA_{dm,t} \;= sft\_va_{dm,t}[shr\_cva_{dm}\, K_{dm,t}^{-rho\_va_{dm}} \;+\; (1-shr\_cva_{dm}) \sum_{hh}(BX_t\, LD_{dm,t}^{hh})^{-rho\_va_{dm}}]^{-\frac{1}{rho\_va_{dm}}} \tag{13}$$

where shr_cva_{dm} is the share parameter of capital in real value added of activity dm, sft_va_{dm} is the shift parameter, and rho_va_{dm} is the exponent parameter and obeys $elas\_va_{dm} = 1/(1+rho\_va_{dm})$. In the case of labour augmented technological growth, activities choose the level of "efficient labour", multiplication of the labour demand and the level of productivity.

The FOCs of this dividend maximisation problem w.r.t. the levels of physical investment, capital stock, and effective labour, respectively, are as follows.

$$q_{dm,t} \;=\; PPSI_t \;+\; 2\, PVA_{dm,t}\, \frac{ADJ_{dm,t}}{PSI_{dm,t}} \tag{14}$$

$$\begin{aligned} q_{dm,t}\,(1+r_t)\,(1\;+\;grw_t) \;=\; q_{dm,t+1}\,(1-\delta_{dm}) \;+\; PVA_{dm,t+1}\frac{ADJ_{dm,t+1}}{K_{dm,t+1}} \\ +\,(1-corptax)\, WK_{dm,t+1} \end{aligned} \tag{15}$$

$$BX_t\, LD_{dm,t}^{hh} \;=\; \left[\frac{PVA_{dm,t}\,(1-shr\_cva_{dm})\,BX_t}{W_t^{hh}\, WFDIST_{dm,t}^{hh}}\right]^{elas\_va_{dm}} sft\_va_{dm}^{elas\_va_{dm}-1}\, VA_{dm,t} \tag{16}$$

For non-dividend maximisers, the investment (by destination) expenditure in period t of an activity is a fixed fraction of its total profits in period t as follows

$$INV_{ndm,t} \;=\; shr\_invdes_{ndm}\, WK_{ndm,t}\, K_{ndm,t} \tag{17}$$

where shr_invdes_{ndm} is a fixed parameter.

The level of investment expenditures determines the level of physical investment demand of a firm ($PSI_{ndm,t}$) which, in turn, determines the level of sectoral capital stock ($K_{ndm,t}$).

$$INV_{ndm,t} \; = \; PPSI_t \; PSI_{ndm,t} \tag{18}$$

$$K_{ndm,t+1} \; (1 \; + \; grw_t) = \; (1-\delta_{ndm}) \; K_{ndm,t} \; + PSI_{ndm,t} \tag{19}$$

The depreciation rate of these firms is arbitrarily set to be equal to 0.05.

The firms in this subset minimise the cost of production by choosing labour input, for the given level of sectoral capital stock. The real value added, $VA_{ndm,t}$, is assumed to be a CES aggregate of sectoral capital stock and sectoral labour input:

$$VA_{ndm,t} \; = sft\_va_{ndm,t} [shr\_cva_{ndm} \; K_{ndm,t}^{-rho\_va_{ndm}} \; + \; (1 - shr\_cva_{ndm}) \; \sum_{hh} (BX_t \; LD_{ndm,t}^{hh})^{-rho\_va_{ndm}}]^{-\frac{1}{rho\_va_{ndm}}} \tag{20}$$

$$BX_t \; LD_{ndm,t}^{hh} \; = \; \left[\frac{PVA_{ndm,t} \; (1 - shr\_cva_{ndm}) \; BX_t}{W_t^{hh} \; WFDIST_{ndm,t}^{hh}} \right]^{elas\_va_{ndm}} \; sft\_va_{ndm}^{elas\_va_{ndm}-1} \; VA_{ndm,t} \tag{21}$$

The FOC of the minimisation problem implies the optimal level of labour demand, equation (21).

3.2.2 Production

The total value of production, the LHS of equation (22), is equal to payments to factors of production, i.e. value-added, production taxes paid to the government and the total cost of intermediate inputs.

$$PX_{a,t} \; QX_{a,t} \; = \; PVA_{a,t} \; VA_{a,t} + \; PRODTAXS_{a,t} \; + PCIN_{a,t} \; CCIN_{a,t} \tag{22}$$

where $QX_{a,t}$ is an activity's total production, $VA_{a,t}$ is real value-added $CCIN_a$ is composite intermediate input demand, $PX_{a,t}$, $PVA_{a,t}$, and $PCIN_{a,t}$ are their prices, respectively, and $PRODTAXS_{a,t}$ is the value of production tax.

The production of activities in the model is represented by a nested structure as shown in Figure 2. To reflect the differences in energy demand compositions, activities are assigned to three distinct groups at two different layers of the nested production structure. The elasticity of substitution parameter, σ, takes the value of 0 for activities which have a quite strict composition of energy demand. On the other hand, its value is either 1.1 or 1.3 for activities whose composition of the energy demand is not dominated by a specific energy commodity.

The activities are assumed to produce a composite product QX which is an aggregate of value added (VA), business energy (BEN), and other inputs (OTI). The value added is a CES aggregate of factors of production which are capital and labour, and the commodity OTI is an aggregate of all intermediate inputs except the energy commodities. For all activities, except the electricity production, the commodity BEN is assumed to be an aggregate of energy electricity ($EElec$), fuel (FUE) and business heating

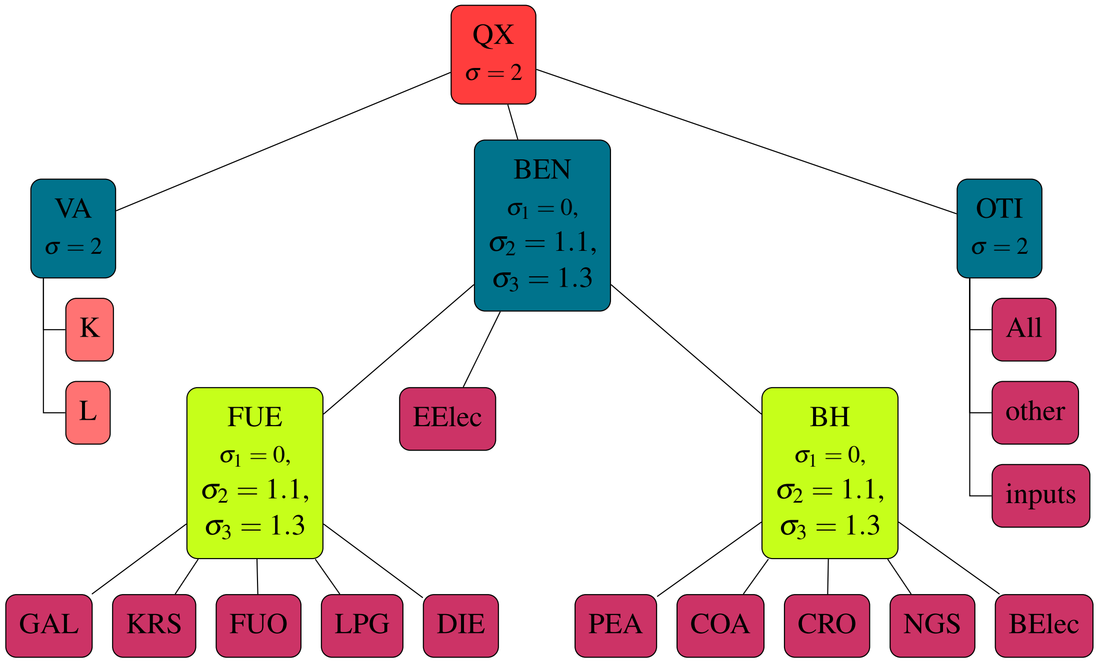

Figure 2: Nested Structure of Production, except Electricity Production

(BH). The composite commodity BH is an aggregate of liquid and solid fuels including coal, peat, crude oil, natural gas supply, and business electricity for heating purposes. On the other hand, the composite commodity FUE is an aggregate of gasoline, kerosene, fuel oil, liquid petroleum gas, and diesel. The electricity demand of activities, except the electricity production, is disaggregated across demands for energy purposes and heating/combustion purposes.[4] The nested structure of production of all activities except electricity production is depicted by Figure 2.

The electricity production activity has a unique production technology and energy demand composition. The activity's business energy, BEN, is assumed to be a CES aggregate of electricity, natural gas, and other energy (OTE) which is a CES aggregate of all remaining energy commodities, as shown in Figure 3. The value of σ is 1.1 for the commodity of BEN while it is 1.3 for the OTE because the electricity production has more flexibility to substitute between liquid and solid fuels than natural gas and electricity's itself.

3.2.3 Multi-product Determination

Each activity can produce multiple products and the levels of output for each product are determined by a revenue maximisation problem. Gross production is a constant elasticity of transformation (CET) aggregate of the products produced by the activity if the activity is not the producer of a single commodity

[4] At this stage, the disaggregation is done by arbitrarily assuming that 40% (60%) of the total sectoral electricity is used for heating/combustion (energy).

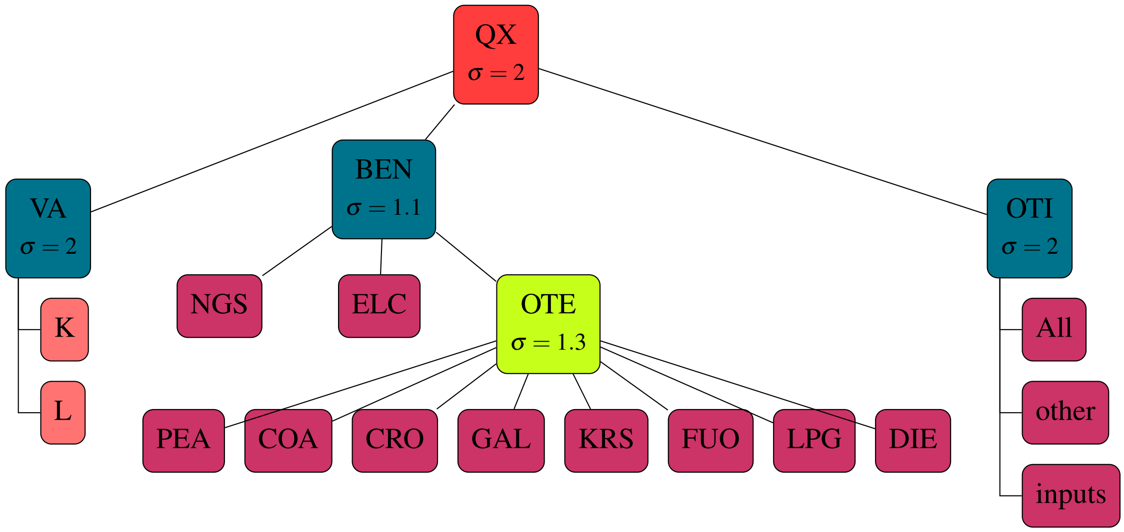

Figure 3: Nested Structure of Electricity Production

and the level of production of a single commodity doe not exceed 95% of total production of the activity.

$$QX_{acet,t} \;=\; sft\_qxac_{acet} \left[\sum_{c} shr\_qxac_{acet,c} \; QXAC_{acet,c,t}^{rho\_qxac_{acet}} \right]^{\frac{1}{rho\_qxac_{acet}}} \tag{23}$$

while total value of output has to be equal to the sum of values of each commodity produced:

$$PX_{acet,t} \; QX_{acet,t} \;=\; \sum_{c} PXAC_{acet,c,t} \; QXAC_{acet,c,t} \tag{24}$$

where $QXAC_{acet,c,t}$ is the volume of production of commodity c by activity $acet$ and $PXAC_{acet,c,t}$ is its price, sft_qxac_{acet} is the shift parameter and shr_qxac_{acet} is the share parameter of commodity c, $elas\_qxac_{acet}$ is the elasticity of transformation and $rho\_qxac_{acet} \;=\; 1/elas\_qxac_{acet}+1$ holds for $elas\_qxac_{acet}$ $\in (0,\infty]$. Each activity maximises (24) subject to (23) and the following FOC determines the level of production:

$$QXAC_{acet,c,t}|_{shr\_qxac_{acet,c}} = \left[\frac{PXAC_{acet,c,t}}{PX_{acet,t} \; shr\_qxac_{acet,c} \; sft\_qxac_{acet}^{rho\_qxac_{acet}}} \right]^{elas\_qxac_{acet}} QX_{acet,t} \tag{25}$$

As the price of commodity c produced by activity $acet$ increases, production increases. Note that equation (25) is defined conditional on a positive production share of commodity c in total output of activity $acet$ on the SAM. This implies that an activity is not able to produce commodities which are not produced in the base case.

For activities which produce a single commodity or the level of production of a single commodity exceeds 95% of total production of the activity, the level of production of each commodity is a fixed share of total output of the activity.

$$QXAC_{aleo,c,t}|_{shr\_qxac_{aleo,c}} = shr\_qxac_{aleo,c}\ QX_{acet,t} \tag{26}$$

where $shr\_qxac_{aleo,c}$ is the share parameter and obeys $\sum_c shr\_qxac_{aleo,c} = 1$. The parameter is equal to 1 for a commodity c if the activity is the unique producer.

3.3 Enterprises

The model economy includes an "enterprises" account. The representative enterprise is assumed to be the owner of all firms. Such an assumption helps to simplify some details of the model and also avoids the need for detailed data which is not available. The enterprises account collects all gross sectoral profits and receives transfers from the government $\overline{GTRENT}_t$ which is fixed in nominal terms, and pays corporate tax to the government, $CORPTAXS_t$. The remaining amount is either saved by the enterprise account, $ESAV_t$, or paid to households as dividend payments, $DISDIV_t$.

$$DISDIV_t = \sum_a WK_{a,t}\ K_{a,t} + \overline{GTRENT_t} - CORPTAXS_t - ESAV_t \tag{27}$$

It is assumed that enterprises savings is a fixed fraction, shr_sav, of net-of-tax profit receipts of the account:

$$ESAV_t = shr\_sav\ (\sum_a WK_{a,t}\ K_{a,t} - CORPTAXS_t) \tag{28}$$

3.4 Government

In this model economy, the government collects direct taxes on labour incomes and on sectoral profits (i.e. corporate tax) (eq. 33), indirect taxes on sales of commodities (eq. 29), carbon tax on energy commodities (eq. 30), export tax on export of commodities (eq. 32), and production tax on production activities (eq. 31). She allocates her total revenues to consumption and to transfers to households, which is fixed in real terms. The government redirects the total carbon tax collections to households in the form of lump-sum transfers. The total government transfers to households and the total carbon tax collection are distributed across households based on the fixed fractions. However, the model algorithm is flexible to evaluate some exclusion rules for the latter income item, e.g. the government may consider excluding the richest households from the revenue cycling scheme.

Sales tax on commodity c is imposed on total domestic supply of the commodity, which is the sum of import and domestic production:

$$SALTAX_{c,t} = stax_c\ (PM_{c,t}\ QM_{c,t} + PD_{c,t}\ QD_{c,t}) \tag{29}$$

where $stax_c$ is ad-valorem sales tax rate on commodity c. Carbon tax is collected on domestic consumption of energy commodities at fixed price of per-tonne equivalent of carbon, $\overline{PCAR}_{c,t}$, which is exogenously determined by the government,

$$CTAXS_{c,t} \;=\; alpha_c\,\overline{PCAR}_{c,t}\,carcon_c\,QS_{c,t} \tag{30}$$

where $carcon_c$ is the carbon content of commodity c which is calibrated by dividing total emission of commodity c by its total consumption for the year of 2014.[5] The parameter $alpha_c$ equates the carbon tax collection of the government on commodity c for the given levels of variables and parameters.

Activities pay production tax on the value of their total production

$$PRODTAXS_{a,t} \;=\; prodtax_a\,PX_{a,t}\,QX_{a,t} \tag{31}$$

where $prodtax_a$ is ad-valorem tax rate on activity a.

Although the domestic sale of electricity is exempted from the carbon tax, its export is subject to a carbon tax. Therefore, the total carbon tax collection of the government includes the export tax on electricity. Since the inclusion of carbon tax affects the domestic purchaser prices of commodities, the carbon tax payments on the consumption of electricity are set to zero. This amount is transferred to the export tax account on the SAM.

$$EXPTAXS_{c,t} \;=\; exptax_c\,PWE_{c,t}\,QE_{c,t}\,ER_t \tag{32}$$

where $exptax_c$ is ad-valorem tax rate on the export of commodity c. The $exptax_c$ is positive for only electricity and zero for the other commodities.

Corporate tax is paid at a uniform rate across activities, $corptax$, by the enterprise account on total profits of activities

$$CORPTAXS_t \;=\; corptax\sum_a WK_{a,t}\,K_{a,t} \tag{33}$$

Total revenue of the government is equal to summation of the four items above.

$$\begin{aligned} GOVREV_t \;=\; &\sum_a PRODTAXS_{a,t} \;+\; \sum_c (SALTAX_{c,t} \;+\; CTAXS_{c,t} \;+\; EXPTAXS_{c,t}) \\ &+\; \sum_{hh} wtax^{hh}\,WINC_t^{hh} \;+\; CORPTAXS_t \end{aligned} \tag{34}$$

The total government consumption of commodities has an autonomous part which is fixed in nominal terms, $\overline{GOVCONA_t}$ and an induced part which is a positive function of the current period's nominal gross domestic product

$$GOVCON_t \;=\; \overline{GOVCONA_t} \;+\; mps\,GDP_t \tag{35}$$

where the parameter mps stands for the government's marginal propensity to spend. A simple ordinary least square (OLS) analysis shows that the value of mps is 0.05 when the first difference of the govern-

5 See Section 7 for details.

ment consumption is regressed on the first difference of GDP. If the level of government consumption is regressed on the level of GDP, the parameter *mps* becomes 0.106. To lower the sensitivity of the induced government expenditures, the first differences are used. The level of government consumption of each commodity is a simply fixed fraction, *shr_gc*, of her total consumption

$$PQD_{c,t}\ CG_{c,t}\ =\ shr\_gc\ GOVCON_t \tag{36}$$

where $CG_{c,t}$ is the government consumption demand for commodity c.

The difference between total revenues and expenditures of the government is public savings:

$$\begin{aligned} GSAV_t\ =\ & GOVREV_t\ -\ GOVCON_t\ -\ \overline{GTRHH_t}\ CPI_t\ - \\ & \overline{GTRENT_t}\ -\ GFINT_t\ ER_t\ -\sum_{hh} RCI_t^{hh} \end{aligned} \tag{37}$$

where $\overline{GTRHH_t}$ is the real value of the total government transfers to households which is distributed across households by fixed parameters, shr_gtrhh^{hh}

$$TR_t^{hh}\ =\ shr\_gtrhh^{hh}\ \overline{GTRHH_t}\ CPI_t \tag{38}$$

The last term on the RHS is the total cost of revenue cycling to households based on the total carbon tax collections and $GFINT_t$ is the interest payments of the government to the rest of the world over the outstanding foreign debt stock, $GFDS_t$, which grows by the level of government savings:

$$GFINT_t\ =\ r_t\ GFDS_t \tag{39}$$

$$GFDS_{t+1}\ =\ GFDS_t\ -GSAV_t\ /ER_t \tag{40}$$

4 Commodities

Commodity supply consists of the production from domestic producers and imports. Commodity demand consists of demand for intermediate usage, household (private) demand, government demand, investment demand, trade and transportation margins demand, and export demand.

Data concerning supply and demand of commodities are retrieved from the Supply and Use Tables (SUTs), where the commodity production per activity and the various demands are given. The SUTs, however, only provide the value of exports by commodities but not which activities' outputs are exported. For this reason, we define a commodity, namely $QXC_{c,t}$, that is a CES composite of domestically produced commodities. Its volume and value are as follows:

$$QXC_{c,t}|_{cp_c\ and\ chn_c} = sft\_qxcs_c \left[\sum_a shr\_qxcs_{a,c}\ QXAC_{a,c,t}^{-rho\_qxcs_c}\right]^{\frac{-1}{rho\_qxcs_c}} \tag{41}$$

$$PXC_{c,t}\ QXC_{c,t}\ =\ \sum_{a} PXAC_{a,c,t}\ QXAC_{a,c,t} \tag{42}$$

where $PXC_{c,t}$ is price of composite commodity $QXC_{c,t}$. Equation (41) is defined over the set cp_c to control for positive values of $QXAC_{a,c,t}$ and over the set chn_c to control for non-homogeneous commodities. For commodities that are not produced domestically, $QXC_{c,t}|_{cpn_c} = 0$ holds while for homogeneous commodities, the volume of the composite commodity $QXC_{c,t}$ is equal to production of activity a

$$QXC_{c,t}|_{ch_c} = \sum_{a} QXAC_{a,c,t} \tag{43}$$

The commodity $QXC_{c,t}$ is either sold in the domestic market or exported. From the demand side, demand is assumed to be a CET aggregate of domestic sales and exports. The volumes of these components are determined by FOCs of a profit maximisation problem in which

$$QXC_{c,t}\ PXC_{c,t}|_{cp_c} =\ PD_{c,t}\ QD_{c,t}\ +\ PE_{c,t}\ QE_{c,t} \tag{44}$$

is maximised subject to

$$QXC_{c,t}|_{ce_c} =\ sft\_qxcd_c \left[shr\_qxcd_c\ QD_{c,t}^{rho\_qxcd_c} + (1 - shr\_qxcd_c)\ QE_{c,t}^{rho\_qxcd_c}\right]^{\frac{1}{rho\_qxcd_c}} \tag{45}$$

where $QD_{c,t}$ is volume of sales in the domestic market and $PD_{c,t}$ is its producer price, $QE_{c,t}$ is volume of exports and $PE_{c,t}$ is its price in domestic currency. The price of composite commodity $(PXC_{c,t})$ is defined over the set of domestically produced commodities because supply of the composite commodity $(QXC_{c,t})$ comes from the domestic producers. If it is not produced by domestic producers, its price is equal to zero, $PXC|_{cpn_c} = 0$. The FOC of this problem yields

$$QE_{c,t}|_{ce_c} =\ \left[\frac{PE_{c,t}\ shr\_qxcd_c}{PD_{c,t}\ (1 - shr\_qxcd_c)}\right]^{elas\_qxcd_c} QD_c \tag{46}$$

The volume of export is a positive function of its price denominated in domestic currency. Equations (45) and (46) are defined over the set of export commodities only. This is because it is highly improbable that commodities are imported solely for the purpose of exporting them. For commodities that are not in the subset of export commodities, the volume of their domestic demands are equal to their supply

$$QD_{c,t}|_{cen_c} = QXC_{c,t} \tag{47}$$

By following the convention in the literature, it is assumed that domestically produced commodities that are sold in the domestic market $(QD_{c,t})$ and imports $(QM_{c,t})$ are imperfect substitutes of each other. These two generate a composite commodity $QS_{c,t}$ via a CES function.

$$QS_{c,t}|_{cp_c\ and\ cm_c} = sft\_arm_c \left[shr\_arm_c\ QD_{c,t}^{-rho\_arm_c} + (1 - shr\_arm_c)\ QE_{c,t}^{-rho\_arm_c} \right]^{\frac{-1}{rho\_arm_c}} \tag{48}$$

The FOC of the following cost minimisation problem yields the optimal volume of import demand, $QM_{c,t}$.

$$PQS_{c,t}\ QS_{c,t} = PD_{c,t}\ QD_{c,t} + PM_{c,t}\ QM_{c,t} \tag{49}$$

$$QM_{c,t}|_{cp_c\ and\ cm_c} = \left[\frac{PD_{c,t}\ (1 - shr\_arm_c)}{PM_{c,t}\ shr\_arm_c} \right]^{elas\_arm_c} QD_{c,t} \tag{50}$$

where $PQS_{c,t}$ is purchaser price of composite commodity c. $PM_{c,t}$ is import price of commodity $QM_{c,t}$ including tariffs and denominated in domestic currency.

Equation (48) is defined over both sets of domestically produced (cp_c) and imported (cm_c) commodities. For commodities either not domestically produced but imported ($PD_{c,t}$ is undefined) or domestically produced but not imported ($PM_{c,t}$ is undefined), total domestic demand is linear summation of imports and domestic sales

$$QS_{c,t}|_{\{cpn_c\ and\ cm_c\}\ OR\ \{cp_c\ and\ cmn_c\}} = QM_{c,t} + QD_{c,t} \tag{51}$$

4.1 Margins

Trade and transportation services are necessary to deliver commodities from factories and docks to markets. Producer prices, $PD_{c,t}$, do not comprise the cost of these margins since these are not part of the production process. These costs are paid by final users of commodities and are included in purchaser prices, $PQS_{c,t}$. Since a commodity is produced by several activities and the cost of trade and transportation margins is paid by consumers, margins are demanded by commodities. Each commodity demands margin m as a fixed fraction, $marg\_d_{m,c}$, of its total composite supply and the total volume of these demands is equal to the total supply of margins, $QSTM_{m,t}$:

$$QSTM_{m,t} = \sum_c marg\_d_{m,c}\ QS_{c,t} \tag{52}$$

where the set m stands for margins and consists of a single element as we have information only on the total of trade and transportation margins in the national accounts. Then, margin demand in terms of commodities is a fixed fraction of this supply, i.e via Leontief technology, and its price is simply equal to the weighted average of the commodities' purchaser prices

$$QDTM_{c,t} = \sum_m marg\_s_{c,m}\ QSTM_{m,t} \tag{53}$$

$$PTM_{m,t} \ = \sum_{c} marg\_s_{c,m} \, PQD_{c,t} \tag{54}$$

where $QDTM_{c,t}$ is margin demand of commodity c, $marg\_s_{c,m}$ is the share parameter of commodity c in total margin supply of m and $PTM_{m,t}$ is margin price.

5 Quantities

The total intermediate demand of commodity c is simply equal to the summation of activities' intermediate demands (55) and total private, i.e. household, demand for commodity c is the summation of all households' demands (56).

$$QINT_{c,t} \ = \sum_{a} INT_{c,a,t} \tag{55}$$

$$TOTPRCON_{c,t} \ = \sum_{hh} CD^{hh}_{c,t} \tag{56}$$

$$\begin{aligned} GDP_t \ = & \sum_{a} (PVA_{a,t} \, VA_{a,t} \ + \ PRODTAXS_{a,t}) \ + \\ & \sum_{c} (SALTAX_{c,t} \ + \ CTAXS_{c,t} \ + \ EXPTAXS_{c,t}) \end{aligned} \tag{57}$$

The total value of the gross domestic product, GDP_t, by the value-added approach is equal to the summation of the value added in each sector and indirect taxes on production activities, sales of commodities and international trade.

6 Prices

Here we discuss the various prices used in the model. The export price of commodity c denominated in domestic currency received by the exporter, $PE_{c,t}$, is equal the product of world price of commodity c, $PWE_{c,t}$, exchange rate, ER_t and export tax rate, $exptax_c$.

$$PE_{c,t}|_{ce_c} = \ PWE_{c,t} \ (1 - exptax_c) \ ER_t \tag{58}$$

If the value of $exptax_c$ is positive (negative), the domestic price is lower (higher) than the international price, so it represents a tax (subsidy). World prices are conventionally defined as fixed parameters without a time subscript. However, due to the quite volatile pattern of international energy prices, the time subscript is added to allow us to give the time sequence of the prices.

The import price of commodity c denominated in domestic currency, $PM_{c,t}$, is equal the product of the world price of commodity c, $PWM_{c,t}$, exchange rate, ER_t, and tariff rate, $tariff_c$.

$$PM_{c,t}|_{cm_c} = \ PWM_{c,t} \ (1 + tariff_c) \ ER_t \tag{59}$$

Since Ireland is a member of the Customs Union, the trade flows between the EU member states are exempted from import tariffs. On the other hand, the main trade partners of Ireland are the UK[6] (one-third and one-sixth of the total merchandise imports and exports, respectively) and the US (one-tenth and more than one-fifth of the total merchandise imports and exports, respectively) but tariff rates are quite low for the majority of commodities. In line with this structure of the foreign trade flows, data on tariff revenues of the government are not officially available. Therefore, tariff revenues and thus the tariff rates on imported commodities are assumed to be zero.

$$PQS_{c,t}\ QS_{c,t}\ =\ PM_{c,t}\ QM_{c,t}\ + PD_{c,t}\ QD_{c,t} \tag{60}$$

The value of total domestic demand by producer prices, PQS, is equal to the values of imports and domestic sales of commodities. However, sales tax, carbon tax, and trade and transportation margins are paid by consumers and they are included in the purchaser price, PQD_c, of commodity c.

$$PQD_{c,t}\ =\ PQS_{c,t}\ (1+stax_c)\ +\ alpha_c\ \overline{PCAR}_{c,t}\ carcon_c\ +\ \sum_m marg\_d_{m,c}\ PTM_{m,t} \tag{61}$$

Equation (62) shows the material balance of commodity supply and demand. The LHS is equal to total supply of composite commodity c that is demanded for several purposes including trade and transportation margins.

$$QS_{c,t}\ =\ QINV_{c,t}\ +\ TOTPRCON_{c,t}\ +\ QINT_{c,t}\ +\ CG_{c,t}\ +\ QDTM_{c,t} \tag{62}$$

The solution of $PXAC_{a,c}$, the price of commodity c produced by activity a, comes from the solution of the problem defined by equations (41, 42). Equation (63) is defined over the set of non-homogeneous commodities and of commodities produced in the base case, i.e. $SAM_{a,c}$ is non-zero. The price of activity commodities for homogeneous commodities produced in the base case is equal to their composite domestic commodity price, (64), and is equal to zero for the other cases.

$$\begin{aligned} PXAC_{a,c}|_{cp_c\ and\ chn_c} =\ & PXC_c\ QXC_c\ shr\_qxcs_{a,c}\ QXAC_{a,c}^{-rho\_qxcs_c-1} \\ & \sum_a \left[shr\_qxcs_{a,c}\ QXAC_{a,c}^{-rho\_qxcs_c}\right]^{-1} \end{aligned} \tag{63}$$

$$PXAC_{a,c,t}|_{ch_c} =\ PXC_{c,t} \tag{64}$$

There are two more prices: the price of composite commodity c produced by domestic producers, PXC_c, and price of value added, PVA_a. These prices are solved by equations (44) and (22), respectively.

Since it is assumed that activities produce multiple products and equation (42) is used in the multiple product determination problem, using it to solve for the price of output, PX_a, is not an option. Therefore,

[6] The weight of the UK in total Irish international trade flows will be more important after full exit of the UK from the EU.

another variable namely $shr\_qxacqx_{a,c}$ is introduced as

$$QXAC_{a,c,t} \;=\; shr\_qxacqx_{a,c}\; QX_{a,t} \tag{65}$$

Note that this equation does NOT imply that multiple product determination is fixed share of total output in which case $shr\_qxacqx_{a,c}$ must be a parameter rather than a variable. Then, the output price is equal to

$$PX_{a,t} \;=\; \sum_{c} shr\_qxacqx_{a,c}\; PXAC_{a,c,t} \tag{66}$$

Price of (physical) investment is uniform across activities due to the fact that we can only observe the breakdown of total investment expenditures across commodities. To make this price activity-specific, we have to know the distribution of the activity's physical investment across commodities. In other words, we need to know the capital composition matrix of activity, which is not readily available in almost all countries.

$$PPSI_{t} \;=\; \prod_{c} PQD_{c,t}^{shr\_invorg_{c}} \tag{67}$$

Equations (68) and (69) define consumer price index (CPI) and producer price index (PPI), respectively. The commodity weights of the CPI are equal to shares of commodities in total private consumption while the weights in the PPI are shares of commodities in total domestic production.

$$CPI_{t} \;=\; \sum_{c} wgt\_cpi_{c}\; PQD_{c,t} \tag{68}$$

$$PPI_{t} \;=\; \sum_{c} wgt\_ppi_{c}\; PD_{c,t} \tag{69}$$

CPI_1, the value of consumer price index in the first period, is chosen as the *numéraire* of the system, i.e. all prices are solved for *relative to* the CPI_1.

7 Energy

The flow of emissions through the economy is captured in I3E by explicitly including energy/carbon commodities. The I3E model includes the following energy commodities: peat, coal, crude oil, gasoline, kerosene, fuel oil, liquid petroleum gas, diesel, electricity, natural gas, and other petroleum products. Crude oil and fuel oil are not subject to private consumption, i.e. households do not demand these energy commodities. The consumption of energy commodities regardless of the purpose causes CO_2 emissions, with the exception of crude oil (which is transformed into other energy commodities) and electricity (which is generated using energy commodities). The parameter $carcon_c$ takes a positive value for these energy commodities, which cause emissions and reflects the amount of emissions released per unit of the carbon commodity in Mt of CO_2. For the other commodities defined in the model, the value of $carcon_c$

is equal to zero. Using this approach the level of emissions needed to produce non-energy commodities can also be calculated. The level of emission for each energy commodity is calculated as

$$emis_{c,t}|_{carcon_c} = carcon_c\ QSc,t \tag{70}$$

The economy-wide total emissions is the summation of emissions per commodity

$$emis\_tot_t = \sum_c emis_{c,t} \tag{71}$$

The economy-wide total emissions consists of emissions caused by activities in the production process (based on the intermediate input demand, eq. 72), households (based on the consumption demand, eq. 73), and government (based on the public demand, eq. 74).

$$emis\_act_{a,t} = \sum_c carcon_c\ INT_{c,a,t} \tag{72}$$

$$emis\_hh_t^{hh} = \sum_c carcon_c\ CD_{c,t}^{hh} \tag{73}$$

$$emis\_gov_t = \sum_c carcon_c\ CGc,t \tag{74}$$

Equation (75) calculates the total residential emissions which includes private demand of energy commodities for residential heating purposes, i.e. the commodities under the nest of residential heating (*RHC*) on Figure 1. Households' total emissions can be distinguished as residential and non-residential emissions in order to measure the effects of policy shocks on their behaviours regarding private consumption for different purposes.

$$emis\_hhres_t^{hh} = emis\_hh_t^{hh} - \sum_{SPRV_c} carcon_c\ CD_{c,t}^{hh} \tag{75}$$

where $SPRV_c$ is a subset of energy commodities demanded for private transportation purposes and comprises *TDie* and *GAL*, under the nest of *PRV* on Figure 1.

8 Economic Growth

Economic growth has three sources: the growth of employment due to population growth, the growth in capital stock driven by investment, and the growth in total factor productivity which is known as the Solow residual. This is the component of the economic growth that is not explained by growth in the factors of production.

As stated by Acemoglu (2009), if the technological progress is not of the Harrod-neutral form with any constant returns to scale (CRS) production function, per capita or aggregate variables cannot grow at a constant rate. This is referred to as a balanced growth path (BGP). In fact, the type of technological progress (capital augmented – Solow-neutral, labour augmented – Hicks-neutral) matters if the CRS production function is in a CES form in which the elasticity of substitution between capital and labour is not unity. If it is equal to 1, which corresponds to a Cobb-Douglas type of production function, each technological progress produces identical results (Acemoglu, 2009, 71–2). Since the value added function is assumed to be in the form of CES, the technical change has to be labour-augmented.

It is assumed that the total population grows at a constant rate, n_t, and the technology, i.e. the productivity of the labour force, grows at a constant rate, g_t.[7] In mathematical terms,

$$
\begin{aligned}
POP_{t+1} &= POP_t \, (1 + n_t) \\
BX_{t+1} &= BX_t \, (1 + g_t)
\end{aligned}
$$

where POP_t is the total population at period t and BX_t is the productivity level of the labour force at period t. In the case of labour–augmented technological growth, the growth of the productivity level of labour implies that the economy operates as if she has more labour (Acemoglu, 2009, 69).

The balanced growth path in a discrete time model requires that a variable should evolve as follows:

$$x_{t+1} = x_t \, (1 + n_t) \, (1 + g_t) = x_t \, (1 + n_t + g_t + n_t \, g_t) = x_t \, (1 + grw_t)$$

where grw_t is the economic growth rate. In the current version, the values of n and grw are retrieved from the medium-run estimates of the macroeconometric forecast model of the ESRI, namely COSMO (COre Structural MOdel for Ireland), Bergin et al. (2017). Accordingly, n_t and grw_t are calibrated as 0.8% and 3.3%. These population and economic growth rates imply 2.48% growth rate of the labour productivity.

9 Saving–Investment

Private, i.e. household, savings are determined endogenously as explained in the previous sections.

$$TOTPRSAV_t = \sum_{hh} SAV_t^{hh} \tag{76}$$

Government savings, $GSAV_t$, are endogenously determined via the government budget constraint and the enterprise savings, $ESAV_t$, are a fixed fraction of the total net-of-corporate tax profits, eq. (28).

In the absence of financial markets, a CGE model is able to solve for one of the following: foreign trade balance/savings, nominal exchange rate, and price index/real exchange rate (Robinson, 1989, 921). Here, foreign savings, $\overline{FSAV_t}$, are assumed to be fixed, and the consumer price index's value in the first

[7] In the current version, all growth rates are uniform across the model horizon. The time subscript is added to allow us to incorporate different growth rates through time.

period is fixed, making it possible for the model to solve for the nominal exchange rate which equates to the foreign exchange movements. The value of total savings in the economy is

$$TOTSAV_t = TOTPRSAV_t + GSAV_t + ESAV_t + \overline{FSAV_t}\, ER_t \tag{77}$$

In the equilibrium, the sum of investment expenditures of activities, i.e. investment by destination, must be equal to the total demand of commodities for investment purposes, i.e. investment by origin.

$$TOTINV_t = \sum_a INV_{a,t} \tag{78}$$

$$PQD_{c,t}\, QINV_{c,t} = shr\_invorg_c\, TOTINV_t \tag{79}$$

where shr_invorg_c is the share of commodity c in total volume of investment and $QINV_c$ is the volume of investment demand for commodity c.

10 Equilibrium

For the economy to be in equilibrium, all markets must be in equilibrium. In the relevant sections, the price and quantity equilibrium conditions were discussed. Here we discuss the labour market and foreign market equilibrium conditions. The labour market equilibrium, (80), solves for wage rate for the fixed supply of labour.

$$\overline{LSUP_t^{hh}} = \sum_a LD_{a,t}^{hh} \tag{80}$$

The foreign market equilibrium, (81), ensures foreign exchange supply and demand and solves for the equilibrium exchange rate.

$$\sum_c PWE_{c,t}\, QE_{c,t} + \sum_{hh} \overline{NFI_t^{hh}} + \overline{FSAV_t} = \sum_c PWM_{c,t}\, QM_{c,t} + GFINT_t \tag{81}$$

Up to this point, the number of endogenous variables and the number of variables are equal to each other, and there is no need to add an equation that shows investment-saving balance. If the system of equations is defined correctly, i.e. if there is no error in the model, investment and savings will be equal to each other by definition. However, rather than dropping this equation, it is conventionally associated with a slack variable, namely Walras. If there is no error in the model, $WALRAS_t = 0$ should hold for every time period t and experiment.

$$TOTSAV_t = TOTINV_t + WALRAS_t \tag{82}$$

Appendix A Lists of Activities and Commodities

Table A.1: Activities

Abbreviation	Name	NACE Codes	σ_{BEN}	σ_{FUE}	σ_{BH}	σ_{OTE}
ACC	Accommodation and hotel services	55–56,79	1.3	0	0	
AGR	Agriculture	1-3	1.3	0	0	
ATS	Air transportation	51	0	1.1	1.1	
BFM	Basic metal manufacturing	24–25	1.3	1.1	1.1	
BPP	Basic pharmaceutical products	21	1.3	0	0	
CHE	Chemicals and chemical products	20	1.3	1.3	1.3	
CON	Construction	41–43	1.3	0	1.3	
EDU	Education sector	85	1.3	1.1	1.3	
FBT	Food, beverage and tobacco	10–12	1.3	1.3	1.3	
FSR	Financial services	64–66,77	1.3	0	0	
HHS	Health sector	86–88	1.3	1.1	1.1	
HTP	High-technology products	26–28	1.3	1.3	1.3	
LTS	Land transportation	49	1.1	0	1.1	
NGS	Natural gas supply		0	1.1	0	
OIN	Other industrial products	17,18,33	1.3	1.3	1.1	
OMN	Other mining products		1.3	1.1	1.3	
ONM	Other non-metallic products	23	1.3	1.1	1.3	
OTM	Other manufacturing	31–32	1.1	1.3	0	
PEA	Peat		1.3	1.1	1.3	
PET	Petroleum		0	0	0	
PUB	Public sector	84	1.3	1.1	1.3	
RES	Real estate services	68	1.3	0	0	
RUP	Rubber and plastic products	22	1.3	1.3	1.1	
SER	Other services	remaining*	1.3	0	0	
TEL	Telecommunication services	61	1.3	0	0	
TEX	Textile	13–15	1.3	1.3	1.1	
TRD	Trade	45–47	1.3	1.3	0	
TRE	Transportation equipment	29–30	1.3	1.3	0	
WAT	Water and sewerage	36,37–39	1.3	1.3	1.1	
WTS	Water transportation	50	0	0	0	
WWP	Wood and wood products	16	1.3	1.1	0	
ELC	Electricity		1.1			1.3

*: It excludes NACE codes 5-9 (Mining, Quarrying and Extraction), 19 (Petroleum Products), and 35 (Electricity and Gas Supply).

Note: The activities without NACE codes are further disaggregated sectors.

Table A.2: Commodities

AGR	Agriculture	BFM	Basic metal manufacturing
PEA	Peat	HTP	High-technology products
COA	Coal	TRE	Transportation equipment
CRO*	Crude oil	ELC	Electricity
OMN*	Other mining products	NGS	Natural gas supply
FBT	Food, beverage and tobacco	WAT	Water and sewerage
TEX	Textile	CON	Construction
WWP	Wood and wood products	TRD	Trade
OIN	Other industrial products	LTS	Land transportation
GAL	Gasoline	WTS	Water transportation
KRS	Kerosene	ATS	Air transportation
FUO*	Fuel oil	ACC	Accommodation and hotel services
LPG	Liquid petroleum gas	TEL	Telecommunication services
DIE	Diesel	FSR	Financial services
OPP	Other petroleum products	RES	Real estate services
OTM	Other manufacturing	PUB	Public services
CHE	Chemicals and chemical products	EDU	Education sector
BPP	Basic pharmaceutical products	HHS	Health sector
RUP	Rubber and plastic products	SER	Other services
ONM	Other non-metallic products		

*: Not subject to private consumption.

Appendix B List of Endogenous Variables

Name	Description
$ADJ_{a,t}$	Adjustment cost
$CD_{c,t}^{hh}$	Consumption by commodities
$CG_{c,t}$	Consumption demand of government by sectors
$CINC_t^{hh}$	Capital (dividend) income
$CORPTAXS_t$	Corporate tax payments
CPI_t	Consumer price index
$CTAXS_{c,t}$	Value of carbon tax revenues
$DISDIV_t$	Total distributed dividends
$DIV_{a,t}$	Sectoral dividends
ER_t	Level of nominal exchange rate
$ESAV_t$	Savings of enterprises
GDP_t	Gross domestic product
$GOVCON_t$	Total government consumptions
$GOVREV_t$	Government revenues
$GSAV_t$	Government savings
INC_t^{hh}	Disposable income
$INT_{c,a,t}$	Intermediate input demand of commodity *c* by activity *a*
$INV_{a,t}$	Investment by destination
$K_{a,t}$	Sectoral capital stock
$KSUP_t$	Total capital stock
$LD_{a,t}^{hh}$	Labour demand of household *hh* by activity *a*
$PD_{c,t}$	Producer price of domestic supply of commodity *c*
$PE_{c,t}$	Export price of commodity *c* (including export tax) in domestic currency
$PM_{c,t}$	Price of import commodity *c* (including import tax) in domestic currency
PPI_t	Producer price index
$PPSI_t$	Price of investment
$PQD_{c,t}$	Purchaser price of composite commodity *c*
$PQS_{c,t}$	Price of domestically sold composite commodity *c*
$PRODTAXS_{a,t}$	Taxes on production payments by activities
$PSI_{a,t}$	Physical investment
$PTM_{m,t}$	Price of margin
$PVA_{a,t}$	Price of sectoral value-added
$PX_{a,t}$	Price of sectoral output
$PXAC_{a,c,t}$	Price of commodity *c* produced by activity *a*

$PXC_{c,t}$	Price of composite domestic commodity c
$q_{a,t}$	Tobin q
$QD_{c,t}$	Total domestic supply of commodity c
$QDTM_{c,t}$	Total demand of margin
$QE_{c,t}$	Total export of commodity c
$QINT_{c,t}$	Total intermediate input demand of commodity c
$QINV_{c,t}$	Investment demand of commodity c
$QM_{c,t}$	Total import demand of commodity c
$QS_{c,t}$	Composite supply of commodity c
$QSTM_{m,t}$	Total supply of margin
$QX_{a,t}$	Sectoral output
$QXAC_{a,c,t}$	Production of commodity c by activity a
$QXC_{c,t}$	Total production of commodity c by domestic producers
RCI_t^{hh}	Revenue cycling income
$SALTAX_{c,t}$	Value of sales tax revenues
SAV_t^{hh}	Private savings
$shr\_qxacqx_{a,c,t}$	Share of commodity c in production of activity a
$TOTINV_t$	Total investment expenditure
$TOTPRCON_{c,t}$	Total private consumption by commodity c
$TOTPRSAV_t$	Total private savings
$TOTSAV_t$	Total savings
TR_t^{hh}	Transfer receipt by households
$VA_{a,t}$	Sectoral value added
W_t^{hh}	Wage rate
$WALRAS_t$	Slack variable for Walras's law
$WFDIST_{a,t}^{hh}$	Sectoral proportion for factor prices
$WINC_t^{hh}$	Wage income
$WK_{a,t}$	Sectoral price of capital
$WTAXS_t^{hh}$	Wage income tax payments

Appendix C List of Exogenous Variables and Parameters

Name	Description
	Exogenous Variables
$\overline{FSAV}_t$	Foreign savings
$\overline{GTRENT}_t$	Transfers from Government to Enterprises
$\overline{GTRHH}_t$	Transfers from Government to Households
$\overline{LSUP}_t^{hh}$	Labour supply
$\overline{NFI}_t^{hh}$	Net factor income from the rest of the world
$\overline{PCAR}_t$	Carbon tax per tonne equivalent of carbon
	Parameters
$adjpar_a$	Adjustment parameter
$corptax$	Corporate tax rate
$delta_a$	Depreciation rate
$elas\_arm_c$	CES elasticity of Armington
$elas\_qxac_a$	CET elasticity of production (QX)
$elas\_qxcd_c$	CET elasticity of QX demand (domestic sales or export)
$elas\_qxcs_c$	CES elasticity of domestic composite commodity (QXC)
$marg\_d_{m,c}$	Margin demand share in total composite commodity
$marg\_s_{c,m}$	Margin supply share in total margin supply
mps	Marginal propensity to spend of Government consumption w.r.t. GDP
POP^{hh}	Population
$prodtax_a$	Taxes on production paid by activities
PWE_c	World export price of commodity c
PWM_c	World import price of commodity c
r_t	Foreign interest rate
rho^{hh}	Time preference rate
rho_arm_c	Armington exponent
rho_qxac_a	CET exponent of production (QX)
rho_qxcd_c	CET exponent of QX demand (domestic sales or export)
rho_qxcs_c	CES exponent of domestic composite commodity (QXC)
sft_arm_c	CES shift parameter of Armington
sft_qxac_a	CET shift parameter of production (QX)
sft_qxcd_c	CET shift parameter of QX demand (domestic sales or export)
sft_qxcs_c	CES shift parameter of domestic composite commodity (QXC)
shr_arm_c	Share parameter of Armington
shr_cinc^{hh}	Share of Household in capital (dividend) income

shr_gc_c	Share of Government demand by commodities
shr_gtrhh^{hh}	Share of household *hh* in Government transfers
shr_invdes_a	Share of investment demand (=Inv. by Destination) in gross profits (positive for NDM)
shr_invorg_c	Share of investment demand of commodity (=Inv. by Origin) in total investment
$shr\_qxac_{a,c}$	CET share parameter of production (QX)
shr_qxcd_c	CET share parameter of QX demand (domestic sales or export)
$shr\_qxcs_{a,c}$	CES share parameter of domestic composite commodity (QXC)
shr_sav	Enterprises saving rate
$stax_c$	Sales tax rate
$theta^{hh}$	Intertemporal elasticity of substitution
wgt_cpi_c	Share of commodity *c* in total private consumption (Weights in CPI)
wgt_ppi_c	Share of commodity *c* in total domestic production (Weights in PPI)
$wtax^{hh}$	Wage tax rate (including SSI premiums)

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Whitaker Square,
Sir John Rogerson's Quay,
Dublin 2
Telephone **+353 1 863 2000**
Email **admin@esri.ie**
Web **www.esri.ie**
Twitter **@ESRIDublin**
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