Public Debt Consolidation and its Distributional Effects

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Public Debt Consolidation and its Distributional Effects*

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Abstract

We build a dynamic general equilibrium model with heterogeneous households, namely Rich and Poor, and capital-skill complementarity structure in the production function, to study aggregate and distributional implications of fiscal consolidation policies when the government uses a rich set of spending and tax instruments. Fiscal policy is conducted through constrained optimized fiscal rules. Our results show that, in the long run, fiscal consolidation enhances both aggregate efficiency and equity; however, it may hurt Rich households depending on which fiscal instrument takes advantage of the fiscal space created. Along the transition, wage inequality significantly increases due to the capital skill complementarity structure of the production function. Specifically, this happens because debt consolidation crowds in capital and this favours Rich (skilled) households. On the other hand, the reduction in interest rates and government bonds lead to a decrease in Rich households’ income coming from capital and government bonds which eventually decrease income inequality. Finally, a rather novel finding is that the combination of asset and skill heterogeneity amplifies the increase in wage inequality in the early phase of fiscal consolidation.

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1 Introduction

The 2008 world financial crisis saw most European countries in a vulnerable fiscal position with high deficits and debts as a share of GDP, above the 3% and 60% of the Stability and Growth Pact limits.\footnote{There is by now a tendency of declining public deficits in the Euro Area. This is reflected not only in statistical indicators but also in the number of countries that are still under the Excessive Deficit Procedure (EDP). The structural deficit in the EU was reduced markedly from 4.3% to 1.0% in the Euro Area. At the country level, in 2010 only five Member States recorded long run deficits below the 3% of GDP reference threshold, while, 22 did so in 2014. See European Commission (2017).} As a result several European governments have been forced to take restrictive fiscal actions, the so called fiscal consolidation.

By now most studies find that fiscal consolidation entails an intertemporal trade-off for the main macroeconomic aggregates, see e.g. Coenen et al. (2008), Schwarzmüller and Wolters (2014), Philippopoulos et al. (2015) and (2017a). That is the early phase of fiscal consolidation requires cuts in spending and/or rises in taxes.\footnote{In addition, consolidation efforts usually come at the expense of growth-friendly spending items such as spending on public investment and education which further harms the prospects of long term growth see European Central Bank (2017) for a discussion on the trade-offs between fiscal consolidation and reforms.} This is the short run fiscal pain. In the long run, once debt reduction has been achieved, the resulting fiscal space can be utilized to increase spending and/or reduce distortionary taxes. This is the long run fiscal gain. However, fiscal consolidation also has important distributional implications. For example, Furceri et al. (2015) find that during fiscal consolidation periods the lowest income and wealth quantiles of the population became worse off in terms of their disposable income.

The aim of this paper is to study the distributional and aggregate implications of fiscal consolidation policy. Since the aggregate effects have been thoroughly studied, our primary focus will be on the distributional effects. In particular, we examine the intertemporal effects of fiscal consolidation on the distribution of wages and income, whether this reform generates an equity efficiency trade off, and if yes, whether the magnitude of this trade off varies over different time horizons (i.e. short, medium and long run). Furthermore, we examine whether any distributional implications depend on the fiscal policy mix, on the source of heterogeneity incorporated in the model and the measure of inequality under consideration.

To study this, we develop a dynamic general equilibrium model with heterogeneous households calibrated to the Euro Area over the period 2001-2015. The model includes two types of households, capital-skill complementarity in the production function in the spirit of Krusell et al. (2000) and endogenous human capital accumulation. Households differ in the type of labor they supply and their access to capital and financial markets.\footnote{There are different ways to introduce heterogeneous agents in DSGE models. For instance, Gali et al. (2007), Forni et al. (2009), Coenen et al. (2013), Schwarzmüller and Wolters (2017) use models in which a share of households does not have access to financial or/and capital markets (Ricardian vs Non-Ricardian). Households can also exhibit heterogeneity in terms of their impatience (patient vs impatient) as in Bilbiie et al. (2013), their labor market status (public vs private sector workers) as in Ardagna (2007) and Economides et al. (2012) or with respect to their education and skills as in Angelopoulos et al. (2014), Angelopoulos et al. (2017) and Gomes (2018).} In a standard model with representative Ricardian households, each agent can smooth out exogenous fiscal policy changes,
like fiscal consolidation, through borrowing or lending. In addition, all households rely on the same sources of income, while, their labor and capital incomes are identical. Thus, there is no room for distributional analysis. To capture distributional implications, we depart from this framework and allow for non-uniform distributions of asset holdings and skills.

In particular, households that save in the form of government bonds, own physical capital and firms and supply skilled labor services are referred to as *Rich*. On the other hand, households that do not have access to financial and capital markets, i.e. they live hand-to-mouth, and supply unskilled labor services are referred to as *Poor*. In line with empirical evidence we assume that the asset and skill distributions are positively related. The heterogeneity in the skill supply of labor force gives rise to the so called skill premium which is the measure of wage inequality in our setup. The skill premium implies that *Rich* households receive relatively higher wages than *Poor* households. Finally, both households can invest time in education and accumulate human capital.

Regarding policy, government has a rather rich set of spending and tax instruments at its disposal. Particularly, government levies consumption, labor and capital taxes to finance productivity-enhancing spending like public investment and spending on education, utility-enhancing expenditures like government consumption, as well as transfer payments targeted to *Rich* and *Poor* households.

Our main policy experiment is fiscal consolidation. That is the economy starts from a steady state with high debt-to-GDP ratio, say 85.3%, which is the EA-19 data average over the fiscal stress period 2008-2014, and travels towards a new reformed steady state with lower debt-to-GDP ratio, say 60%. We experiment with various reformed economies varying the fiscal instrument which reaps the benefit of fiscal space after debt consolidation. Government can achieve the transition from the status quo to the new reformed steady state by implementing alternative fiscal consolidation policy mixes. Following most of the literature on debt consolidation we follow a rule-based approach to policy. In addition, we focus on constrained optimized fiscal policy rules so as our results do not depend on ad hoc differences in feedback policy coefficients. To do this, we follow Schmitt-Grohé and Uribe (2004, 2007) so that we limit our attention to a family of simple fiscal rules. This means that all the fiscal instruments can respond to the gap between public debt and the target of public debt as shares of output.

The structure of our model enables us to assess aggregate and distributional effects of fiscal consolidation in the long run and along the transition. To do this, we utilize the key endogenous variables of the model. For example, to quantify aggregate effects, we employ variables such as output and social welfare, while, to measure distributional effects, we find it natural to use variables like the net income of *Rich* and *Poor* households, inequality in net incomes and wages,\(^4\)

\(^4\)The reason that we employ these two different measures of inequality is the following. The vehicle of our analysis is the household budget constraint, which is the most straightforward way to understand how the different dimensions of inequality are linked with endogenous choices, e.g. labor supply and saving decisions, access to financial markets and fiscal policy changes. For a systematic and comprehensive analysis see Krueger et. al. (2010) and Heathcote et. al. (2010).
household specific welfare as well as relative welfare.

Our main results are as follows. In the long run, fiscal consolidation always enhances both efficiency and equity. Thus, once debt reduction has been achieved and fiscal space has been created, output increases, while, both wage and income inequality decrease. Looking at welfare, in contrast, fiscal consolidation is not always Pareto efficient. For example, such reform can be harmful for Rich households when government uses the post consolidation fiscal space to increase non productive government consumption or transfers targeted to Poor households. On the other hand, we find that fiscal consolidation is Pareto efficient, i.e. both Rich and Poor households get better off in the reformed steady state, when the government uses the resulting fiscal space to finance an increase in spending on public education and/or to finance a cut in distortionary taxation.

Along the transition, the government should cut non productive government consumption to reduce debt to its new lower target, while, keeping all the remaining fiscal instruments constant at their historical averages. This induces a temporary output contraction, however due to the crowding in effects, aggregate output quickly recovers above its pre consolidation level. Thus fiscal consolidation is costly on impact but efficient in the medium/long run.

On the other hand, the different driving forces of inequality in the model lead to interesting distributional implications that may differ across the various measures of inequality. In a nutshell, the skill premium increases, while, income inequality decreases. In terms of welfare, fiscal consolidation hurts only Poor households in the short run, while, it benefits Rich households.

More specifically, wage inequality significantly increases in the short/medium run due to the capital skill complementarity structure of the production function. The logic is the following, fiscal consolidation results in an increase in the physical capital stock and a reduction in relative skill supply which drive upward the wages of Rich households and increases the gap with the wage of Poor households in the short run.

However, income inequality decreases due to the asset heterogeneity between Rich and Poor households. Since fiscal consolidation induces a sharp reduction in government bonds and real rates, Rich households, who own assets and capital, see a decrease in their income coming from government bonds and capital. The latter leads to the deterioration in Rich households’ total net income vis-à-vis Poor households’ total net income despite the opposite movement in their wages and labor incomes.

A rather novel finding of this paper is that the combination of asset and skill heterogeneity amplifies the increase in wage inequality in the early phase of fiscal consolidation. Poor households rely only on labor income since they do not own assets and capital. Thus, in the transition phase of consolidation, they need to increase their labor supply to compensate the loss in their labor income due to the reduction in their wages. The increase in their hours worked pushes further downward Poor households’ wages, and thus, widens the inequality with respect to Rich households’ wages. Finally, in terms of welfare, fiscal consolidation is not Pareto efficient in the short run. Although it seems to enhance social welfare, this increase arises solely from welfare
gains accruing to *Rich* households, while the welfare of *Poor* households decreases.

This paper contributes to the vast literature on fiscal consolidation by focusing mainly on its distributional implications. First, this paper is closely related to papers that study the aggregate and distributional implications of fiscal consolidation in dynamic general equilibrium models under ad hoc policy, e.g. Coenen et al. (2008), Economides et al. (2012), Almeida et al. (2013), Schwarzmüller and Wolters (2017) and Roubanis (2018).

Second, our work is related to papers that compute optimal fiscal consolidation policies in DSGE models, e.g. Bi and Kumhof (2011), Cantore et al. (2012), Philippopoulos et al. (2015), (2017a) and (2017b) and Cardani et al. (2018). Third, our work is also related to papers that study other types of fiscal policy reforms in dynamic general equilibrium models with heterogeneous agents, like Garcia-Mila et al. (2010), Angelopoulos et al. (2013), Angelopoulos et al. (2014), Angelopoulos et al. (2017), Gomes (2018), Michaud and Rothert (2018), however, they do not focus on the distributional effects of fiscal consolidation.

In this paper, we employ a joint heterogeneity setup including asset and skill heterogeneity to assess fiscal consolidation. Moreover, we compute constrained optimized fiscal policy rules for a rich set of (utility- and productivity-enhancing) spending and tax instruments. Thus, we provide a systematic framework to assess the distributional implications, along with the aggregate effects, of fiscal consolidation policy.

The structure of the paper proceeds as follows. Section 2 presents the model and solves for the decentralized competitive equilibrium. Section 3 develops our calibration strategy and solves for the status quo steady state solution. Section 4 describes the main policy experiment, while, Section 5 presents our results. Section 6 analyzes the underlying transmission mechanism and Section 7 conducts sensitivity analysis. Section 8 concludes the paper. Algebraic details and additional results are in an Appendix.

## 2 The model

### 2.1 Informal description of the model

We develop a closed economy dynamic general equilibrium model which consists of households, firms and a government. The key feature of the model is household heterogeneity. Households differ in two aspects. First, in the type of labor they supply and second in their access to financial and capital markets. Thus we incorporate *ex ante* skill and wealth heterogeneity. In particular households that have access to capital and financial markets, supply skilled labor services and own private firms are referred to as *Rich*. Households that do not participate in capital and financial markets and supply unskilled labor services, thus, only consume their after-tax labor

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5 On the other hand, Forni et al. (2010), Cogan et al. (2013), Erceg and Lindé (2013), Pappa et al. (2015) and Economides et al. (2017) focus on the aggregate implications of debt consolidation under ad hoc policy.

6 See e.g. He and Liu (2008) and Angelopoulos et al. (2014) and the references therein on the interaction among various fiscal policy reforms and distribution.
income referred to as Poor.\footnote{We follow Michaud and Rothert (2018) by referring to the two types of households as Rich and Poor.} In addition, both household types can accumulate human capital using a human capital production function à la Lucas while they derive utility from public consumption.

On the production side, firms use physical and public capital, skilled and unskilled labor in order to produce an homogeneous good. In the production sector we incorporate a nested CES-Cobb Douglas production function similar to Krusell et al. (2000) which exhibits capital-skill complementarity. As is known, this feature gives rise to the so called skill premium.

The government has a rather rich set of fiscal policy instruments at its disposal. In particular, it issues public debt and levies consumption, labor and capital income taxes to finance its stream of public expenditures, namely spending on public education and investment, government consumption and transfer payments which are allowed to be allocated unevenly between households. Finally, we assume that all agents have perfect foresight.

\section{2.2 Population composition}

The population size, $N$, is exogenous and constant. It is comprised by two types of households, i.e. Rich households indexed by the subscript $R = 1, 2, \ldots, N_R$ and Poor households indexed by the subscript $P = 1, 2, \ldots, N_P$ where $N_R > N_P$ and $N_P + N_R = N$ is the total size. No mobility or occupational change is possible between the two types. There are also $f = 1, 2, \ldots, N_f$ firms. For notational convenience, we assume also that $N_f = N_R$.\footnote{Following the related literature, e.g. Angelopoulos et al. (2014) and Michaud and Rothert (2018), we work with a discrete number for population and its segments. We report that our results do not depend on this assumption.}

\subsection{2.3 Rich households}

Each Rich household, $R$, maximizes its discounted lifetime utility given by:

\begin{equation}
V_{R,0} = \sum_{t=0}^{\infty} \beta^t U_{R,t} (c_{R,t}, z_{R,t}, g^c_t) \tag{1}
\end{equation}

where $c_{R,t}$ and $z_{R,t}$ are consumption and leisure of each household, $g^c_t$ is per capita utility-enhancing government consumption\footnote{Notice that $g^c_t \equiv \frac{G^c_t}{N}$ where $G^c_t$ is aggregate utility-enhancing government consumption.} and $0 < \beta < 1$ is the discount rate. The period utility function $U_{R,t}(.)$ is increasing and strictly concave in all its arguments. For our numerical solutions the period utility function is given by:

\begin{equation}
U_{R,t} (c_{R,t}, z_{R,t}, g^c_t) = \mu_1 \log (c_{R,t} + \xi g^c_t) + \mu_2 \log (1 - e_{R,t} - l_{R,t}) \tag{2}
\end{equation}

where $\mu_1, \mu_2$ are preference parameters, $\xi$ measures the degree of substitutability between private and public consumption, e.g. if $\xi > 0$ ($< 0$) private and public consumption are substitutes...
(complements). Households are endowed with a normalized time unit:

\[ z_{R,t} + e_{R,t} + l_{R,t} = 1 \]  

(3)

where \( l_{R,t} \) and \( e_{R,t} \) is time devoted to labor and education respectively. The within period budget constraint of each Rich household, \( R \), is:

\[(1 + \tau_t^c) c_{R,t} + i_{R,t} + d_{R,t} = (1 - \tau_t^k)(r_t k_{R,t} + \pi_{R,t}) + (1 - \tau_t^l) w_{R,t} l_{R,t} h_{R,t} + tr_{R,t} + r_t^b b_{R,t} \]  

(4)

where \( i_{R,t} \), is private investment in physical capital, \( k_{R,t} \), the beginning of period physical capital, \( b_{R,t} \), is dividends received from private firms, \( w_{R,t} \) is the wage rate earned by Rich households, \( tr_{R,t} \equiv \frac{TR_{R,t}}{NR} \) is public transfers to each Rich household and \( 0 < \tau_t^k, \tau_t^l, \tau_t^e < 1 \), are tax rates on capital income, labor income and consumption respectively. Rich households supply skilled labor services while Poor households supply unskilled labor services to firms. Thus, Rich households receive a relatively higher wage \( w_{R,t} > w_{P,t} \) than Poor households (for more details see section 2.6).\(^{10}\) To allow for productive education expenditures we use a human capital production function as in Lucas (1988) and Glomm and Ravikumar (1997).\(^{11}\) Therefore, individual human capital is augmented by time spent in education, \( e_{R,t} \), and by public spending on education, \( g_{R,t}^e \), which is a fixed share \( \omega \) of per capita public spending on education, \( g_t^e \).\(^{12}\) The law of motion of human capital of Rich household, \( R \), is:

\[ h_{R,t+1} = (1 - \delta^h) h_{R,t} + B_R \left[(e_{R,t})^\theta (g_{R,t}^e)^{1-\theta}\right]^{\chi_R} \]  

(5)

where \( B_R > 0, \theta \in (0,1), \chi_R < 1 \) are productivity parameters, and \( \delta^h \) is the depreciation rate of human capital. Following He and Liu (2008), \( \chi_R < 1 \), captures decreasing returns to scale in the production of new human capital. The laws of motion of physical capital and government bonds for each \( R \) are:

\[ k_{R,t+1} = (1 - \delta^k) k_{R,t} + i_{R,t} \]  

(6)

\[ b_{R,t+1} = b_{R,t} + d_{R,t} \]  

(7)

Each Rich household in any given period \( t \), chooses \( c_{R,t}, e_{R,t}, l_{R,t}, k_{R,t+1}, b_{R,t+1} \) and \( h_{R,t+1} \) to maximize its lifetime utility subject to the constraints (4) (in which we incorporate (6) and

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\(^{10}\)Throughout the rest of the paper labor provided by Rich households referred to as skilled labor and labor provided by Poor households referred to as unskilled labor. As we explain in section 3.1 there is adequate empirical evidence associating wealth with skills.

\(^{11}\)Alternatively, Trabandt and Uhlig (2011) use a learning by doing specification by including hours of work and education as inputs. On the contrary, in our model, as in Daniel and Gao (2015), we allow for a production function that combines a time input and a good input so as to assess the effects of public education spending as an additional productivity enhancing fiscal instrument.

\(^{12}\)Similarly, \( g_t^e \equiv \frac{G_t^e}{N} \) where \( G_t^e \) is total public spending on education.
(7)) and (5) taking factor prices and policy as given. Defining as $\lambda_{R,t}$ and $\psi_{R,t}$ the Lagrange multipliers associated with (4) and (5) respectively. The first-order conditions are given in Appendix A.1.

2.4 Poor households

Each Poor household, $P$, maximizes its expected discounted lifetime utility given by:

$$V_{P,0} = \sum_{t=0}^{\infty} \beta^t U_{P,t}(c_{P,t}, z_{P,t}, g_t^P)$$  \hfill (8)

The period utility function $U_{P,t}(.)$ is increasing and strictly concave in all its arguments where we use the same functional form for preferences and the same normalized time unit as above.\(^{13}\)

The within period budget constraint of each Poor household is given by:

$$(1 + \tau_t^P) c_{P,t} = (1 - \tau_t^P) w_{P,t} l_{P,t} + t r_{P,t}$$  \hfill (9)

where $w_{P,t}$ is the wage rate received by Poor households, $t r_{P,t} = \frac{T R_{P,t}}{N_P}$ is public transfers to each Poor household. Contrary to Rich households, Poor households do not have access to financial and capital markets thus the only source of income is the labor income.\(^{14}\) In addition they supply unskilled labor, and thus receive lower wages than Rich households. As before, the law motion of human capital of each household of type, $P$, is:

$$h_{P,t+1} = (1 - \delta^h) h_{P,t} + B_P \left[ (e_{P,t})^\theta (g_{t}^P)^{1-\theta} \right]^{x_P}$$  \hfill (10)

where $g_{t}^P \equiv (1 - \omega) g_t^P$ is the amount of total public spending on education services enjoyed by each $P$.\(^{15}\) Each Poor household, $P$, maximizes its lifetime utility in any given period $t$ by choosing $c_{P,t}, e_{P,t}, l_{P,t}$, and $h_{P,t+1}$ subject to the constraints (9) and (10) taking factor prices and policy as given. Defining as $\lambda_{P,t}$ and $\psi_{P,t}$ the Lagrange multipliers associated with (9) and (10) respectively. The first-order conditions are given in Appendix A.2.

2.5 Firms

There are $f = 1, 2, \ldots, N^f$ identical firms owned by the Rich households. Each firm, $f$, acts competitively taking prices as given. Firm’s objective is to maximize their profits:

$$\pi_f^t = y_f^l - r_t^l k_f^l - w_{R,t} l_{R,t}^f - w_{P,t} l_{P,t}^f$$  \hfill (11)

\(^{13}\)Notation and functional forms of Poor households are analogous to Rich households.

\(^{14}\)In section 7.2 we relax this assumption.

\(^{15}\)This is meant to be not only formal education (i.e. secondary or tertiary education spending), but could resemble other types of educational programmes such as vocational training, on-the-job learning, continuing professional development programmes among others. This type of investment is of special importance for the less skilled or less wealthy members in the society since it increases their productivity and labor earnings.
where \( y_f^t \) is firm \( f \)'s output. Firms utilize four factors inputs to produce an homogeneous good, i.e. physical capital, \( k_f^t \), skilled labor services rented from Rich households, \( l_{R,t}^f \), unskilled labor services rented from Poor households, \( l_{P,t}^f \), and per capita public capital, \( k_{g,t}^f \). Production is given by the following constant returns to scale (CRS) and constant elasticity of substitution (CES) function similar to Krusell et al. (2000):

\[
y_f^t = A \left[ m \left(l_{P,t}^f \right)^\sigma + (1 - m) \left( \rho \left(k_f^t \right)^v + (1 - \rho) \left(l_{R,t}^f \right)^v \right)^\frac{\sigma}{v} \right]^{\frac{1}{v}} (k_{g,t}^f)^{1-\alpha} \tag{12}
\]

where \( A > 0 \) is scale parameter, \( 0 < \alpha, \rho, m < 1 \) are factor inputs share parameters and \( \sigma, v \leq 1 \) are parameters governing factor elasticities (see below for more details). Each firm \( f \) maximizes its profits (11) subject to its production function (12) by choosing \( k_f^t, l_{R,t}^f, l_{P,t}^f \). First order conditions are given in Appendix A.3.

### 2.6 Skill premium

In our setup the skill premium is defined as the ratio of the wage rate earned by the Rich household over the wage rate earned by the Poor household. Combining the first order conditions with respect to \( l_{R,t}^f \) and \( l_{P,t}^f \) in the Appendix A.3 gives the following condition for the skill premium:

\[
\frac{w_{R,t}}{w_{P,t}} = (1 - \rho) \frac{1 - m}{m} \left[ \rho \left( \frac{k_f^t}{l_{R,t}^f} \right)^v + (1 - \rho) \right]^{\frac{\sigma}{v} - 1} \tag{13}
\]

The different roles in the production function for skilled (Rich) and unskilled (Poor) labor give rise to the so called skill premium meaning that \( \frac{w_{R}}{w_{P}} > 1 \). In section 3.1 we calibrate the associated parameters in the production function so that the implied factor input elasticities and the resulting skill premium are in line with empirical studies. The elasticities of substitution between physical capital and skilled labor and between skilled and unskilled labor is \( \frac{1}{1-\sigma} \) whereas the elasticity between capital and skilled labor is \( \frac{1}{1-v} \). This formulation implies that as long as \( \sigma > v \) the production function exhibits capital-skill complementarity. Moreover, this specification implies that the skill premium will be \textit{ceteris paribus} increasing in physical capital to skilled labor ratio, \( \frac{k_{R,t}^f}{l_{R,t}^f} \) (known as the capital-skill complementarity effect) and decreasing in the skilled to unskilled labor ratio, \( \frac{l_{R,t}^f}{l_{P,t}^f} \) (known as the relative skill supply effect). For more details see in Lindquist (2004) and Dolado et al. (2018).\(^{16}\)

\(^{16}\)Notice that \( \frac{\partial (\frac{w_{R,t}}{w_{P,t}})}{\partial (\frac{k_f^t}{l_{R,t}^f})} > 0 \) as long as \( \sigma > v \) and \( 0 < \rho, m < 1 \) while \( \frac{\partial (\frac{w_{R,t}}{w_{P,t}})}{\partial (\frac{l_{R,t}^f}{l_{P,t}^f})} < 0 \) as long as \( \sigma < 1 \) and \( 0 < \rho, m < 1 \).
### 2.7 Government

The within-period government budget constraint is given by (in aggregate terms): \[
G_t^c + G_t^i + G_t^e + TR_{R,t} + TR_{P,t} + (1 + r_t^b)B_t = B_{t+1} + T_t
\]

where \(G_t^c\) is total public spending on utility-enhancing government consumption, \(G_t^i\) is total spending on public investment \(G_t^e\) is total public spending on education, \(TR_{R,t}\) and \(TR_{P,t}\) are respectively transfers to Rich and Poor households, \(B_t \equiv N_R b_{R,t}\) denotes the beginning-of-period stock of government bonds, and \(T_t\) denotes total tax revenues defined as: \[
T_t \equiv \tau_t^c (N_R c_{R,t} + n_P c_{P,t}) + \tau_t^b N_R (r_t k_{R,t} + \pi_{R,t}) + \tau_t^f (N_P w_{P,t} l_P h_{P,t} + N_R w_{R,t} l_{R,t} h_{R,t})
\]

Each period \(t\) government sets eight fiscal instruments, i.e. five public spending instruments, namely utility-enhancing spending, public education, public investment and agent specific transfers to Rich and Poor households and three tax instruments, namely capital, labor and consumption taxes. We assume, unless otherwise stated, that the residual policy instrument is public debt. The law motion of aggregate public capital is given by:

\[
k_{t+1}^g = (1 - \delta^g) k_t^g + G_t^i
\]

For notational convenience, in what follows, we use per capita quantities denoted with small case letters for example, \(k_t^g \equiv \frac{K_t^g}{N_t}\), while we define spending instruments in terms of their output shares, i.e. \(s_t^g \equiv \frac{G_t^g}{N_t^g}\), \(s_t^b \equiv \frac{G_t^b}{N_t^b}\), \(s_t^e \equiv \frac{G_t^e}{N_t^e}\), \(s_t^f \equiv \frac{G_t^f}{N_t^f}\), \(s_t^{TR} \equiv \frac{TR_{R,t} + TR_{P,t}}{N_t^R + N_t^P}\) and total tax revenues in per capita terms, \(\tau_t \equiv \frac{T_t}{N_t}\). Also, we express the number of Rich, Poor households and firms in terms of their total population share, i.e. \(n_R \equiv \frac{N_R}{N}, n_P \equiv \frac{N_P}{N} = 1 - n_R, n_f \equiv \frac{N_f}{N} = n_R\) respectively.

### 2.8 Fiscal policy rules

Fiscal policy sets its spending-tax instruments following simple fiscal policy rules. In particular, we allow all the main policy instruments \(\phi_t = \{s_t^g, s_t^b, s_t^e, s_t^{TR}, \tau_t, \tau_t^c, \tau_t^b, \tau_t^f\}\) to react to the public debt to GDP ratio, \(q_{t-1} \equiv \frac{n_R b_{R,t-1}}{n_t^g y_{t-1}}\), as deviation from a target according to a simple linear rule:

\[
\phi_t - \phi^* = \gamma_q (q_{t-1} - q^*)
\]

where \(\phi^*, q^*\) denote fiscal policy targets and \(\gamma_q\) are feedback policy coefficients. If \(\phi_t\) is a spending instrument then, \(\gamma_q \leq 0\), and \(\gamma_q \geq 0\) if \(\phi_t\) is a tax instrument (see equations (60)-(67) in Appendix).
2.9 Decentralized Competitive Equilibrium (DCE)

DCE is defined as a sequence of allocations, prices and policies such that: (i) both household types maximize their lifetime welfare, (ii) firms maximize their profits, (iii) goods, capital, labor, dividends and bond markets clear, (iv) all constraints are satisfied, (v) policymakers follow fiscal rules in Section 2.8.

We end up with a non-linear dynamic equilibrium system summarized by 39 equations in 39 unknowns \( \{y_t^f, c_{R,t}, c_{P,t}, l_{R,t}, d_{S,t}, k_{R,t+1}, h_{R,t+1}, b_{R,t+1}, k_{P,t+1}, c_{P,t}, l_{P,t}, r_t^b, \pi_{R,t}, \lambda_{R,t}, \lambda_{P,t}, \psi_{R,t}, \psi_{P,t}, k_t^g, r_t, w_{R,t}, w_{P,t}, q_t, g_t^c, g_t^e, \theta_{R,t}, \theta_{P,t}, g_{P,t}^c, g_{P,t}^e, s_t^g, s_t^h, s_t^{trP}, s_t^{trR}, \tau_t^c, \tau_{R,t}, \tau_{P,t} \} \). This is given the initial conditions for the state variables and the values of the feedback fiscal policy coefficients in the associated fiscal policy rules. We present the clearing market conditions and the full equilibrium system in the Appendices B and C respectively.\(^{17}\)

2.10 Plan of the rest of the paper

In the rest of the paper we work as follows. First, using common structural parameter values and fiscal policy data from the EA-19 over the period 2001-2015 we solve for the steady state solution of the model in Section 3. We explain our calibration strategy in Section 3.1. The long run solution is computed in Section 3.2. Throughout the paper we refer to this solution as the status quo economy. In our policy experiments we use this solution as point of departure in order to evaluate debt consolidation policies under constrained optimized fiscal rules. Our fiscal policy experiment is described in Section 4.

Second, we compute various steady state reformed economies in which public debt to GDP ratio is lower. In the reformed economies thanks to public debt reduction one fiscal instrument can adjust in the reformed steady state to reap the benefit of the fiscal adjustment (fiscal gain). We study various reformed economies depending on which fiscal instrument adjusts in the new steady state. Aggregate and distributional long run effects are computed in Section 5.1.

Third, we compute the transitional dynamics from the status quo economy with high debt to GDP ratio to a reformed economy with lower public debt to GDP ratio. Aggregate and distributional effects in the transition are presented in Section 5.2.

In Section 6 we analyze the transmission mechanism of debt consolidation under the constrained optimized fiscal policy. Finally, we conduct a sensitivity analysis in Section 7.

\(^{17}\) As the DCE system presented in the Appendix C shows, policy instruments distort first order conditions and resource constraints in several ways. These distortions are referred to as wedges in the related macroeconomic literature (see e.g. Chari et al., 2007). In particular, because of consumption taxes, labor taxes, capital taxes, government spending on infrastructure and government spending on education, there are wedges in agents’ labor supply and investment decisions, while, because of government spending on infrastructure, education, utility-enhancing services and transfer payments, there are also wedges in budget constraints and resource constraints. This is typical of economic environments without lump-sum policy instruments. Here, however, the aim is to investigate the implications of debt consolidation by using the full model in hand, rather than to identify the role of each friction/wedge in a historical episode.
3 Calibration and status quo long run equilibrium

In this section we discuss our calibration choices and present the steady state solution of the model.

3.1 Parameter values and fiscal policy data

In Table 1a we report the values of the structural parameters. In Table 1b we report the fiscal policy instruments values using fiscal data averages for the EA-19 over the period 2001-2015. Parameter values and fiscal policy instruments are chosen so that the model’s long run solution targets various observed key macroeconomic ratios of the EA-19 economy (see Table 2). We employ data from the following resources: AMECO database of the European Commission, Eurostat COFOG (Classification of Functions of Government), OECD Education at a Glance (EAG), LFS (labor Force Survey), EU-SILC (Social Income and Living Conditions), Household Finance and Consumption Survey (HFCS), SES (Structure of Earnings). Below we analyze in detail our calibration strategy.

Population shares

In the present model households differ along two dimensions, i.e. access to financial/capital markets and skills. We set \( n_P = 0.3 \) and \( n_R = 0.7 \) so that 30% of total population does not participate in capital and financial markets which is within the range reported by Coenen et al. (2008). This is in line with data on household savings in HFCS which reports that the asymmetric savings distribution is also reflected in income distribution. For instance, in the Euro Area the richest 20% income group holds over 60% of total savings. Regarding the skills distribution, as is defined by the International Standard Classification of Education (ISCED), data from Eurostat indicate that in the EA-19 a range from 30% to 35% has at least attained lower secondary education (i.e. 10 years of education) while the rest has attained at least post-secondary non-tertiary and tertiary education which roughly matches our parameter choices for \( n_R \) and \( n_P \). Finally, data from EU-SILC report that high income groups as well as high savings groups in the population exhibit relatively higher educational attainment rates. Thus, we deem there exists enough evidence to associate savings and income with skills and education.

Preferences and parameters common to all agents

The time discount factor \( \beta \) is set so as to give an annual real interest rate equal to 2.25% which is consistent with data on EA-19 (see AMECO database). The preference parameters \( \{\mu_1, \mu_2, \xi\} \) are calibrated so that the weighted average of skilled and unskilled hours worked is 0.232. It also implies that in steady state Rich households devote more time to labor relative to Poor (see Table 2 in the next subsection).\(^\text{18}\) We set the depreciation rates of physical and public capital \( \{\delta^k, \delta^g\} \) equal to 6%, as in Coenen

\(^{18}\) This is in accordance with the Eurostat LFS data which reports that the average hours worked per week of full time employment is 41.5 hours out 168 hours of the whole week. This gives an average labor time of 0.247 which is almost equal to our calculated value. The same survey reports that workers occupied in skilled professions (e.g. managers, professionals, engineers) record higher weekly hours worked than those of unskilled professions (e.g. clerical stuff, technicians etc). Our numerical solution is consistent with these data.
et al. (2008). Given that there is not a clear consensus on the magnitude of the depreciation of human capital we assume $\delta^h = \delta^k = \delta^\theta = 6\%$.

**Production** We normalize the scale parameter $A$ to 1. We calibrate the parameter, $v = -0.553$, which governs the elasticity of substitution between capital and skilled labor, $\frac{1}{\nu} = 0.6439$, and the parameter, $\sigma = 0.323$, which governs the elasticity of substitution between capital/skilled labor and unskilled labor, $\frac{1}{\nu} = 1.4771$, jointly so as to match the target of 1.55 for the value of skill premium which is consistent with the data.\(^{19}\) Our calibrated values lie within the range of values estimated in Krusell et al. (2000) and Maliar et al. (2017). Then, we set the remaining parameters of the production function, $\{\rho, m\}$, so that the model’s status quo solution is consistent with data on factor inputs shares such as labor income share, capital income share. The choice of the parameter $\alpha$ along with the depreciation rate of physical and public capital imply a physical capital to GDP ratio of 2.97 and a public capital to GDP ratio of 0.51.

**Human capital** Next, we set the parameters governing the production of new human capital of each household type. The sets of parameters $\{B_R, B_P\}$ and $\{x_R, x_P\}$ both relate to technology and ability in the creation of new human capital and skills. For this reason, similar to He and Liu (2008) and Angelopoulos, Benecchi and Malley (2017) we normalize $B_R$ and $B_P$ equal to 1 and let $x_R = 0.40 > x_P = 0.35$ to capture any differences in ability between the two household types. In essence, this choice reflects the standard Mincerian logic which assumes that *rich* households due to their higher education status, obtain higher returns. We set the value of the elasticity parameter of education time with respect to new human capital $\theta$ equal to 0.8 so that $1 - \theta = 0.2$ as in Blankenau et al. (2004). This implies that households devote 13% on average of their time endowment to skill enhancing activities. Note that in the model both time spent on education and public education spending are meant to be post-formal schooling. This implies that both agents have already acquired a minimum of 10 years of education.\(^{20}\)

**Policy** To set the long run values of fiscal variables we employ data from Eurostat for EA-19 over 2001-2015. In particular, we set the long run values of tax instruments equal to the associated 2001-2015 average effective average tax rates, i.e. effective tax rate on consumption is equal to 19.8%, effective tax rate on labor is equal to 46.75% and effective tax rate on capital is equal to 36.8%.\(^{21}\) We set public debt to GDP ratio in the status quo economy equal to

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\(^{19}\)The value of the skill premium typically ranges from 1.45 to 1.80 depending on the country, timeframe and method of measurement (for an overview of inequality measures see Krueger et al. (2008) while for measurement issues see Crivellaro (2016) and OECD Education at a Glance reports (2008-2018)). For our target value we use OECD’s Relative Earnings by educational attainment indicator.

\(^{20}\)Data on the share of time that individuals spend on education are not available at this level of heterogeneity. Thus, we use as proxy the following formula. Skilled individuals devote on average at least 7 additional years to post-formal schooling educational activities. Under the assumption that the average retirement age is 63 years old and that working life begins at 16 years old, this implies 47 years of working life duration or an additional 14.8% of their time devoted to education ($7/47$ years). Similarly assuming that unskilled agents will devote 3 additional years to obtain at least an upper-secondary degree this implies an additional 12% of their time endowment on post-formal education activities.

\(^{21}\)Effective tax rates are constructed following the methodology in Mendoza et al. (1994) more details on the methodology and the associated database are provided in Kostarakos and Varthalitis (2019).
85.3% which is consistent with EA-19 data over the period of fiscal stress between 2008-2014.\textsuperscript{22} Regarding public spending instruments we set transfers as a share of GDP equal to 25.4%.\textsuperscript{23} Recall that as pointed out above we assume that transfers are unevenly distributed between the two household types favouring \textit{Poor} households. Given the difficulty to retrieve the exact share allocated to each household type we assume that \textit{Poor} households receive double the amount of transfers in nominal terms relative to \textit{Rich} households. The rest of the public expenditure sub-components have been retrieved from Eurostat COFOG database which breaks down public spending per functional use. This provides an elegant disentanglement of total public spending into the main spending components of our model. For instance, public spending on education, \( s^{ge} \), is set at 1\%, which is close to the post-formal schooling public spending on education.\textsuperscript{24} For simplicity we assume that this share is equally allocated between the two household types, i.e. we set \( \omega = 0.5 \). Spending on public investment as a share of GDP, \( s^{gi} \), is set at 3.1\%, based on data reported in the Economic Affairs function of the COFOG database. Finally, we allow the output share of government consumption, \( s^{gc} \), to be residually determined.\textsuperscript{25}

\textsuperscript{22}Particularly we use the time series "General government consolidated gross debt - Excessive deficit procedure based on ESA 2010".

\textsuperscript{23}This includes the items of social benefits and other current transfers as those are defined in Eurostat National Accounts Indicators (ESA 2010).

\textsuperscript{24}This includes mainly government expenditure on tertiary education, post-secondary education other than tertiary and upper secondary education level.

\textsuperscript{25}In our model the definition of government consumption follows the COFOG database definition, and thus, includes a broad range of government functions such as general public services, public order and defence, recreation and culture, environmental protection, household and community amenities, health.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \beta \leq 1$</td>
<td>time discount factor</td>
<td>0.978</td>
</tr>
<tr>
<td>$0 &lt; \xi &lt; 1$</td>
<td>public consumption weight in composite consumption</td>
<td>0.100</td>
</tr>
<tr>
<td>( \mu_1 &gt; 0 )</td>
<td>preference weight in the utility</td>
<td>0.400</td>
</tr>
<tr>
<td>( \mu_2 &gt; 0 )</td>
<td>preference weight in the utility</td>
<td>0.600</td>
</tr>
<tr>
<td>$0 \leq \delta^k \leq 1$</td>
<td>depreciation rate of physical capital</td>
<td>0.060</td>
</tr>
<tr>
<td>$0 \leq \delta^h \leq 1$</td>
<td>depreciation rate of human capital</td>
<td>0.060</td>
</tr>
<tr>
<td>$0 \leq \delta^g \leq 1$</td>
<td>depreciation rate of public capital</td>
<td>0.060</td>
</tr>
<tr>
<td>$0 &lt; n_R &lt; 1$</td>
<td>population share of Rich</td>
<td>0.700</td>
</tr>
<tr>
<td>$0 &lt; n_P &lt; 1$</td>
<td>population share of Poor</td>
<td>0.300</td>
</tr>
<tr>
<td>$0 &lt; \alpha &lt; 1$</td>
<td>share of composite input</td>
<td>0.980</td>
</tr>
<tr>
<td>( \sigma &lt; 1 )</td>
<td>capital and skilled labor to unskilled labor substitution</td>
<td>0.323</td>
</tr>
<tr>
<td>( v &lt; 1 )</td>
<td>capital to skilled labor substitution</td>
<td>-0.553</td>
</tr>
<tr>
<td>$0 &lt; m &lt; 1$</td>
<td>labor share of Rich</td>
<td>0.200</td>
</tr>
<tr>
<td>$0 &lt; \rho &lt; 1$</td>
<td>share of physical capital in the composite input</td>
<td>0.680</td>
</tr>
<tr>
<td>$0 \leq \theta \leq 1$</td>
<td>elasticity of education time</td>
<td>0.800</td>
</tr>
<tr>
<td>$0 \leq 1 - \theta \leq 1$</td>
<td>elasticity of public education spending</td>
<td>0.200</td>
</tr>
<tr>
<td>$A &gt; 0$</td>
<td>scale parameter</td>
<td>1.000</td>
</tr>
<tr>
<td>$B_R &gt; 0$</td>
<td>human capital technology parameter of Rich</td>
<td>1.000</td>
</tr>
<tr>
<td>$B_P &gt; 0$</td>
<td>human capital technology parameter of Poor</td>
<td>1.000</td>
</tr>
<tr>
<td>$0 &lt; x_R &lt; 1$</td>
<td>returns to scale for new human capital of Rich</td>
<td>0.400</td>
</tr>
<tr>
<td>$0 &lt; x_P &lt; 1$</td>
<td>returns to scale for new human capital of Poor</td>
<td>0.350</td>
</tr>
</tbody>
</table>
### Table 1b: Fiscal policy instruments

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^k$</td>
<td>capital tax rate</td>
<td>0.368</td>
</tr>
<tr>
<td>$\tau^l$</td>
<td>labor tax rate</td>
<td>0.467</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>consumption tax rate</td>
<td>0.198</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Public spending</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^g^e$</td>
</tr>
<tr>
<td>$s^g^c$</td>
</tr>
<tr>
<td>$s^g^i$</td>
</tr>
<tr>
<td>$s^{tr}$</td>
</tr>
</tbody>
</table>

#### 3.2 Status quo steady state solution

The steady state solution of the model, when we use the parameter values and the policy instruments of Tables 1a-1b, is reported in Table 2. In what follows, we refer to this steady state solution as the status quo economy and will serve as the point of departure for the fiscal policy experiments studied in the next sections. The implied numerical solution replicates some key macroeconomic ratios observed in the EA-19 like consumption as a share of output, physical capital as a share of output, debt as a share of output and skill premium.\(^{26}\)

---

\(^{26}\)Note that the values in Table 2 are defined as follows: $\bar{r} \equiv (1 - \tau^k)(r - \delta^k) \equiv r^b$, $c \equiv \frac{n^bcR^b+n^cR^c}{n^fY}$, $kR \equiv \frac{n^kR^k}{n^fY}$, $q \equiv \frac{n^qR^q}{n^fY}$. 

Table 2: Status quo solution

<table>
<thead>
<tr>
<th>Main variables</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c_R)</td>
<td>consumption of Rich</td>
</tr>
<tr>
<td>(c_P)</td>
<td>consumption of Poor</td>
</tr>
<tr>
<td>(l_R)</td>
<td>skilled labor</td>
</tr>
<tr>
<td>(l_P)</td>
<td>unskilled labor</td>
</tr>
<tr>
<td>(e_R)</td>
<td>time in education of Rich</td>
</tr>
<tr>
<td>(e_P)</td>
<td>time in education of Poor</td>
</tr>
<tr>
<td>(\bar{r})</td>
<td>real return to physical capital</td>
</tr>
<tr>
<td>(y)</td>
<td>output</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Key ratios</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{c}{y})</td>
<td>consumption as share of GDP</td>
<td>0.5580</td>
</tr>
<tr>
<td>(\frac{k_R}{y})</td>
<td>physical capital as share of GDP</td>
<td>2.9600</td>
</tr>
<tr>
<td>(q)</td>
<td>debt as a share of GDP</td>
<td>0.8530</td>
</tr>
<tr>
<td>(\frac{w_R}{w_P})</td>
<td>skill premium</td>
<td>1.5500</td>
</tr>
</tbody>
</table>

4 Fiscal policy experiment

In this section, we define the fiscal policy experiments. The economy starts from its status quo steady state, as computed in Table 2, and travels towards a new reformed steady state with lower public debt-to-GDP ratio. Since public debt-to-GDP ratio is lower in the new reformed economy, the government can exploit the resulting fiscal space to increase public spending and/or reduce taxes. On the other hand, the opposite should happen in the transition to the new reformed steady state since such a debt reduction unavoidably requires cuts in spending and/or rises in taxes in the short and medium term.

Specifically, in the new reformed steady state, the debt-to-output ratio is reduced from 85.3%, which is the EA-19 data average over the fiscal stress period 2008-2014, to 60%; this exogenous reduction allows one spending (tax) instrument to increase (decrease) residually. Note that the 60% policy target is chosen simply to reflect the criteria set by the Maastricht Treaty. The government can achieve the transition from the status quo to the new reformed steady state by implementing different fiscal policy mixes following the fiscal feedback rules given by equations (60)-(67) in the Appendix C. We will compute constrained optimized fiscal policy rules so as our results will not depend on ad hoc differences in feedback policy coefficients. To do this, we follow Schmitt-Grohé and Uribe (2004, 2007) so that we limit our attention to fiscal rules that satisfy the following conditions. First, they are constrained, i.e. the fiscal instruments can react to a small number of easily observable macroeconomic indicators. Second, this reaction guarantees
local determinacy of the rational expectations equilibrium.\textsuperscript{27} Third, they are optimized, i.e. policymakers choose the feedback policy coefficients to maximize a welfare criterion. The welfare criterion is the weighted conditional welfare of the *Rich* and *Poor* households as defined in (1) and (8) respectively, i.e. \( W_0 = \bar{n}V_{R,0} + (1 - \bar{n})V_{P,0} \), where \( \bar{n} \) denotes the weight assigned to *Rich* households’ lifetime welfare. In our main experiments, we assume a Benthamite government meaning that we set the weights in the government’s objective function equal to the associated population shares, i.e. \( \bar{n} = n_R \). Notice that welfare is computed conditional on the initial conditions which are given by the status quo solution as computed in Table 2.\textsuperscript{28}

5 Main Results

In this section, we present the aggregate and distributional implications of fiscal consolidation as defined above. Subsection 5.1 focuses on the reformed steady state, while, transition results are in subsection 5.2. To quantify aggregate effects, we will employ variables such as output, \( y \), and social welfare, \( W \), while, to measure distributional effects, we find it natural to use variables like the net income\textsuperscript{29} of *Rich* and *Poor* households, \( y^\text{net}_R \) and \( y^\text{net}_P \) respectively, the relative net income (defined as the ratio of the *Rich* household’s net income to the net income of the *Poor* household, \( \frac{y^\text{net}_R}{y^\text{net}_P} \)), the skill premium (which is typically defined as \( \frac{w_R}{w_P} \)), household specific welfares, \( V_R \) and \( V_P \), and the relative welfare (defined as the ratio of the *Rich* household’s discounted lifetime utility to the discounted lifetime utility of the *Poor* household, \( \frac{V_R}{V_P} \)).

5.1 Aggregate and distributional implications of debt consolidation in the reformed steady state

In this subsection, we present the new reformed steady state. As in e.g. Coenen et al. (2008) and Philippopoulos et al. (2015, 2017a, 2017b), once the debt-to-output ratio has been reduced, the government can increase spending or/and decrease taxes by taking advantage of the fiscal space created; this is the so called long run fiscal gain of debt consolidation.

Table 3 presents the key endogenous variables of the model capturing aggregate and distributional effects. In this table, we vary the residual fiscal instrument that adjusts in the long run and present the associated values of output, \( y \), household specific net incomes, \( y^\text{net}_R \) and \( y^\text{net}_P \), the relative net income, \( \frac{y^\text{net}_R}{y^\text{net}_P} \), and skill premium, \( \frac{w_R}{w_P} \). All values in Table 3 are reported

\textsuperscript{27}In addition, when necessary, we further restrict the vector of policy coefficients to lie within a space that delivers plausible values for fiscal instruments, i.e. \( s^g,s^b,s^{trp},s^{sprp} \) > 0 and \( 0 < \tau^r,\tau^b,\tau^n < 1 \).

\textsuperscript{28}In particular, we take a second order approximation to both the equilibrium conditions and the welfare criterion. First, we compute a second order approximation of both conditional welfare and the decentralized equilibrium around the reformed steady state as functions of the vector of feedback policy coefficients. Then, we use an optimization routine like \texttt{fminsearch} to compute the values of the feedback policy coefficients that maximize the conditional welfare criterion. For more details see Philippopoulos et al. (2017a) and (2017b). Dynare and Matlab routines are available upon request.

\textsuperscript{29}Net income is defined as gross income minus all types of taxes, i.e. net income of *Rich* households is \( y^\text{net}_{R,t} \equiv (1 - \tau^l) (\tau r_{R,t} + \pi_t) + (1 - \tau^l) w_R I_{R,t} h_{R,t} + \tau^b h_{R,t} + \tau^c c_{R,t} \), and of *Poor* households is \( y^\text{net}_{P,t} \equiv (1 - \tau^l) w_P I_{P,t} h_{P,t} + \tau^b h_{P,t} + \tau^c c_{P,t} \).
as percentage deviations from their status quo values. A positive (negative) value implies an increase (decrease) vis-à-vis its status quo value. The first column of Table 3 reports which fiscal instrument adjusts in the new steady state taking advantage of the post-consolidation fiscal space, while, in the last column we compute the magnitude of the associated fiscal adjustment. The fiscal instruments which adjust in the new reformed economies are the output share of government consumption, $s^{g}$, the output share of government investment, $s^{i}$, the output share of public transfers to Poor households, $s^{trp}$, the output share of public spending on education, $s^{e}$, the tax rates on consumption, $\tau^{c}$, labor income, $\tau^{l}$, and capital income, $\tau^{k}$. To understand the mechanisms of each reform, we experiment with one spending/tax policy instrument at a time keeping the others constant at their status quo values.

Table 3: Steady state output and distributional effects in the various reformed economies (as % deviations from status quo)

<table>
<thead>
<tr>
<th>Fiscal Instr.</th>
<th>$y$</th>
<th>$y_{R}^\text{net}$</th>
<th>$y_{P}^\text{net}$</th>
<th>$w_{R}^\text{net}$</th>
<th>$w_{P}^\text{net}$</th>
<th>$\Delta \text{Inst}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^{g}$</td>
<td>0.0069</td>
<td>-0.0042</td>
<td>0.0058</td>
<td>-0.0100</td>
<td>-0.0077</td>
<td>0.0047</td>
</tr>
<tr>
<td>$s^{i}$</td>
<td>0.0124</td>
<td>0.0006</td>
<td>0.0111</td>
<td>-0.0095</td>
<td>-0.0069</td>
<td>0.0047</td>
</tr>
<tr>
<td>$s^{trp}$</td>
<td>0.0009</td>
<td>-0.0034</td>
<td>0.0128</td>
<td>-0.0162</td>
<td>-0.0129</td>
<td>0.0056</td>
</tr>
<tr>
<td>$s^{e}$</td>
<td>0.0029</td>
<td>0.0168</td>
<td>0.0261</td>
<td>-0.0093</td>
<td>-0.0120</td>
<td>0.0047</td>
</tr>
<tr>
<td>$\tau^{c}$</td>
<td>0.0079</td>
<td>0.0052</td>
<td>0.0153</td>
<td>-0.0101</td>
<td>-0.0074</td>
<td>-0.0101</td>
</tr>
<tr>
<td>$\tau^{l}$</td>
<td>0.0158</td>
<td>0.0137</td>
<td>0.0217</td>
<td>-0.0080</td>
<td>-0.0051</td>
<td>-0.0095</td>
</tr>
<tr>
<td>$\tau^{k}$</td>
<td>0.0175</td>
<td>0.0119</td>
<td>0.0150</td>
<td>-0.0031</td>
<td>-0.0005</td>
<td>-0.0133</td>
</tr>
</tbody>
</table>

A key message from Table 3 is that debt consolidation is always output enhancing and, perhaps more interestingly, progressive in the long run. That is, output, $y$, increases (see column 2 in Table 3), while relative net income (income inequality), $y_{R}^\text{net}$, and skill premium (wage inequality), $w_{R}^\text{net}$, decrease (see column 4 and 5 in Table 3 respectively) vis-a-vis their status quo value in all reformed economies. Thus, fiscal consolidation can improve both equity and efficiency in the long run.

Regarding output, debt consolidation is more productive in the long run when the government increases its productive spending (like public spending on education or investment) or reduces distortionary taxation. Actually, the best instrument to use in terms of output, as well as in terms of net income of all individuals, is public spending on education.

The relative income, $y_{R}^\text{net}$, decreases in all reformed economies which means that debt consolidation benefits more the Poor households. This is mostly driven by the sharp decrease in the income coming from government bonds earned by Rich households which gets lower under debt consolidation.\footnote{To save space, we do not report results for public transfers to Rich household, $s^{trp}$. Note that for Rich households the Ricardian equivalence holds, thus, the long run effects are trivial.} The reduction in the net income of Rich households is less striking when the government reduces income taxes (see the last two rows in Table 3) for two reasons. First,\footnote{These findings are in line with Schwarzmüller and Wolters (2015). However, they focus on consumption inequality rather than net income and wages.}
lower income taxes imply higher wealth; recall that *Rich* households earn capital income while they receive higher wages than *Poor* households.\(^{32}\) Second, the resulting increase in output requires additional physical capital and, due to the complementarity between physical capital and skilled labor, more skilled than unskilled labor, which again favors the *Rich* households. These effects moderate the adverse effect on the net income of *Rich* households from the decrease in income coming from bond holdings. In terms of wage inequality (see column 4 in Table 3), debt consolidation always reduces the skill premium in the long run.\(^{33}\)

Similar studies have examined the effects of fiscal consolidation on aggregate welfare (see e.g. Bi and Kumhof (2011) and Philippopoulos et al. (2017a) and (2017b)). To this end, in Table 4, we compute the percentage deviations of social welfare, \(W\), *Rich* and *Poor* households’ discounted lifetime steady state welfare, \(V_R\) and \(V_P\) respectively, as well as the relative welfare, \(\frac{V_R}{V_P}\).

<table>
<thead>
<tr>
<th>Fiscal Instr.</th>
<th>(W)</th>
<th>(V_R)</th>
<th>(V_P)</th>
<th>(\frac{V_R}{V_P})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s^g)</td>
<td>-0.0077</td>
<td>-0.0225</td>
<td>0.0174</td>
<td>-0.0407</td>
</tr>
<tr>
<td>(s^g_i)</td>
<td>0.0042</td>
<td>-0.0089</td>
<td>0.0264</td>
<td>-0.0363</td>
</tr>
<tr>
<td>(s^{tr}P)</td>
<td>0.0047</td>
<td>-0.0158</td>
<td>0.0394</td>
<td>-0.0574</td>
</tr>
<tr>
<td>(s^g_e)</td>
<td>0.0781</td>
<td>0.0769</td>
<td>0.0801</td>
<td>-0.0035</td>
</tr>
<tr>
<td>(\tau^c)</td>
<td>0.0131</td>
<td>0.0009</td>
<td>0.0337</td>
<td>-0.0339</td>
</tr>
<tr>
<td>(\tau^l)</td>
<td>0.0235</td>
<td>0.0145</td>
<td>0.0387</td>
<td>-0.0252</td>
</tr>
<tr>
<td>(\tau^k)</td>
<td>0.0271</td>
<td>0.0214</td>
<td>0.0368</td>
<td>-0.0161</td>
</tr>
</tbody>
</table>

Welfare results in Table 4 are consistent with the findings in Table 3. In particular, fiscal consolidation enhances social welfare and also reduces the welfare gap between *Rich* and *Poor* households in all reformed economies. Similarly to the results in Table 3, the increase in government spending on education yields the highest welfare gains both for the society and for each income class of households. Also, using the fiscal space created by debt consolidation to finance an increase in spending on public education and/or to finance a cut in distortionary taxation

\(^{32}\)Table E.1 in Appendix E presents the various sources of income by household type as percentage deviations from their status quo value.

\(^{33}\)The net effect on skill premium depends on which of the capital-skill complementarity or the relative skill supply effect dominates. On the one hand, the increase in output requires more physical capital pushing skill premium upwards. On the other hand, relative skill supply increases pushing skill premium in the opposite direction. In our experiments the latter effect is stronger.

\(^{34}\)In Table 4 a positive (negative) value denotes a welfare improvement (deterioration) vis-à-vis its status quo value. Since we assume logarithmic utility function, welfare takes negative values, i.e. \(W, V_R, V_P < 0\). For illustrative purposes, we define the percentage deviations of, say social welfare measure, from its status quo value as \(\frac{(W^\phi - (W^{SQ}))/W^{SQ}}{(W^{SQ})}\), where \(W^\phi\) and \(W^{SQ}\) denote social steady state welfare in any reformed economy and status quo economy respectively. In contrast the relative welfare is a positive number, \(\frac{V_R}{V_P} > 0\). In Table 4, we report \(\frac{(V_R^{SQ} - (V_P^{SQ}))}{(V_P^{SQ})}\). As before, a positive (negative) value means that the welfare gap increases (decreases).
is Pareto efficient, i.e., both Rich and Poor households get better off in the reformed economy. On the other hand, fiscal consolidation is not always Pareto efficient. For example, although increases in government consumption, public transfers targeted to Poor households and public investment can also enhance the aggregate output, while, the last two also improve the social welfare, they are not Pareto efficient. Any small social welfare gains arise solely because of the increase in Poor households’ welfare; the results in Table 4 show that the welfare of the Rich is reduced.

5.2 Aggregate and distributional implications of debt consolidation along the transition

In this section, we focus our analysis on the transition implications of public debt consolidation. The economy departs from its status quo steady state and moves towards a new reformed economy with lower debt to GDP ratio equal to 60%.\(^{35}\) This requires fiscal policy to use one (or more) fiscal instruments to react to debt deviations from its new target. We focus our analysis on constrained optimized fiscal policy rules as analyzed in section 4.

The optimized policy mix suggest that fiscal policy should cut government consumption sharply to consolidate its debt while at the same time keep constant distortionary taxation and productive public spending. The resulting optimized values of feedback policy coefficients reported in the note of Table 5 suggest that increasing distortionary income taxation and/or decreasing productive spending to reduce public debt is not recommended. These results are consistent with findings in similar studies see in Philippopoulos et al. (2015) and (2017b).

Table 5 computes the average percentage deviations from their status quo value of output, \(y\), relative net income, \(\frac{y\text{net}}{y\text{net}}\), and skill premium, \(\frac{wR}{wP}\), over various time horizons under the constrained optimized fiscal policy.\(^{36}\)

<table>
<thead>
<tr>
<th>Variable</th>
<th>2 years</th>
<th>4 years</th>
<th>6 years</th>
<th>10 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>0.002</td>
<td>0.013</td>
<td>0.0155</td>
<td>0.016</td>
</tr>
<tr>
<td>(\frac{y\text{net}}{y\text{net}})</td>
<td>-0.037</td>
<td>-0.033</td>
<td>-0.031</td>
<td>-0.027</td>
</tr>
<tr>
<td>(\frac{wR}{wP})</td>
<td>0.0443</td>
<td>0.0277</td>
<td>0.0208</td>
<td>0.0138</td>
</tr>
</tbody>
</table>

Note: The constrained optimized policy coefficients are
\[\gamma^{c} = \gamma^{l} = \gamma^{k} = \gamma^{g} = \gamma^{g}^{q} = \gamma^{g}^{tr} = \gamma^{g}^{tr} = 0\]

\(^{35}\)To save space, we present results for the transition to a reformed economy in which government consumption is the fiscal instrument that reaps the benefit of debt reduction. In section 7.3 we present the associated transitional dynamics when the economy travels towards the reformed economies where public spending on education or capital tax are the residual fiscal instruments. Results for the rest of the reformed economies reported in Table 3 are available upon request.

\(^{36}\)Table 5 presents results from model based simulations generated by the first order accurate approximation of the equilibrium system.
Results in Table 5 imply that fiscal consolidation is efficient in the medium and long run meaning that aggregate output, $y$, increases above its pre-reform value after the second year. Also, the relative net income, $\frac{y_{\text{net}}}{y_{\text{P}}}$, is reduced over all time horizons. This is attributed to the sharp reduction of the Rich households’ income coming from bonds over all time horizons and to the temporary reduction in their capital income. However, wage inequality, $\frac{w_{\text{R}}}{w_{\text{P}}}$, widens for a prolonged period. As we also discuss below, debt consolidation results in an increase in physical capital stock which due to the capital-skill complementarity channel in our model increases skill premium.

Similarly, Table 6 presents social welfare and household specific welfare as percentage deviations from a reference regime\(^{37}\) under the constrained optimized fiscal consolidation policy over various time horizons.\(^{38}\) A positive value implies that the associated welfare criterion increases vis-à-vis the reference regime.

<table>
<thead>
<tr>
<th>Table 6: Welfare over various time horizons with optimized policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare</td>
</tr>
<tr>
<td>Social</td>
</tr>
<tr>
<td>Rich</td>
</tr>
<tr>
<td>Poor</td>
</tr>
</tbody>
</table>

Note: Feedbacks as in Table 5.

Welfare results in Table 6 suggest that fiscal consolidation is social welfare enhancing over all time horizons (see second row), however, it does not produce a Pareto efficient outcome in the short run. In particular, in the first four years this reform hurts Poor households and benefits Rich households. As can be seen in Table 6, Poor households’ welfare is reduced over the first four years (see fourth row), while, Rich households welfare increases in short/medium run (see third row). That is debt consolidation is Pareto efficient only in the medium and long run. To understand the logic of the results in Tables 5 and 6 we discuss the underlying transmission mechanism in the next section.

6 The underlying transmission mechanism

In this section we present the dynamic paths of the key endogenous variables of the model during the transition phase of fiscal consolidation. This provides insight into the transmission channel of fiscal consolidation policy in our economy and allows us to understand the logic of our results in Section 5. As before, the economy departs from its status quo steady state and travels to one of the reformed economies reported in Section 5.1. Figure 1 depicts the dynamic paths of the

\(^{37}\) We can use multiple reference regimes, for simplicity and comparability with results in Table 5, we compute percentage deviations from a reference regime in which the economy stays at its status quo steady state forever.

\(^{38}\) Table 6 presents results from model-based simulations generated by the second order accurate approximation of the equilibrium system and the welfare criteria.
main endogenous variables under the constrained optimized fiscal policy, while, in the long run, the government uses the resulting fiscal space to increase government consumption.

Figure 1: Dynamic paths of the main endogenous variables with optimized policy

Notes: Endogenous variables are expressed as percentage deviations from their status quo value. Total consumption is defined as \( c = n_R c_R + n_P c_P \).

We start with aggregate effects. Fiscal consolidation implemented via the constrained optimized fiscal policy (i.e. via government consumption cuts\(^{39}\)) decreases aggregate demand which induces a temporary output contraction (see the dynamic path of \( y \)). Subsequently, the debt adjustment and the sharp reduction in government consumption cause a reduction in real interest rates, \( r^b \) and \( r \). This crowds in total private consumption, \( c \), and investment, \( i_R \), whereas the latter results in the rise of the physical capital stock; see the dynamic path of \( k_R \). Thus, output recovers above its pre consolidation value and converges to its new higher value in the reformed steady state. The increase in the capital stock is significant and prolonged as the economy converges to a reformed steady state with higher capital stock.

We now turn to the distributional implications of fiscal consolidation. First, due to the capital-skill complementarity channel, the rise in physical capital induce an increase in the wages of Rich (skilled) households,\(^{40}\) see the dynamic path of \( w_R \). Thus, the skill premium (wage

\(^{39}\)To save space we present the dynamic paths of fiscal policy instruments and other endogenous variables in Figure E.1 in the Appendix E.

\(^{40}\)At the same time, Rich households substitute labor with leisure in the very short run (see Figure E.1 in the Appendix E) and thus relative skill supply, \( \frac{t_R c_R h_R}{t_P c_P h_P} \), decreases in equilibrium. Both relative skill supply and capital skill complementarity effects push upward skill premium.
inequality), \( \frac{w_R}{w_P} \), increases in the transition phase of fiscal consolidation. On the other hand, relative net income, \( \frac{y_{net}^R}{y_{net}^P} \), falls over the transition to the reformed steady state and converges to its new lower value. The relative net income is a function of various endogenous variables of the model, thus, to understand its dynamic path, we decompose it into the various sources of income of Rich and Poor households. Results in Figure 1 indicate that the decrease in the relative income can be mostly attributed to the sharp decrease in the income coming from government bonds (see the dynamic paths of \( r^b b_R \)), the temporary reduction in the capital income due to the fall in the return on capital (see the dynamic path of \( r^k k_R \) respectively) and the higher income taxes paid by Rich households. On the other hand, the labor income of Rich households, \( w_R h_R \), increases relatively more than the labor income of Poor households, \( w_P h_P \), due to the capital-skill complementarity effect.

Finally, to understand the welfare results in Table 6, we focus on the variables that enter the utility functions of Rich and Poor households. Consumption of Rich households, \( c_R \), increases due to the crowding in effects induced by debt consolidation and the sharp fall in interest rates. Also, since the labor income of Rich households increases (mostly due to the increase in their wages), these households substitute hours worked with leisure. Both effects enhance the welfare of Rich households in the short/medium run.

On the other hand, debt consolidation puts downward pressure to the wages of Poor (unskilled) households, \( w_P \), and thus they reduce consumption, \( c_P \), on impact; recall that they cannot smooth their consumption over time. In addition, in the short run, due to lower wages, they need to increase their labor supply to boost their labor income. Both consumption and labor dynamics contribute to the welfare losses experienced by Poor households in the short run.

7 Sensitivity analysis

In this section we conduct a sensitivity analysis. In Sections 7.1 and 7.2, we focus on the key sources of inequality in our model to shed light on the different channels through which they affect our main results. In section 7.3, we discuss transitional dynamics towards alternative reformed economies. In Section 7.4, we discuss similarities of constrained optimized fiscal policy with a Ramsey-type policy in our model. Finally, Section 7.5 reports sensitivity tests with respect to other key structural parameters of the model. In all the exercises conducted in this section, we compute the constrained optimized fiscal policy rules as stated in Section 4 under the new calibration and/or modelling changes. Here, we report that the optimized policy mix remains the same.

\footnote{In an earlier version of this paper we show that income inequality can increase in the short run under an ad hoc (non optimized) spending based fiscal consolidation policy see in Sakkas and Varthalitis (2018).}

\footnote{Although government imposes identical tax rates to both households, distortionary taxes are analogous to income. This implies that a relatively higher income increase is accompanied with higher taxes.}
7.1 Capital-skill complementarity

In this section we focus on the structural parameter that governs the capital-skill complementarity (CSC), i.e. $v < 1$, in equation (13) in section 2.5. This parameter governs the elasticity of substitution between physical capital and skilled labor, $\frac{1}{1-v}$. A smaller (larger) value of $v$ (i.e. a smaller (larger) value of $\frac{1}{1-v}$) corresponds to a higher (lower) degree of complementarity between skilled labor and physical capital.

Figure 2 presents the dynamic paths of the key endogenous variables of the model in three economies when we vary the degree of capital-skill complementarity. First, the blue lines present the dynamic responses from Section 6 (we include these for comparison); recall that the value of $v$ in the benchmark calibration is equal to $-0.553$ (which means $\frac{1}{1-v} = 0.6439$). Second, the yellow lines present the dynamic responses in an economy which exhibits lower capital-skill complementarity (labeled as low-CSC), for example we set $v = 0.1 > -0.55$. Notice that this value implies a higher elasticity of substitution between capital and skilled labor with respect to the benchmark economy, i.e. $\frac{1}{1-v} = 1.11 > 0.6439$. Third, the red lines simulate an economy which exhibits higher capital-skill complementarity (labeled as high-CSC), for example we set $v = -1 < -0.55$. Similarly, this value implies a lower elasticity of substitution between capital and skilled labor with respect to the benchmark economy, i.e. $\frac{1}{1-v} = 0.5 < 0.6439$.

Figure 2: Dynamic paths of the main endogenous variables under various degrees of capital-skill complementarity

Notes: See notes in Figure 1.
Comparing the dynamic paths of aggregate output, $y$, relative income, $\frac{y^{\text{net}}}{y^P}$, and skill premium, $\frac{w_R}{w_P}$, we infer that debt consolidation’s aggregate and distributional effects remain qualitatively similar across the three economies (compare benchmark, blue lines, with high-CSC, red lines, and low-CSC, yellow lines).

However, as expected, the capital-skill complementarity channel plays a key role in the dynamics of wage inequality (see the dynamic path of $\frac{w_R}{w_P}$ across the three economies). A larger degree of capital-skill complementarity (i.e. lower elasticity of substitution between capital and skilled labor, $\frac{1}{\sigma}$) favors Rich households in the early phase of fiscal consolidation in terms of wage and labor income. Particularly, as the degree of capital-skill complementarity increases, the wage of Rich households, $w_R$, increases more, while the wage of Poor households, $w_P$, decreases relatively more and thus wage inequality, $\frac{w_R}{w_P}$, widens. Similarly, the increase in the labor income of Rich households, $w_RlRh_R$, is larger, while, the increase in the labor income of Poor household, $w_plPh_P$, is smaller as we increase the degree of capital skill complementarity.

The logic of these results is the following. A larger degree of capital-skill complementarity (i.e. a lower value of $v$) in the production function results in, ceteris paribus, a larger reduction in the marginal product of capital, see the dynamic path of $r$. This subsequently leads to a larger rise in the physical capital stock in the short run, $k_R$, and thus, due to the higher capital-skill complementarity, a higher rise in the wage of Rich (skilled) households, $w_R$.

On the other hand, the short run reduction in the relative income, $\frac{y^{\text{net}}}{y^P}$, is relatively bigger (smaller) on impact in the high (low)-CSC economy, while, in the medium run, the dynamic paths are almost identical with the one of the benchmark economy. This mostly happens due to the larger (smaller) reduction in real rates, see the dynamic paths of $r$ and $r^b$, which imply a larger (smaller) decrease in capital income, $rk_R$, and income coming from government bonds, $r^bR$, for Rich households.

### 7.2 Access to capital/financial markets

In this section we relax the assumption that Poor households have no access to financial/capital markets. In particular, we allow both Rich and Poor households to accumulate physical capital and save in the form of government bonds subject to transactions costs as in e.g. Angelopoulos et al. (2014). The role of transactions costs is twofold, first to guarantee a well defined steady state solution and ensure stationarity in the transition, second to allow us to maintain heterogeneity in asset holdings between Rich and Poor households. To do this, we assume that Poor households face consistently higher transactions costs than Rich households which implies that Rich households have consistently higher wealth than Poor households. To save space we present modelling details in Appendix D. For illustrative purposes, here we report that the associated parameters which measure the size of these transactions costs for capital and government bonds are denoted as $\chi_R, \phi_R$ and $\chi_P, \phi_P$ for Rich and Poor households respectively.

To understand the role of asset heterogeneity in our main results, in this section, we present results from three economies varying the degree of financial constraints faced by Poor households.
As we increase the magnitude of the associated transactions costs for Poor households, asset heterogeneity increases since the Poor reduce their asset holdings and rely more on labor income.

In Figure 3, first, we simulate an economy in which we allow Poor households to accumulate physical capital and save in the form of government bonds. As said, to maintain asset heterogeneity they are subject to relatively higher transaction costs than Rich households, for example we set $\chi_R = \phi_R = 0.001 < \chi_P = \phi_P = 0.01$. Second, we simulate an economy in which we further increase asset heterogeneity between the Rich and the Poor. That is Poor households can still accumulate physical capital and save in the form of government bonds but now are subject to much higher transaction costs than Rich households, for example we set $\chi_R = \phi_R = 0.001 < \chi_P = \phi_P = 0.1$. The latter economy is closer to the economy presented in Section 6, where Poor households live hand to mouth. Finally, for comparison, we also include results from Section 6.

Figure 3 presents the dynamic responses of the key endogenous variables in the three economies under the constrained optimized fiscal rules.

Figure 3: Dynamic paths of the main endogenous variables under various degrees of asset heterogeneity

Notes: See notes in Figure 1. Total physical capital and private investment are $k \equiv n_Rk_R + n_Pk_P$ and $i \equiv n_Ri_R + n_Pi_P$ respectively. We set $\chi_R = \phi_R = 0.001$ in the economies represented with red and yellow lines.

Figure 3 shows that the degree of asset heterogeneity does not alter significantly the aggregate effects of fiscal consolidation, however, as expected, it affects its distributional implications.
Perhaps, a rather novel finding is that the degree of asset heterogeneity affects significantly wage inequality $\frac{w_R}{w_P}$. That is the higher the transactions costs faced by Poor households, i.e. the larger the inequality in asset distribution, the larger the increase in wage inequality in the short run phase of fiscal consolidation. This is apparent when we compare the impact response of $\frac{w_R}{w_P}$ in the benchmark economy, where, Poor households have zero access in financial markets (blue lines) with the economies where we allow them partial access to financial markets subject to high/low transactions costs (see red/yellow lines respectively).

In the latter economies, the negative effect of debt consolidation on Poor households is less severe since they do not solely rely on labor income. As a result they do not need to increase labor supply and substitute leisure with labor to the same extent as in the benchmark economy. For that reason, the equilibrium wage of Poor households, $w_P$, increases in the short run which leads to a smaller increase in wage inequality vis-à-vis the benchmark economy.

Finally, as expected, the relative net income, $\frac{y_{netR}}{y_{netP}}$, reduces less as we decrease asset heterogeneity. Now, since Poor households earn income from government bonds and capital, their income decreases analogously to the income of Rich households coming from the same sources (due to reasons discussed in Section 6) causing a smaller reduction in the relative income (income inequality).

### 7.3 Transition towards alternative reformed economies

In section 5.1, we showed that the long run aggregate and distributional effects of debt consolidation depend on the fiscal instrument which adjusts in the new reformed steady state to exploit the post consolidation fiscal space. So far in our analysis of the transmission mechanism, we present results from the case in which fiscal policy adjusts government consumption in the new reformed steady state. In Figure 4, we present the dynamic paths of key endogenous variables of the model when the economy departs from the status quo steady state, as above, and converges to two of the alternative reformed economies discussed in Section 5.1. That is, fiscal policy implements the constrained optimized fiscal policy rules computed as in Section 4, while, the economy converges to two alternative reformed steady states, namely, where the government increases public spending on education, $s^e$, or decreases capital tax, $\tau^k$. For notational convenience, in this section we label each of these policy experiments using the fiscal instrument utilized in the new reformed steady state.

As above, for comparison, the blue lines present the dynamic responses of Section 6 (labeled as $s^e$). In addition, the red lines present the dynamic responses for the case in which the government exploits the fiscal space to increase public spending on education in the new reformed steady state (labelled as $s^e$). Finally, yellow lines present the dynamic responses for the case in which the government cuts capital taxes in the new reformed steady state (labelled as $\tau^k$). We choose these reformed economies simply because they deliver the best steady state welfare and output outcomes (see Tables 3 and 4 in Section 5.1).
We report that the constrained optimized fiscal policy does not depend on the reformed steady state at which the economy travels to. Also, Figure 4 indicates that our key results analyzed in Section 6 remain the same. However, some new results are worth mentioning. First, the larger long run aggregate benefit of fiscal consolidation, arising from a more efficient allocation of the post consolidation fiscal space, can be brought forward in the short/medium run. This is reflected in the dynamic paths of endogenous variables like output, aggregate consumption and physical capital. Regarding output, $y$, the recession on impact is smaller, while, the rebound in the medium run is larger when the fiscal space is used more efficiently. In addition, the increase in aggregate consumption, $c$, and physical capital, $k_R$, are larger and more persistent when the economy travels to the reformed economies studied in this section (compare the blue lines, $s^g^c$, with the red, $s^g^e$, and yellow lines, $k^k$).

This effect works through the expectation channel, that is, households and firms anticipate the positive effects from the increase in productive spending on education or the reduction in capital taxes and adjust their behaviour in the short run. However, investment in physical and human capital take time to materialize, and thus the positive effects are more evident in the medium/long run.

Short run dynamic responses in Figure 4 are also consistent with findings in Section 5.1. That is, as highlighted in Section 5.1, using the fiscal space created by debt consolidation to finance an increase in spending on education or to finance a cut in distortionary capital tax yields relatively better outcomes than the benchmark economy for both Rich and Poor households in
the short and medium run as well as in the long run. For example, *Rich* households experience a larger increase in their capital income in the medium run (see $r_k R$) and labor income in the short/medium run (see $w_p l_P h_P$) and, at the same time, *Poor* households also experience a larger increase in their labor income (see $w_p l_P h_P$).

A clear cut message is that using the fiscal space to finance policies that induce long run positive supply side effects benefits both households.

### 7.4 Discussion of the Ramsey approach

In this paper, in accordance with most of the related literature on debt consolidation (see the Introductory section), we have focused on constrained optimized state-contingent policy rules. That is, the values of policy instruments over time can deviate from their trend values by reacting optimally to a number of macroeconomic indicators, but these trend values are set as in the data averages rather than chosen optimally. Alternatively, one could compute Ramsey-type optimal policy, meaning completely optimal state-contingent policy rules. Our approach can be justified by the fact that institutional and political constraints do not allow for a fully optimal Ramsey-type policy, especially when one considers fiscal policy (see also e.g. Schmitt-Grohé and Uribe, 2005, Kirsanova et al., 2007, Philippopoulos et al., 2017b). Nevertheless, it is worth pointing out that the policy message coming from our constrained optimized rules is in accordance with the typical Ramsey recipe. Namely, under both approaches, the optimal design of fiscal policy takes the form of higher productivity-enhancing spending and/or lower capital taxes in the long run, while, in the short, any required increase in revenues should be achieved by changes in the least distorting fiscal instrument which, in our model (as is common in the debt consolidation literature), is cuts in non-productive public spending.

### 7.5 Other parameters

We conduct sensitivity analysis with respect to other structural parameters of the model, namely the degree of complementarity/substitutability between private and public consumption, $0 \leq \xi < 0.2$, human capital technology parameters, $0.9 < B_P \leq B_R < 1.1$, parameters that govern the returns to scale for *Rich* and *Poor* human capital, $0.25 < x_P \leq x_R < 0.5$, the weights in the government’s objective function, i.e. $0.5 < \bar{\pi} < 0.8$. We report that our main results do not change (the full set of results is available upon request from the authors).

### 8 Conclusions and possible extensions

In this paper, we assessed the aggregate and distributional implications of fiscal consolidation policy using a dynamic general equilibrium model with heterogeneous agents. Since the main results have been summarized in the Introduction, we close with some possible extensions.

A possible extension is to depart from the closed economy setup and study similar questions in an open economy model allowing for international mobility of capital and labor (i.e. migration).
This could leave room for studying cross-border effects. Potential frictions in international capital and labor mobility could be an additional source of heterogeneity among different types of households. Thus, fiscal consolidation policy could affect aggregate and distributional outcomes through additional channels. We leave these extensions for future work.
References


32


[27] European Commission. 2014. Public Finances in the EMU.


33


A First order conditions

A.1 Rich households

Each Rich household in any given period \( t \), chooses \( c_{R,t}, e_{R,t}, l_{R,t}, k_{R,t+1}, b_{R,t+1}, h_{R,t+1} \) to maximize its lifetime utility subject to the constraints (4) (in which we incorporate (6) and (7)) and (5) taking factor prices and policy as given. Defining as \( \lambda_{R,t} \) and \( \psi_{R,t} \) the Lagrange multipliers associated with (4) and (5) respectively. The first order conditions are:

\[
\lambda_{R,t} = \frac{\mu_1}{(1 + \tau_t^e) (c_{R,t+1} + \xi g_{t+1}^e)} \tag{18}
\]

\[
\frac{\mu_2}{1 - e_{R,t} - l_{R,t}} = \psi_{R,t} x_R B_R \theta (e_{R,t})^{\theta - 1} \left[ (e_{R,t})^\theta (g_{R,t})^{1-\theta} \right] x^{R-1} \tag{19}
\]

\[
\frac{\mu_2}{1 - e_{R,t} - l_{R,t}} = \lambda_{R,t}(1 - \tau_t^l) w_{R,t} h_{R,t} \tag{20}
\]

\[
\frac{1}{(1 + \tau_t^e) (c_{R,t} + \xi g_t^e)} = \frac{\beta \left[ 1 - \delta^k + (1 - \tau_{t+1}^k) r_{t+1} \right]}{(1 + \tau_{t+1}^k) (c_{R,t+1} + \xi g_{t+1}^e)} \tag{21}
\]

\[
\frac{1}{(1 + \tau_t^e) (c_{R,t} + \xi g_t^e)} = \frac{\beta \left[ 1 + \tau_{t+1}^l \right]}{(1 + \tau_{t+1}^l) (c_{R,t+1} + \xi g_{t+1}^e)} \tag{22}
\]

\[\psi_{R,t} = \beta \lambda_{R,t+1}(1 - \tau_{t+1}^l) w_{R,t+1} l_{R,t+1} + \beta \psi_{R,t+1}(1 - \delta^h) \tag{23}\]

A.2 Poor households

Each Poor household, \( P \), maximizes its lifetime utility in any given period \( t \) by choosing \( c_{P,t}, e_{P,t}, l_{P,t}, h_{P,t+1} \) subject to the constraints (9) and (10) taking factor prices and policy as given. Defining as \( \lambda_{P,t} \) and \( \psi_{P,t} \) the Lagrange multipliers associated with (9) and (10) respectively. The first order conditions are:

\[
\lambda_{P,t} = \frac{\mu_1}{(1 + \tau_t^e) (c_{P,t+1} + \xi g_{t+1}^e)} \tag{24}
\]

\[
\frac{\mu_2}{1 - e_{P,t} - l_{P,t}} = \psi_{P,t} x_P B_P \theta (e_{P,t})^{\theta - 1} \left[ (e_{P,t})^\theta (g_{P,t})^{1-\theta} \right] x^{P-1} \tag{25}
\]

\[
\frac{\mu_2}{1 - e_{P,t} - l_{P,t}} = \lambda_{P,t}(1 - \tau_t^l) w_{P,t} h_{P,t} \tag{26}
\]

\[\psi_{P,t} = \beta \lambda_{P,t+1}(1 - \tau_{t+1}^l) w_{P,t+1} l_{P,t+1} + \beta \psi_{P,t+1}(1 - \delta^h) \tag{27}\]

A.3 Firms

Each firm \( f \) maximizes its profits (11) subject to its production function (12) by choosing \( k_f^l, l_{R,f}^l, l_{P,f}^l \). The first order conditions are:
\[ r_t = A (1 - m) \left( \rho \left( k^f_t \right)^v + (1 - \rho) \left( l^f_{R,t} \right)^v \right)^{\sigma - 1} \rho \left( k^f_t \right)^{v - 1} \times \left[ m \left( l^f_{P,t} \right)^\sigma + (1 - m) \left( \rho \left( k^f_t \right)^v + (1 - \rho) \left( l^f_{R,t} \right)^v \right)^\sigma \right]^{\frac{\sigma - 1}{\sigma}} \left( k^g_t \right)^{1 - \alpha} \] (28)

\[ w_{R,t} = A (1 - m) \left( \rho \left( k^f_t \right)^v + (1 - \rho) \left( l^f_{R,t} \right)^v \right)^{\sigma - 1} \rho \left( k^f_t \right)^{v - 1} \times \left[ m \left( l^f_{P,t} \right)^\sigma + (1 - m) \left( \rho \left( k^f_t \right)^v + (1 - \rho) \left( l^f_{R,t} \right)^v \right)^\sigma \right]^{\frac{\sigma - 1}{\sigma}} \left( k^g_t \right)^{1 - \alpha} \] (29)

\[ w_{P,t} = A \sigma m \left( l^f_{P,t} \right)^{\sigma - 1} \left[ m \left( l^f_{P,t} \right)^\sigma + (1 - m) \left( \rho \left( k^f_t \right)^v + (1 - \rho) \left( l^f_{R,t} \right)^v \right)^\sigma \right]^{\frac{\sigma - 1}{\sigma}} \left( k^g_t \right)^{1 - \alpha} \] (30)

Notice that each firm, \( f \), makes extraordinary profits given by \( \pi^f_t = (1 - \alpha) y^f_t \) as in Guo and Lansing (1997).

### B Market clearing conditions

Market clearing conditions in the capital market, the dividends market, the labor (Rich-skilled and Poor-unskilled) market, the government bonds market are respectively (in per capita terms):

\[ n^f k^f_t = n_R k_{R,t} \] (31)

\[ n^f \pi^f_t = n_R \pi_{R,t} \] (32)

\[ n^f l^f_{R,t} = n_R l_{R,t} h_{R,t} \] (33)

\[ n^f l^f_{P,t} = n_P l_{P,t} h_{P,t} \] (34)

\[ b_t \equiv \frac{B_t}{N} = n_R b_{R,t} \] (35)

The economy’s aggregate resource constraint in per capita terms is given by:

\[ n_R \left[ c_{R,t} + k_{R,t+1} - (1 - \delta^k) k_{R,t} \right] + n_P c_{P,t} + g^i_t + g^c_t + g^e_t = n^f y^f_t \] (36)

where we express the number of Rich and Poor households in terms of their population share shares \( n_R \equiv \frac{N_R}{N} \), \( n_P \equiv \frac{N_P}{N} = 1 - n_R \).

### C Full equilibrium system

The full equilibrium system in per capita terms is given in detail by the following 39 equations in 39 unknowns which are \( \{ y^f_t, c_{R,t}, c_{P,t}, i_{R,t}, d_{R,t}, k_{R,t+1}, h_{R,t+1}, h_{P,t+1}, b_{R,t+1}, e_{R,t}, e_{P,t}, l_{R,t}, \)
\[ l_{P,t}, r_{h}^{i}, \pi_{R,t}, \lambda_{R,t}, \lambda_{P,t}, \psi_{R,t}, \psi_{P,t}, k_{R}^{e}, r_{l}, w_{R,t}, w_{P,t}, g_{l}, g_{c}^{e}, g_{l}^{c}, tr_{R,t}, tr_{P,t}, g_{R,l,t}, g_{P,t}, s_{R}^{e}, s_{l}^{e}, s_{R}^{c}, s_{l}^{c}, s_{t}^{tr,p}, s_{t}^{tr,r}, \tau_{l,t}, \tau_{l}^{k} \] given the initial condition for the state variables and the feedback policy coefficients:

\[
\lambda_{R,t} = \frac{\mu_{1}}{(1 + \tau_{l}^{k}) (c_{R,t+1} + \xi g_{R}^{e})} \tag{37}
\]

\[
\frac{\mu_{2}}{1 - e_{R,t} - l_{R,t}} = \psi_{R,t} x_{R} B \theta (c_{R,t} \theta - 1) (e_{R,t})^{\theta} (g_{R,t}^{e})^{1 - \theta} x_{R}^{-1} \tag{38}
\]

\[
\frac{\mu_{2}}{1 - e_{R,t} - l_{R,t}} = \lambda_{R,t} (1 - \tau_{l}^{k}) w_{R,t} h_{R,t} \tag{39}
\]

\[
1 \frac{1}{(1 + \tau_{l}^{k}) (c_{R,t} + \xi g_{R}^{e})} = \beta \left[1 - \delta^{k} + (1 - \tau_{l}^{k}) r_{l+1} \right] \tag{40}
\]

\[
1 \frac{1}{(1 + \tau_{l}^{k}) (c_{R,t} + \xi g_{R}^{e})} = \beta \left[1 + r_{l}^{b} \right] \tag{41}
\]

\[
k_{R,t+1} = (1 - \delta^{k}) k_{R,t} + i_{R,t} \tag{42}
\]

\[
b_{R,t+1} = b_{R,t} + d_{R,t} \tag{43}
\]

\[
\beta \lambda_{R,t+1} (1 - \tau_{l+1}^{k}) w_{R,t+1} l_{R,t+1} + \beta \psi_{R,t+1} (1 - \delta^{h}) \tag{44}
\]

\[
h_{R,t+1} = (1 - \delta^{h}) h_{R,t} + B_{R} \left[ (e_{R,t})^{\theta} (g_{R,t}^{e})^{1 - \theta} \right]^{x_{R}} \tag{45}
\]

\[
(1 + \tau_{l}^{k}) c_{R,t} + i_{R,t} + d_{R,t} = (1 - \delta^{k}) (r_{l} k_{R,t} + \pi_{R,t}) + (1 - \tau_{l}^{k}) w_{R,t} l_{R,t} h_{R,t} + t_{R,t} + r_{l}^{b} b_{R,t} \tag{46}
\]

\[
\lambda_{P,t} = \frac{\mu_{1}}{(1 + \tau_{l}^{k}) (c_{P,t+1} + \xi g_{P}^{e})} \tag{47}
\]

\[
\frac{\mu_{2}}{1 - e_{P,t} - l_{P,t}} = \psi_{P,t} x_{P} B \theta (c_{P,t} \theta - 1) (e_{P,t})^{\theta} (g_{P,t}^{e})^{1 - \theta} x_{P}^{-1} \tag{48}
\]

\[
\frac{\mu_{2}}{1 - e_{P,t} - l_{P,t}} = \lambda_{P,t} (1 - \tau_{l}^{k}) w_{P,t} h_{P,t} \tag{49}
\]

\[
\psi_{P,t} = \beta \lambda_{P,t+1} (1 - \tau_{l+1}^{k}) w_{P,t+1} l_{P,t+1} + \beta \psi_{P,t+1} (1 - \delta^{h}) \tag{50}
\]

\[
h_{P,t+1} = (1 - \delta^{h}) h_{P,t} + B_{P} \left[ (e_{P,t})^{\theta} (g_{P,t}^{e})^{1 - \theta} \right]^{x_{P}} \tag{51}
\]

\[
n_{R} \left[ c_{R,t} + k_{R,t+1} (1 - \delta^{k}) k_{R,t} \right] + n_{P} c_{P,t} + g_{c}^{e} + g_{l}^{c} + g_{l}^{e} = n^{f} g_{l}^{f} \tag{52}
\]

\[
(s_{R}^{e} + s_{l}^{e} + s_{t}^{tr,p} + s_{t}^{tr,r}) n^{f} g_{l}^{f} + (1 + \tau_{l}^{b}) n_{R} b_{R,t} = n_{R} b_{R,t+1} \tag{53}
\]

\[
(\tau_{l}^{k} n_{R} c_{P,t} + n_{P} c_{P,t}) + \tau_{l}^{k} n_{R} (r_{l} k_{R,t} + \pi_{R,t}) + \tau_{l}^{k} n_{P} w_{P,t} l_{P,t} h_{P,t} + n_{R} w_{R,t} l_{R,t} h_{R,t} \tag{54}
\]

\[k_{l+1}^{k} = (1 - \delta^{k}) k_{l}^{g} + g_{l}^{g} \tag{55}
\]

\[n^{f} g_{l}^{f} = A \left[ m \left( (n_{P} l_{P,t} h_{P,t})^{e} + (1 - m) (\rho (n_{R} k_{R,t})^{v} + (1 - \rho) (n_{R} l_{R,t} h_{R,t})^{v}) \right)^{\alpha} (k_{l}^{g})^{1 - \alpha} \right] \tag{55}
\]
\[ r_t = A\alpha (1 - m) \left( \rho (n_R k_t R, t) \right)^\nu + (1 - \rho) \left( n_R l_t R, h_t R, t \right)^\nu \beta^{-1} \rho (n_R k_t R, t)^{\nu - 1} \]
\[ \times \left[ m \left( n_P l_P, h_P, t \right)^\sigma + (1 - m) \left( \rho (n_R k_t R, t) \right)^\nu + (1 - \rho) \left( n_R l_t R, h_t R, t \right)^\nu \right]^{\frac{\nu}{\beta}} (k_t^\rho)^{1 - \alpha} \quad (56) \]
\[ w_{R,t} = A\alpha (1 - m) \left( \rho (n_R k_t R, t) \right)^\nu + (1 - \rho) \left( n_R l_t R, h_t R, t \right)^\nu \beta^{-1} \rho (n_R k_t R, t)^{\nu - 1} \]
\[ \times \left[ m \left( n_P l_P, h_P, t \right)^\sigma + (1 - m) \left( \rho (n_R k_t R, t) \right)^\nu + (1 - \rho) \left( n_R l_t R, h_t R, t \right)^\nu \right]^{\frac{\nu}{\beta}} (k_t^\rho)^{1 - \alpha} \quad (57) \]
\[ w_{P,t} = A\alpha m \left( n_P l_P, h_P, t \right)^\sigma \left[ m \left( n_P l_P, h_P, t \right)^\sigma + (1 - m) \left( \rho (n_R k_t R, t) \right)^\nu + (1 - \rho) \left( n_R l_t R, h_t R, t \right)^\nu \right]^{\frac{\nu}{\beta}} (k_t^\rho)^{1 - \alpha} \quad (58) \]
\[ \pi_{R,t} = (1 - \alpha) y_t^f \quad (59) \]

The fiscal feedback policy rules are:

\[ s_t^{ge} - s_t^{ge} = \gamma_q^{ge} (q_{t-1} - q) \quad (60) \]
\[ s_t^{gi} - s_t^{gi} = \gamma_q^{gi} (q_{t-1} - q) \quad (61) \]
\[ s_t^{tp} - s_t^{tp} = \gamma_q^{tp} (q_{t-1} - q) \quad (62) \]
\[ s_t^{tr} - s_t^{tr} = \gamma_q^{tr} (q_{t-1} - q) \quad (63) \]
\[ s_t^{ge} - s_t^{ge} = \gamma_q^{ge} (q_{t-1} - q) \quad (64) \]
\[ \tau_t^{e} - \tau_t^{e} = \gamma_q^{c} (q_{t-1} - q) \quad (65) \]
\[ \tau_t^{l} - \tau_t^{l} = \gamma_q^{l} (q_{t-1} - q) \quad (66) \]
\[ \tau_t^{k} - \tau_t^{k} = \gamma_q^{k} (q_{t-1} - q) \quad (67) \]

where

\[ q_{t-1} = \frac{n_R b_{R,t}}{n f y_t^{f,-1}} \quad (68) \]
\[ s_t^{gi} = \frac{g_t^{i}}{n f y_t^{f}} \quad (69) \]
\[ s_t^{ge} = \frac{g_t^{c}}{n f y_t^{f}} \quad (70) \]
\[ s_t^{gi} = \frac{g_t^{c}}{n f y_t^{f}} \quad (71) \]
\[ s_t^{tr} = \frac{n_f t_{r,P,t}}{n f y_t^{f}} \quad (72) \]
D Allowing access to capital/financial markets for Poor households

In this Appendix we relax the assumption that Poor households do not have access to capital/financial markets. To do this, we allow both Rich and Poor households to accumulate physical capital and borrow or save in government bonds. As is known, under perfect capital markets and with common discount factors, the allocation of the aggregate stock of capital and bonds to different types of individual investors cannot be pinned down by the equilibrium conditions. To overcome this problem we assume that both types of households face intermediation or transaction costs due to imperfections in capital and financial markets as in e.g. Benigno (2009), Angelopoulos et al. (2014) and Economides et al. (2014). Transaction costs guarantee a well-defined long run solution and induce stationarity during the transition. In addition, they allow us to maintain household heterogeneity in asset holdings (see below). Here we only present the new equations and variables associated with the maximization problem of Poor households, and any equations that have changed relative to the model in the main text.

Now, each Poor household, $P$, maximizes its expected discounted lifetime utility subject to the following within period budget constraint which substitutes equation (9) in Section 2.4:

$$ (1 + r_t^P) c_{P,t} + i_{P,t} + d_{P,t} = (1 - r_t^P) r_t k_{P,t} + (1 - r_t^P) w_{P,t} I_{P,t} + h_{P,t} + b_{P,t} + r_t^b b_{P,t} $$

(76)

where $i_{P,t}$ is private investment in physical capital of Poor households, $d_{P,t}$, is savings in the form of government bonds of Poor households, $k_{P,t}$, is the beginning of period physical capital owned by Poor households and, $b_{P,t}$, is the beginning of period government bonds owned by Poor households. The laws of motion of physical capital and government bonds of each Poor household are:

$$ k_{P,t+1} = (1 - \delta^k) k_{P,t} + i_{P,t} - \frac{\chi_P}{2} \left( \frac{k_{P,t}}{n^I y_t} \right)^2 $$

(77)

$$ b_{P,t+1} = b_{P,t} + d_{P,t} - \frac{\phi_P}{2} \left( \frac{b_{P,t}}{n^I y_t} \right)^2 $$

(78)

the quadratic terms as in e.g. Benigno (2009), $\frac{\chi_P}{2} \left( \frac{k_{P,t}}{n^I y_t} \right)^2$ and $\frac{\phi_P}{2} \left( \frac{b_{P,t}}{n^I y_t} \right)^2$, capture transaction costs while the parameters $\chi_P, \phi_P \geq 0$ measure the size of these transactions costs. Poor

$$ s_t^{R} = \frac{n_R r_t}{n^I y_t} $$

(73)

$$ g_{R,t}^e = \omega g_t^e $$

(74)

$$ g_{P,t}^e = (1 - \omega) g_t^e $$

(75)
households choose $k_P, b_P$ which yield two new first order conditions:

$$\frac{1}{(1 + \tau_t^r) (c_P, t + \xi_t)} = \frac{\beta \left[ 1 - \delta^k + (1 - \tau_{t+1}^k) \tau_{t+1} - \chi_P \frac{k_{P,t+1}}{(1 + \xi_{t+1})} \right]}{\left( 1 + \tau_{t+1}^r \right) \left( c_{P,t+1} + \xi_{t+1}^c \right)}$$  \hspace{1cm} (79)

$$\frac{1}{(1 + \tau_t^b) (c_P, t + \xi_t)} = \frac{\beta \left[ 1 + \phi^b + \phi_P \frac{b_{P,t+1}}{(1 + \xi_{t+1})} \right]}{\left( 1 + \tau_{t+1}^r \right) \left( c_{P,t+1} + \xi_{t+1}^c \right)}$$  \hspace{1cm} (80)

Similarly, the problem of Rich households is modified as follows. Rich household face similar quadratic transactions costs and thus the laws of motion of physical capital and government bonds for each Rich household changes from equations (6) and (7) to:

$$k_{R,t+1} = (1 - \delta^k) k_{R,t} + i_{R,t} - \frac{\chi_R}{2} \left( \frac{k_{R,t}^c}{n^f y_t} \right)^2$$  \hspace{1cm} (81)

$$b_{R,t+1} = b_{R,t} + d_{R,t} - \frac{\phi_R}{2} \left( \frac{b_{R,t}^c}{n^f y_t} \right)^2$$  \hspace{1cm} (82)

where the quadratic terms $\frac{\chi_R}{2} \left( \frac{k_{R,t}^c}{n^f y_t} \right)^2$ and $\frac{\phi_R}{2} \left( \frac{b_{R,t}^c}{n^f y_t} \right)^2$ capture transaction costs and the parameter $\chi_R, \phi_R \geq 0$ measure the size of these transactions costs.

The first order conditions of Rich households with respect to $k_R, b_R$, change from equations (19) and (21) in Appendix A.1 to:

$$\frac{1}{(1 + \tau_t^r) (c_R, t + \xi_t)} = \frac{\beta \left[ 1 - \delta^k + (1 - \tau_{t+1}^k) \tau_{t+1} - \chi_P \frac{k_{R,t+1}}{(1 + \xi_{t+1})} \right]}{\left( 1 + \tau_{t+1}^r \right) \left( c_{R,t+1} + \xi_{t+1}^c \right)}$$  \hspace{1cm} (83)

$$\frac{1}{(1 + \tau_t^b) (c_R, t + \xi_t)} = \frac{\beta \left[ 1 + \phi^b + \phi_P \frac{b_{R,t+1}}{(1 + \xi_{t+1})} \right]}{\left( 1 + \tau_{t+1}^r \right) \left( c_{R,t+1} + \xi_{t+1}^c \right)}$$  \hspace{1cm} (84)

For comparison with the benchmark economy, in the numerical solution of section 7.2 we set $\chi_R < \chi_P$ and $\phi_R < \phi_P$ such that the total wealth of Rich households is higher than that of Poor households. We report that our main results do not depend on the exact values of the transaction costs parameters.

Finally, the resource constraint of the economy changes from equation (36) in Appendix C to:

$$n_R \left[ c_{R,t+1} + k_{R,t+1} - (1 - \delta^k) k_{R,t} + \frac{\chi_R}{2} \left( \frac{k_{R,t}}{n^f y_t} \right)^2 + \frac{\phi_R}{2} \left( \frac{b_{R,t}}{n^f y_t} \right)^2 \right] +$$

$$n_P \left[ c_{P,t+1} + k_{P,t+1} - (1 - \delta^k) k_{P,t} + \frac{\chi_P}{2} \left( \frac{k_{P,t}}{n^f y_t} \right)^2 + \frac{\phi_P}{2} \left( \frac{b_{P,t}}{n^f y_t} \right)^2 \right] + g_t^i + g_t^i + g_t = n^f \ y_t$$  \hspace{1cm} (85)

while the government budget constraint changes from equation (53) in Appendix C to:
\[(s_t g^e + s_{t} g^i + s_t tr_R + s_t tr_P)n_t y_t + (1 + r_t^b)(n_R b_{R,t} + n_P b_{P,t}) = n_R b_{R,t+1} + n_P b_{P,t+1} \]
\[\tau_t^c (n_RC_{R,t} + n_PC_{P,t}) + \tau_t^k n_R (r_t k_{R,t} + \pi_{R,t} + n_P k_{P,t}) + \tau_t^l (n_P w_{P,t} l_{P,t} h_{P,t} + n_R w_{R,t} l_{R,t} h_{R,t}) \]

(86)

The new market clearing conditions for physical capital and bonds are
\[n^f k_{t} = n_R k_{R,t} + n_P k_{P,t}\]
and \[b_t = n_R b_{R,t} + n_P b_{P,t}\] which substitute equations (31) and (35) in Appendix B respectively.

To sum up, in the DCE system presented in Appendix C we add four new endogenous variables, \(k_{P,t}, d_{P,t}, b_{P,t}\) and \(i_{P,t}\) and four new equations (77), (78), (79) and (80). In addition, the equations (81), (82), (83), (84), (85), (86) substitute equations (42), (43), (40), (41), (52), (53) respectively.

E Tables and Figures

Table E.1: Steady state labor and capital incomes
(as % deviations from status quo)

<table>
<thead>
<tr>
<th>Fiscal Instr.</th>
<th>(w_{R,R} h_R)</th>
<th>(w_{P,L} p_{P})</th>
<th>(r_{k})</th>
<th>(r_{b,B})</th>
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<td>(s_t g^e)</td>
<td>0.0071</td>
<td>0.0039</td>
<td>0.0076</td>
<td>-0.3449</td>
</tr>
<tr>
<td>(s_t g^i)</td>
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<td>0.0088</td>
<td>0.0122</td>
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<tr>
<td>(s_t g^e)</td>
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<td>0.0232</td>
<td>0.0289</td>
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<tr>
<td>(\tau_t^c)</td>
<td>0.0081</td>
<td>0.0050</td>
<td>0.0086</td>
<td>-0.3439</td>
</tr>
<tr>
<td>(\tau_t^l)</td>
<td>0.0159</td>
<td>0.0138</td>
<td>0.0162</td>
<td>-0.3360</td>
</tr>
<tr>
<td>(\tau_t^k)</td>
<td>0.0238</td>
<td>0.0108</td>
<td>0.0116</td>
<td>-0.3343</td>
</tr>
</tbody>
</table>

Figure E.1: Dynamic path of other endogenous variables under the optimized fiscal policy