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Model Of Strategic Electrolysis Firms In Energy, Ancillary Services And Hydrogen Markets

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OR in energy; Energy markets; EPEC; Hydrogen economy; Stackelberg model.

Abstract

This work analyses the trading of strategic merchant hydrogen technologies in energy and ancillary services markets. The hydrogen firms trade in two markets: 1) a joint hydrogen and energy/reserves day-ahead market and 2) the balancing settlements market. We contrast the co-optimized markets with trading in an energy-only market. Trading both energy and ancillary services leads hydrogen firms to produce and use more hydrogen, leading to less reliance on fossil fuels and an increase in the revenue streams of the electrolysis-based firms. The problem is formulated as a stochastic multi-leader-multi-follower model. Each leader firm solves a bi-level Stackelberg problem. The upper-level is the Nash game among strategic firms. The lower-level is an instance of a Generalized Nash Equilibrium of the followers.

1 Introduction

Energy source diversification will play a fundamental role in the transition to a net-zero emissions economy (Dincer and Acar, 2018). Hydrogen produced via electrolysis can assist in this diversification by balancing the variability of renewable energy availability and by providing ancillary system services (AS) to the power system (Staffell et al., 2019). Electrolyser technologies are moving towards financial viability (Hou et al., 2017; Kopp et al., 2017), though policy supports are still anticipated as being necessary to enable strong hydrogen technology penetration (Moore and Shabani, 2016). Revenue diversification is an alternative mechanism that could improve the technology's competitiveness (Liu and Mancarella, 2021). Previous studies, e.g., Bakhtiari and Naghizadeh (2018) and Haggi et al. (2021), show that hydrogen technologies can enhance the reliability of the power system while Caumon et al. (2015) considers this possibility as another revenue stream to firms with electrolysers and fuelcells units. Many viability assessments are however based solely on the energy value of hydrogen, e.g., Noussan et al. (2021), studies a dedicated off-grid solar or wind power plants, rather than considering potential revenue streams from the provision of ancillary services to the power system, as well as exploiting arbitrage opportunities in the hydrogen, power and system services markets.

1.1 Literature Review

Within the context of the energy transition, the objective of this paper is to examine the viability of hydrogen technologies in energy and balancing markets. To-date, the literature has mainly focused on isolated aspects of hydrogen integration into the wider energy system.

The co-optimization of energy and hydrogen is studied in Tao et al. (2021); Wu et al. (2021); Pan et al. (2020); Park Lee et al. (2018); Mehrjerdi et al. (2019); Li et al. (2020). Other studies have focused on integrated wind-electrolyser systems, e.g., Olateju et al. (2016); Zhang and Wan (2014); Xu et al. (2019), while other papers focus on integrated electrolyser-fuelcell systems, e.g., Zahedi et al. (2021); Nojavan et al. (2017); Garcia-Torres and Bordons (2015); Rouholamini et al. (2017). In contrast, this paper considers independent merchant firms in a competitive market.

Previous research that considers potential of the complementarity of renewable generators and hydrogen technologies to increase profits, e.g., Liu and Mancarella (2021); Zahedi et al. (2021); Alkano and Scherpen (2018); Wu et al. (2020) have neglected many of the technical constraints, including hydrogen storage or the power grid topology. In contrast, this model explicitly considers hydrogen storage and power transmission constraints.

Finally, the role of new hydrogen technologies in providing reserves has been studied in McPherson et al. (2018); Naughton et al. (2019); Dadkhah et al. (2020); Samani et al. (2020); Garcia-Torres and Bordons (2015); Wu et al. (2019). However, these studies rely on exogenous electricity pricing and therefore cannot capture the impact of strategic trading.

Overall, the previous work have tacitly considered price-taking firms. A more general formulation would allow firms to trade strategically to set market clearing prices. This can be accomplished with a Stackelberg model. The work in Shams et al. (2021); Shafiekhani et al. (2019); Li et al. (2018); Nasrolahpour et al. (2018) implement a bi-level (Stackelberg) market model comprising of strategic and competitive firms, however the modelling approach is either of single-leader-single-follower, multi-leader-single-follower or single-leader-multi-follower problem.

1.2 Contributions of this Research

From this analysis of the literature, two major gaps emerge: the first is the assumption of perfectly competitive or price-taking behaviour by firms, and the second is the consideration of hydrogen costs and revenues in day-ahead energy markets only.

This paper addresses these gaps by focusing on merchant strategic hydrogen (H_2) firms participating in a joint competitive electricity, reserves and hydrogen market and the balancing market. We use a Stackelberg formulation, i.e., a Nash game among leader firms and a generalized Nash equilibrium among the followers.

The market is modeled as a multi-leader multi-follower stochastic problem recast as two coupled bi-level optimization problems. Each bi-level problem is expressed as a mathematical program with equilibrium constraints (MPEC). Specifically, the leader firms in the upper level problem (ULP) are firms with electrolyser and fuelcell units, respectively. The lower-level problems (LLP) consist of the social-welfare maximizing Day-Ahead Market (DAM) and Balancing Market (BLM). The collection of MPECs is optimized as an equilibrium problem with equilibrium constraints (EPEC). The DAM is a joint hydrogen and energy/reserves market. We consider trading by the firms in the day-ahead market only and the integrated energy, reserves and balancing market. This isolates the impact of ancillary service provision in addition to day ahead market activity.

The original contributions of this paper are therefore threefold. The first is the development of a model of strategic hydrogen producers and consumers, who can influence and respond to endogenously-determined prices, within an energy market framework. The second contribution is the application of this model to a joint energy and hydrogen market, in order to determine the equilibrium decisions of all players. Finally, we make a considerable contribution to the literature by explicitly considering the impacts of participation in ancillary services and balancing markets on market equilibria, compared to an energy-only market paradigm. In this manner, we test the importance of joint optimisation of all three markets when determining market equilibria, and quantify the profit-making opportunities for hydrogen firms in providing ancillary services to the electricity system.

The remainder of this paper is structured as follow. Section 2 presents the modelling framework and solution methodology. Section 3 presents the case study used to apply the model. Section 4 presents and discusses the various results obtained. Section 5 concludes.

2 Modelling and Solution Methodology

The strategic firms are modelled as leaders in the bi-level problems. In this research, the two strategic firms in the ULPs are a profit-maximising electrolyser and a profit-maximising fuelcell. The electrolyser firm buys electricity from the day ahead market which it uses to produce hydrogen, and sells hydrogen to the fuelcell firm, as well as to hydrogen forklifts (FCFL). In addition to the electrolyser, a steam methane reformer (SMR) produces and sells grey hydrogen. The fuel cell firm buys hydrogen which it can use to produce and sell electricity in the day ahead market. Both leader firms can also buy and sell upwards and downwards reserves in a balancing market. Both the FCFL and the SMR are modelled as price-taking firms as part of the LLPs.

We consider two LLPs; a Day-Ahead Market (DAM) problem and a Balancing Market (BLM) problem. The LLPs comprise competitive energy firms that sell electricity in the day ahead market, and buy and sell reserves in the balancing market. We assume both markets are sufficiently competitive or well-regulated to ensure that price-making behaviour by energy firms is not possible, and so the LLPs are modelled as competitive markets. This is the aim of most modern regulated electricity markets. Consequently, both the DAM and the BLM problems are solved via social welfare maximisation/cost minimisation. The leader firms play a Nash game between them while anticipating the reactions of the competitive follower firms. The LLPs are convex so they can be replaced by their corresponding KKT conditions. This yields a complementarity problem representing an equilibrium among the followers. The Fortuny-Amat McCarl method is used to linearise the complementarity conditions. Thus the ULPs depend on all primal and dual variables of the LLPs, which makes a well-posed MPEC. The simultaneous games among the leaders yields an EPEC problem (Gabriel et al., 2013).

To solve the EPEC we implemented the Diagonalization method. This is based on the Gauss-Seidel algorithm for numerically solving simultaneous equations. For further details see Gabriel et al. (2013, p.266).

Table 1
Parameters.

$\eta_{\{e,f\}}$	Efficiency of {electrolyser, fuel cell} units
$\mathscr{P}_{\{e,f\}}^{\{\text{INI,MIN,MAX}\}}$	Initial, min., max. tank pressure [J/m^3]
Φ_k	Probability of scenario k
$B_{d,t}^{EN}$	Energy bid of d [€/MWh]
B_{nm}	Susceptance of line nm
$B_{t,d_{\text{H2}}}^{EN}$	H_2 bid of d_{H2} [€/MWh]
$C_{e/f}^{ch/dis}$	Operating cost of ch./dis. [€/{MWh,kg}]
F_{nm}^{max}	Transmission capacity of line nm
$L_{t,k}$	Net load
LHV	Lower heat value of hydrogen [J/kg]
M^{H2}	Molar mass of hydrogen [kg/mol]
R	Molar gas constant [J/molK]
$O_{g,t}^{EN}$	Energy offer of g [€/MWh]
$P_{\{e,f\}}^{\{\text{MAX,MIN}\}}$	Rated capacity, min. consumption [MW]
$R_e^{ch,UP}, R_e^{ch,DN}$	Max. up/down ch. reserve cap. [MW]
$R_f^{dis,UP}, R_f^{dis,DN}$	Max. up/down dis. reserve cap. [MW]
$T^{\{e,f\}}$	Temperature inside hydrogen tank [K]
VOLL	Value of Lost Load [€/MWh]

Table 2
Indices and Sets.

Ψ_n	Set of units located at node n
Θ_n	Set of nodes connected to node n
d	Power consumer, 1 - N_d
d_{H2}	H_2 load, 1 to $N_{d_{\text{H2}}}$
e	Electrolyser unit, 1 - N_e
f	Fuelcell unit, 1 - N_f
g	Power producer, 1 - N_g
h	SMR plant, 1 - N_d
k	Scenario, 1 - N_k
n,m	node
t	Time index, 1 - $\mathscr{T}$

2.1 Lower-Level Problems

The structure of the LLPs resemble existing markets in that the day-ahead market (DAM) and balancing market (BLM) clear sequentially Nasrolahpour et al. (2018). In the bi-level model the LLPs become constraints in the ULPs. The scheduled and committed quantities of power $p^{\text{DA}}_{t,e}, p^{\text{DA}}_{t,f}$, hydrogen $Q_{t,f}$, $Q_{t,e}, r^{ch,UP}_{t,e}$ and reserve $r^{ch,DN}_{t,e}, r^{dis,UP}_{t,f}, r^{dis,DN}_{t,f}$ are endogenously determined in the DAM. Similarly, for each scenario k, the power bought and sold in the balancing market $q^{dis,UP}_{t,f,k}, q^{dis,DN}_{t,f,k}, q^{ch,UP}_{t,e,k}, q^{ch,DN}_{t,e,k}$ are determined.

In order to generalize the model, the leader firms, power producers and loads can each commit reserves which can be drawn on to provide balancing energy. Tables 1 and 2 describe the model parameters and indices and sets, respectively. The hat (ˆ) variables represent offers/bids submitted by the leader firms and are exogenous the LLPs (i.e, considered as parameters in the LLPs), but are determined in the ULPs.

2.1.1 Day-Ahead Market

The DAM is a joint settlement of hydrogen, energy and reserves. The objective function (1) maximizes social welfare. The primal variables of the DAM problem (1) - (30) are in the set $^{\text{DA}}_{\text{LL}}$ and comprise the day ahead cleared power from all firms (leaders and followers) $(p^{\text{DA}}_{t,e}, p^{\text{DA}}_{t,f}, p_{d,t}, p_{g,t})$, the hydrogen bought and sold by all firms $(Q_{t,f}, Q_{t,e}, Q_{t,h}, Q_{t,d_{\text{H2}}})$ and the upwards and downwards reserves sold by all firms $(r^{ch,UP}_{t,e}, r^{ch,DN}_{t,e}, r^{dis,UP}_{t,f}, r^{dis,DN}_{t,f}, r^{UP}_{g,t}, r^{DN}_{g,t}, r^{UP}_{d,t}, r^{DN}_{d,t})$. Table 3 describes the DAM primal and dual variables.

Table 3
Primal and dual variables for the Day-Ahead Market LLP.

Primal	$p^{\mathrm{DA}}_{t,e}, p^{\mathrm{DA}}_{t,f}$	Scheduled demand/discharge of e/f
	$p_{d,t}, p_{g,t}$	Scheduled demand/generation of d/g
	$Q_{t,f}, Q_{t,e}$	Scheduled H_2 demand/generation of f/e
	$Q_{t,d_{\mathrm{H2}}}, Q_{t,h}$	Scheduled H_2 demand/generation of d_{H2}/h
	$r^{ch,UP}_{t,e}, r^{ch,DN}_{t,e}$	Committed up/down charge res. of e
	$r^{dis,UP}_{t,f}, r^{dis,DN}_{t,f}$	Committed up/down discharge res. of f
	$r^{UP}_{g,t}, r^{DN}_{g,t}$	Committed up/down res. of g
	$r^{UP}_{d,t}, r^{DN}_{d,t}$	Committed up/down res. of d
	$\delta_{t,n}$	Voltage angle of node n
Dual	$\lambda^{EN}_{t,n}, \lambda^{\mathrm{H2}}_{t}, \lambda^{UP}_{t}, \lambda^{DN}_{t}$	Power, hydrogen, UP and DN balance
	$\xi_{t,n=1}, \underline{\xi}_{t,n}/\overline{\xi}_{t,n}$	Voltage angle at t=1, voltage angle bounds
	$\underline{\nu}_{t,n,m}, \overline{\nu}_{t,n,m}$	Transmission capacity bounds
	$\underline{\mu}_{g,t}, \overline{\mu}_{g,t}$	Power capacity bounds of g
	$\underline{\mu}_{d,t}, \overline{\mu}_{d,t}$	Power capacity bounds of d
	$\underline{\mu}_{t,h}, \overline{\mu}_{t,h}$	H_2 capacity bounds of h
	$\underline{\mu}_{d_{\mathrm{H2}},t}, \overline{\mu}_{d_{\mathrm{H2}},t}$	H_2 capacity bounds of d_{H2}
	$\underline{\mu}^{\mathrm{DA}}_{t,e}, \overline{\mu}^{\mathrm{DA}}_{t,e}$	Power commitment bounds of e
	$\underline{\mu}^{\mathrm{Q}}_{t,e}, \overline{\mu}^{\mathrm{Q}}_{t,e}$	H_2 commitment bounds of e
	$\underline{\mu}^{\mathrm{Q}}_{t,f}, \overline{\mu}^{\mathrm{Q}}_{t,f}$	H_2 commitment bounds of f
	$\underline{\mu}^{\mathrm{DA}}_{t,f}, \overline{\mu}^{\mathrm{DA}}_{t,f}$	Power commitment bounds of f
	$\underline{\mu}^{\mathrm{ch,UP}}_{t,e}, \overline{\mu}^{\mathrm{ch,UP}}_{t,e}$	Upward reserve commitment of e
	$\underline{\mu}^{\mathrm{ch,DN}}_{t,e}, \overline{\mu}^{\mathrm{ch,DN}}_{t,e}$	Downward reserve commitment of e
	$\underline{\mu}^{\mathrm{dis,UP}}_{t,f}, \overline{\mu}^{\mathrm{dis,UP}}_{t,f}$	Upward reserve commitment of f
	$\underline{\mu}^{\mathrm{dis,DN}}_{t,f}, \overline{\mu}^{\mathrm{dis,DN}}_{t,f}$	Downward reserve commitment of f
	$\mu^{\mathrm{DA,UP}}_{t,e}$	Max. reserve commitment of e
	$\mu^{\mathrm{DA,DN}}_{t,f}$	Max. reserve commitment of f
	$\underline{\mu}^{\mathrm{UP}}_{t,g}, \overline{\mu}^{\mathrm{UP}}_{t,g}$	Upward reserve commitment of g
	$\underline{\mu}^{\mathrm{DN}}_{t,g}, \overline{\mu}^{\mathrm{DN}}_{t,g}$	Downward reserve commitment of g
	$\mu^{\mathrm{UP}}_{t,g}, \mu^{\mathrm{DN}}_{t,g}$	Max. up/down reserve of g
	$\underline{\mu}^{\mathrm{UP}}_{t,d}, \overline{\mu}^{\mathrm{UP}}_{t,d}$	Upward reserve commitment of d
	$\underline{\mu}^{\mathrm{DN}}_{t,d}, \overline{\mu}^{\mathrm{DN}}_{t,d}$	Downward reserve commitment of d
	$\mu^{\mathrm{UP}}_{t,d}, \mu^{\mathrm{DN}}_{t,d}$	Max. up/down reserve of d

$$
\begin{aligned}
\max_{\substack{\mathrm{DA} \\ \mathrm{LL}}} \{ & Electricity\ sales \\
& + Hydrogen\ sales \\
& - Electricity\ procurement \\
& - Hydrogen\ procurement \\
& - Up\ reserves\ procurement \\
& - Down\ reserves\ procurement \}
\end{aligned} \tag{1}
$$

where

$$
Electricity\ sales = \sum_{t=1}^{\mathscr{T}} \left\{ \sum_{e=1}^{N_e} \hat{b}^{ch}_{t,e} p^{\mathrm{DA}}_{t,e} + \sum_{d=1}^{N_d} B^{EN}_{d,t} p_{d,t} \right\} \tag{1a}
$$

$$
Hydrogen\ sales = \sum_{t=1}^{\mathscr{T}} \left\{ \sum_{f=1}^{N_f} \hat{b}^{\mathrm{H2}}_{t,f} Q_{t,f} + \sum_{d_{\mathrm{H2}}=1}^{N_{d_{\mathrm{H2}}}} B^{EN}_{t,d_{\mathrm{H2}}} Q_{t,d_{\mathrm{H2}}} \right\} \tag{1b}
$$

$$
Electricity\ procurement = \sum_{t=1}^{\mathscr{T}} \left\{ \sum_{f=1}^{N_f} \hat{o}^{dis}_{t,f} p^{\mathrm{DA}}_{t,f} + \sum_{g=1}^{N_g} O^{EN}_{g,t} p_{g,t} \right\} \tag{1c}
$$

$$
\begin{aligned}
Up\ reserves\ procurement = \sum_{t=1}^{\mathscr{T}} \Bigg\{ & \sum_{e=1}^{N_e} \hat{o}^{ch,UP}_{t,e} r^{ch,UP}_{t,e} + \sum_{f=1}^{N_f} \hat{o}^{dis,UP}_{t,f} r^{dis,UP}_{t,f} \\
& + \sum_{d=1}^{N_d} B^{RS}_{d,t} r^{UP}_{d,t} + \sum_{g=1}^{N_g} O^{RS}_{g,t} r^{UP}_{g,t} \Bigg\}
\end{aligned} \tag{1d}
$$

$$
\begin{aligned}
Down\ reserves\ procurement = \sum_{t=1}^{\mathscr{T}} \Bigg\{ & \sum_{e=1}^{N_e} \hat{o}^{ch,DN}_{t,e} r^{ch,DN}_{t,e} + \sum_{f=1}^{N_f} \hat{o}^{dis,DN}_{t,f} r^{dis,DN}_{t,f} \\
& + \sum_{d=1}^{N_d} B^{RS}_{d,t} r^{DN}_{d,t} + \sum_{g=1}^{N_g} O^{RS}_{g,t} r^{DN}_{g,t} \Bigg\}
\end{aligned} \tag{1e}
$$

Equation (1) is optimized subject to the network and firms' capacity constraints, reserves requirements and balancing supply and demand.

The dual variables for each constraint are shown after the colon. (2)-(5) is the DC linearization of the OPF problem. (2) represents the supply and demand balance. (3) defines the reference node. (4) are the lower and upper bounds of the voltage angle, $\delta_{t,n}$. (5) defines the power transmission capacity between nodes n and m.

$$
\sum_{d \in \Psi_n} p_{d,t} + \sum_{e \in \Psi_n} p^{\mathrm{DA}}_{t,e} - \sum_{f \in \Psi_n} p^{\mathrm{DA}}_{t,f} - \sum_{g \in \Psi_n} p_{g,t} = \sum_{m \in \Theta_n} B_{nm} (\delta_{t,n} - \delta_{t,m}) : \lambda^{EN}_{t,n};\ \forall t, \forall n \tag{2}
$$

$$\delta_{t,n=1} = 0: \ \xi_{t,n=1}; \ \forall t \tag{3}$$

$$-\pi \leq \delta_{t,n} \leq \pi: \ \underline{\xi}_{t,n}, \overline{\xi}_{t,n}; \ \forall t, \forall n \tag{4}$$

$$-F_{nm}^{max} \leq B_{nm}(\delta_{t,n} - \delta_{t,m}) \leq F_{nm}^{max}: \ \underline{\nu}_{t,n,m}, \overline{\nu}_{t,n,m}; \ \forall t, \forall n, \forall m \in \Theta_n \tag{5}$$

Constraints (6) and (7) bound the generators' offered power and the loads' power bids, respectively. (8) ensures the cleared power is at most the electrolyser firm's bid, $\hat{p}_{t,e}^{\text{DA}}$, and (9) constraints the FC firm's cleared power to be at most $\hat{p}_{t,f}^{\text{DA}}$.

$$0 \leq p_{g,t} \leq P_g^{max}: \ \underline{\mu}_{g,t}, \ \overline{\mu}_{g,t}; \ \forall g, \forall t \tag{6}$$

$$0 \leq p_{d,t} \leq P_d^{max}: \ \underline{\mu}_{d,t}, \ \overline{\mu}_{d,t}; \ \forall d, \forall t \tag{7}$$

$$0 \leq p_{t,e}^{\text{DA}} \leq \hat{p}_{t,e}^{\text{DA}}: \ \underline{\mu}_{t,e}^{\text{DA}}, \ \overline{\mu}_{t,e}^{\text{DA}}; \ \forall e, \forall t \tag{8}$$

$$0 \leq p_{t,f}^{\text{DA}} \leq \hat{p}_{t,f}^{\text{DA}}: \ \underline{\mu}_{t,f}^{\text{DA}}, \ \overline{\mu}_{t,f}^{\text{DA}}; \ \forall f, \forall t \tag{9}$$

Equation (10) is the H_2 supply and demand balance constraint. The variables $Q_{t,d_{\text{H2}}}$ and $Q_{t,h}$ represent hydrogen demand and supply, respectively, from competitive firms, in this case the FCFL (for hydrogen demand) and the SMR (for hydrogen supply). (11)-(12) constrain the hydrogen production and consumption from competitive firms, respectively. (13) constrains the cleared hydrogen from the electrolyser firm to its offer, $\hat{Q}_{t,e}$, and (14) constrains the FC firm's cleared hydrogen to its bid $\hat{Q}_{t,f}$.

$$\sum_{d_{\text{H2}}=1}^{N_{d_{\text{H2}}}} Q_{t,d_{\text{H2}}} + \sum_{f=1}^{N_f} Q_{t,f} - \sum_{e=1}^{N_e} Q_{t,e} - \sum_{h=1}^{N_h} Q_{t,h} = 0: \lambda_t^{\text{H2}}; \ \forall t \tag{10}$$

$$0 \leq Q_{t,h} \leq Q_{t,h}^{max}: \ \underline{\mu}_{t,h}, \ \overline{\mu}_{t,h}; \ \forall h, \forall t \tag{11}$$

$$0 \leq Q_{t,d_{\text{H2}}} \leq Q_{t,d_{\text{H2}}}^{max}: \ \underline{\mu}_{t,d_{\text{H2}}}, \ \overline{\mu}_{t,d_{\text{H2}}}; \ \forall d_{\text{H2}}, \forall t \tag{12}$$

$$0 \leq Q_{t,e} \leq \hat{Q}_{t,e}: \ \underline{\mu}_{t,e}^{\text{Q}}, \ \overline{\mu}_{t,e}^{\text{Q}}; \ \forall e, \forall t \tag{13}$$

$$0 \leq Q_{t,f} \leq \hat{Q}_{t,f}: \ \underline{\mu}_{t,f}^{\text{Q}}, \ \overline{\mu}_{t,f}^{\text{Q}}; \ \forall f, \forall t \tag{14}$$

Constraints (15) and (16) are the upward and downward reserves requirements, respectively. (17)-(20) limit the cleared reserves to the offers submitted by the upper-level firms. (21) caps the electrolyser firm's upward reserves to the cleared power and (22) caps the FC firm's downward reserves to its scheduled power. (23)-(30) set the the limits on commitment reserves from generators and loads.

$$\sum_{e=1}^{N_e} r_{t,e}^{ch,UP} + \sum_{f=1}^{N_f} r_{t,f}^{dis,UP} + \sum_{d=1}^{N_d} r_{d,t}^{UP} + \sum_{g=1}^{N_g} r_{g,t}^{UP} = R_t^{UP}: \ \lambda_t^{UP}; \ \forall t \tag{15}$$

$$\sum_{e=1}^{N_e} r_{t,e}^{ch,DN} + \sum_{f=1}^{N_f} r_{t,f}^{dis,DN} + \sum_{d=1}^{N_d} r_{d,t}^{DN} + \sum_{g=1}^{N_g} r_{g,t}^{DN} = R_t^{DN} :\ \lambda_t^{DN};\ \forall t \tag{16}$$

$$0 \le r_{t,e}^{ch,UP} \le \hat{r}_{t,e}^{ch,UP} :\ \underline{\mu}_{t,e}^{\text{ch,UP}},\ \overline{\mu}_{t,e}^{\text{ch,UP}};\ \forall e, \forall t \tag{17}$$

$$0 \le r_{t,e}^{ch,DN} \le \hat{r}_{t,e}^{ch,DN} :\ \underline{\mu}_{t,e}^{\text{ch,DN}},\ \overline{\mu}_{t,e}^{\text{ch,DN}};\ \forall e, \forall t \tag{18}$$

$$0 \le r_{t,f}^{dis,UP} \le \hat{r}_{t,f}^{dis,UP} :\ \underline{\mu}_{t,f}^{\text{dis,UP}},\ \overline{\mu}_{t,f}^{\text{dis,UP}};\ \forall f, \forall t \tag{19}$$

$$0 \le r_{t,f}^{dis,DN} \le \hat{r}_{t,f}^{dis,DN} :\ \underline{\mu}_{t,f}^{\text{dis,DN}},\ \overline{\mu}_{t,f}^{\text{dis,DN}};\ \forall f, \forall t \tag{20}$$

$$r_{t,e}^{ch,UP} \le p_{t,e}^{\text{DA}} :\ \mu_{t,e}^{\text{DA,UP}};\ \forall e, \forall t \tag{21}$$

$$r_{t,f}^{dis,DN} \le p_{t,f}^{\text{DA}} :\ \mu_{t,f}^{\text{DA,DN}};\ \forall f, \forall t \tag{22}$$

$$0 \le r_{g,t}^{UP} \le R_g^{UP} :\ \underline{\mu}_{t,g}^{\text{UP}},\ \overline{\mu}_{t,g}^{\text{UP}};\ \forall g, \forall t \tag{23}$$

$$0 \le r_{g,t}^{DN} \le R_g^{DN} :\ \underline{\mu}_{t,g}^{\text{DN}},\ \overline{\mu}_{t,g}^{\text{DN}};\ \forall g, \forall t \tag{24}$$

$$p_{g,t} + r_{g,t}^{UP} \le P_g^{max} :\ \mu_{t,g}^{\text{UP}};\ \forall g, \forall t \tag{25}$$

$$r_{g,t}^{DN} - p_{g,t} \le 0 :\ \mu_{t,g}^{\text{DN}};\ \forall g, \forall t \tag{26}$$

$$0 \le r_{d,t}^{UP} \le R_d^{UP} :\ \underline{\mu}_{t,d}^{\text{UP}},\ \overline{\mu}_{t,d}^{\text{UP}};\ \forall d, \forall t \tag{27}$$

$$0 \le r_{d,t}^{DN} \le R_d^{DN} :\ \underline{\mu}_{t,d}^{\text{DN}},\ \overline{\mu}_{t,d}^{\text{DN}};\ \forall d, \forall t \tag{28}$$

$$p_{d,t} + r_{d,t}^{DN} \le P_d^{max} :\ \mu_{t,d}^{\text{DN}};\ \forall d, \forall t \tag{29}$$

$$r_{d,t}^{UP} - p_{d,t} \le 0 :\ \mu_{t,d}^{\text{UP}};\ \forall d, \forall t \tag{30}$$

2.1.2 Balancing Market

In the second LLP, the power bought and sold in the BLM for N_k net load deviation scenarios are determined. The reserves cleared in the DAM become capacity available in the balancing market. This is analogous to the balancing guidelines in EU Commission (2017). While these reserves are variables of the DAM problem, they are exogenous to the BLM problem. The available capacity is used to meet the balancing energy requirements in the N_k scenarios. The balancing market is modelled via cost minimisation. (31) minimizes the expected imbalance cost across the N_k scenarios, each with probability Φ_k.

The primal variables of the BLM problem (31)-(41) are in the set $^{\text{BL}}_{\text{LL}}$ and comprise the reserves dispatched from all firms (leaders and followers) $\{\ell_{d,t,k}, q_{g,t,k}^{UP}, q_{g,t,k}^{DN}, q_{t,f,k}^{dis,UP}, q_{t,f,k}^{dis,DN}, q_{t,e,k}^{ch,UP}, q_{t,e,k}^{ch,DN}, q_{d,t,k}^{UP}, q_{d,t,k}^{DN}\}$. The dual variables for each constraint are shown after the colon. Table 4 describes the BLM primal and dual variables.

Table 4
Primal and dual variables for the Balancing Market LLP.

Primal	$\ell_{d,t,k}$	Forced power shedding of load d
	$q^{UP}_{g,t,k}, q^{DN}_{g,t,k}$	Up/down energy from reserves of g
	$q^{dis,UP}_{f,t,k}, q^{dis,DN}_{f,t,k}$	Dis. energy from up/down reserves of f
	$q^{ch,UP}_{e,t,k}, q^{ch,DN}_{e,t,k}$	Ch. energy from up/down reserves of e
	$q^{UP}_{d,t,k}, q^{DN}_{d,t,k}$	Up/down energy from reserves of d
Dual	$\lambda^{BL}_{t,k}$	Balancing market equilibrium
	$\underline{\rho}^{\mathrm{UP}}_{g,t,k}, \overline{\rho}^{\mathrm{UP}}_{g,t,k}$	Upward reserve energy bounds for g
	$\underline{\rho}^{\mathrm{DN}}_{g,t,k}, \overline{\rho}^{\mathrm{DN}}_{g,t,k}$	Downward reserve energy bounds for g
	$\underline{\rho}^{\mathrm{UP}}_{d,t,k}, \overline{\rho}^{\mathrm{UP}}_{d,t,k}$	Upward reserve energy bounds for d
	$\underline{\rho}^{\mathrm{DN}}_{d,t,k}, \overline{\rho}^{\mathrm{DN}}_{d,t,k}$	Downward reserve energy bounds for d
	$\underline{\rho}^{\mathrm{dis,UP}}_{f,t,k}, \overline{\rho}^{\mathrm{dis,UP}}_{f,t,k}$	Up dis. reserve energy bounds for f
	$\underline{\rho}^{\mathrm{dis,DN}}_{f,t,k}, \overline{\rho}^{\mathrm{dis,DN}}_{f,t,k}$	Down dis. reserve energy bounds for f
	$\underline{\rho}^{\mathrm{ch,UP}}_{e,t,k}, \overline{\rho}^{\mathrm{ch,UP}}_{e,t,k}$	Up ch. reserve energy bounds for e
	$\underline{\rho}^{\mathrm{ch,DN}}_{e,t,k}, \overline{\rho}^{\mathrm{ch,DN}}_{e,t,k}$	Down ch. reserve energy bounds for e
	$\underline{\rho}_{d,t,k}, \overline{\rho}_{d,t,k}$	Load shedding bounds for d

$$
\begin{aligned}
\min_{\substack{\mathrm{BL}\\\mathrm{LL}}} \{ & Forced\ load\ not\ served \\
& + Cost\ of\ activating\ load\ reserves \\
& + Cost\ of\ activating\ generators\ reserves \\
& + Cost\ of\ activating\ electrolyser\ reserves \\
& + Cost\ of\ activating\ fuelcell\ reserves \}
\end{aligned}
\tag{31}
$$

where

$$Forced\ load\ not\ served = \sum_{t=1}^{\mathcal{T}} \left\{ \sum_{d=1}^{N_d} \left\{ \sum_{k=1}^{N_k} \Phi_k \text{VOLL}\ \ell_{t,d,k} \right\} \right\} \tag{31a}$$

$$Cost\ of\ activating\ load\ reserves = \sum_{t=1}^{\mathcal{T}} \left\{ \sum_{d=1}^{N_d} \left\{ \sum_{k=1}^{N_k} \Phi_k B^{EN}_{d,t} (q^{UP}_{d,t,k} - q^{DN}_{d,t,k}) \right\} \right\} \tag{31b}$$

$$Cost\ of\ activating\ generators\ reserves = \sum_{t=1}^{\mathcal{T}} \left\{ \sum_{g=1}^{N_g} \left\{ \sum_{k=1}^{N_k} \Phi_k O^{EN}_{g,t} (q^{UP}_{g,t,k} - q^{DN}_{g,t,k}) \right\} \right\} \tag{31c}$$

$$Cost\ of\ activating\ electrolyser\ reserves = \sum_{t=1}^{\mathcal{T}} \left\{ \sum_{e=1}^{N_e} \left\{ \sum_{k=1}^{N_k} \Phi_k \hat{b}^{ch}_{t,e} (q^{ch,UP}_{t,e,k} - q^{ch,DN}_{t,e,k}) \right\} \right\} \tag{31d}$$

$$Cost\ of\ activating\ fuelcell\ reserves = \sum_{t=1}^{\mathcal{T}} \left\{ \sum_{f=1}^{N_f} \left\{ \sum_{k=1}^{N_k} \Phi_k \hat{o}^{dis}_{t,f} (q^{dis,UP}_{t,f,k} - q^{dis,DN}_{t,f,k}) \right\} \right\} \tag{31e}$$

Each of the terms in (31) are expected values. The optimization of (31) is subject to the load deviation at time t, i.e, under scenario k when $L_{t,k} > 0$, demand exceeds generation, and conversely when $L_{t,k} < 0$. Equations (31d) and (31e) contain the electrolyser's energy bid and the fuel cell energy offer, respectively, which are exogenous to the BLM problem but are determined in the ULPs.

Equation (32) is the balancing of net load deviation and dispatched reserves.

$$\sum_{d=1}^{N_d} (\ell_{t,d,k} + q^{UP}_{d,t,k} - q^{DN}_{d,t,k}) + \sum_{g=1}^{N_g} (q^{UP}_{g,t,k} - q^{DN}_{g,t,k}) + \sum_{e=1}^{N_e} (q^{ch,UP}_{t,e,k} - q^{ch,DN}_{t,e,k})$$
$$+ \sum_{f=1}^{N_f} (q^{dis,UP}_{t,f,k} - q^{dis,DN}_{t,f,k}) = L_{t,k} : \lambda^{BL}_{t,k};\ \forall t, \forall k \tag{32}$$

Constraints (33)-(40) impose non-negative quantities and limit the maximum balancing energy cleared to the reserves committed in the DAM for all firms. Lastly, (41) sets the the load shedding limit.

$$0 \le q^{UP}_{g,t,k} \le r^{UP}_{g,t} : \underline{\rho}^{\text{UP}}_{g,t,k}, \overline{\rho}^{\text{UP}}_{g,t,k};\ \forall g, \forall t, \forall k \tag{33}$$
$$0 \le q^{DN}_{g,t,k} \le r^{DN}_{g,t} : \underline{\rho}^{\text{DN}}_{g,t,k}, \overline{\rho}^{\text{DN}}_{g,t,k};\ \forall g, \forall t, \forall k \tag{34}$$
$$0 \le q^{UP}_{d,t,k} \le r^{UP}_{d,t} : \underline{\rho}^{\text{UP}}_{d,t,k}, \overline{\rho}^{\text{UP}}_{d,t,k};\ \forall d, \forall t, \forall k \tag{35}$$
$$0 \le q^{DN}_{d,t,k} \le r^{DN}_{d,t} : \underline{\rho}^{\text{DN}}_{d,t,k}, \overline{\rho}^{\text{DN}}_{d,t,k};\ \forall d, \forall t, \forall k \tag{36}$$
$$0 \le q^{dis,UP}_{t,f,k} \le r^{dis,UP}_{t,f} : \underline{\rho}^{\text{dis,UP}}_{t,f,k}, \overline{\rho}^{\text{dis,UP}}_{t,f,k};\ \forall f, \forall t, \forall k \tag{37}$$
$$0 \le q^{dis,DN}_{t,f,k} \le r^{dis,DN}_{t,f} : \underline{\rho}^{\text{dis,DN}}_{t,f,k}, \overline{\rho}^{\text{dis,DN}}_{t,f,k};\ \forall f, \forall t, \forall k \tag{38}$$
$$0 \le q^{ch,UP}_{t,e,k} \le r^{ch,UP}_{t,e} : \underline{\rho}^{\text{ch,UP}}_{t,e,k}, \overline{\rho}^{\text{ch,UP}}_{t,e,k};\ \forall e, \forall t, \forall k \tag{39}$$
$$0 \le q^{ch,DN}_{t,e,k} \le r^{ch,DN}_{t,e} : \underline{\rho}^{\text{ch,DN}}_{t,e,k}, \overline{\rho}^{\text{ch,DN}}_{t,e,k};\ \forall e, \forall t, \forall k \tag{40}$$

$$0 \leq \ell_{d,t,k} \leq p_{d,t} : \underline{\rho}_{d,t,k}, \overline{\rho}_{d,t,k}; \; \forall d, \forall t, \forall k \tag{41}$$

2.2 Upper-Level Problems

The market prices in the ULPs, namely the day ahead electricity price ($\lambda_{t,n}^{EN}$), the hydrogen price (λ_t^{H2}), the upwards reserve price (λ_t^{UP}), the downwards reserve price (λ_t^{DN}) and the balancing market price ($\lambda_{t,k}^{BL}$) are exogenous to the ULPs and endogenously determined in the LLPs.

2.2.1 Electrolyser Firm

Equation (42) is the electrolyser firm's objective function, which maximises the firm's expected profit. Its revenue streams consist of selling i) hydrogen, $Q_{t,e}$, at the less the discharge cost, C_e^{dis} (42b); ii) reserves, $r_{t,e}^{ch,\{UP,DN\}}$ (42c) and (42d); and iii) upward balancing energy (by reducing charging) $q_{t,e,k}^{ch,UP}$ for each k plus the avoided cost of charging, C_e^{ch} (42e). The firm's expenses are i) electricity procurement in the day ahead market, $p_{t,e}^{\mathrm{DA}}$, plus the cost of charging (42a) and ii) electricity purchased from the balancing market, as a result of being called on to provide upward reserve, $q_{t,e,k}^{ch,DN}$ (42f). Table 5 describes the variables in the ULP (42)-(52).

Table 5
Primal variables for the Electrolyser firm ULP.

$\hat{b}_{t,e}^{ch}$	Energy bid [€/MWh]
$\hat{o}_{t,e}^{\mathrm{H2}}$	Hydrogen offer [€/kg]
$\hat{o}_{t,e}^{ch,UP}, \hat{o}_{t,e}^{ch,DN}$	Up/Down reserve capacity offer [€/MWh]
$\hat{p}_{t,e}^{\mathrm{DA}}$	Power bid [MW]
$\mathscr{P}_{t,e}$	Pressure of H_2 tank of ESR e [J/m^3]
$\hat{Q}_{t,e}$	Hydrogen gas offer [kg]
$\hat{r}_{t,e}^{ch,UP}, \hat{r}_{t,e}^{ch,DN}$	Up/Down ch. reserve capacity offer [MW]

$$\begin{aligned}
\Pi_{\mathrm{ESR}} = \max_{\substack{e \\ \mathrm{UL}}} \{ & -Electricity\ and\ Charging\ Cost \\
& + Hydrogen\ Revenue\ less\ Discharge\ Cost \\
& + Up\ Reserve\ Revenue \\
& + Down\ Reserve\ Revenue \\
& + Expected\ Balancing\ Market\ Revenue \\
& - Expected\ Balancing\ Market\ Costs \}
\end{aligned} \tag{42}$$

where

$$Electricity\ and\ Charging\ Cost = \sum_{t=1}^{\mathscr{T}} \left\{ \sum_{e \in \Psi_n} (\lambda_{t,n}^{EN} + C_e^{ch}) p_{t,e}^{\text{DA}} \right\} \tag{42a}$$

$$Hydrogen\ Revenue\ less\ Discharge\ Cost = \sum_{t=1}^{\mathscr{T}} \left\{ \sum_{e=1}^{N_e} \left\{ (\lambda_t^{\text{H2}} - C_e^{dis}) Q_{t,e} \right\} \right\} \tag{42b}$$

$$Up\ Reserve\ Revenue = \sum_{t=1}^{\mathscr{T}} \left\{ \sum_{e=1}^{N_e} \left\{ \lambda_t^{UP} r_{t,e}^{ch,UP} \right\} \right\} \tag{42c}$$

$$Down\ Reserve\ Revenue = \sum_{t=1}^{\mathscr{T}} \left\{ \sum_{e=1}^{N_e} \left\{ \lambda_t^{DN} r_{t,e}^{ch,DN} \right\} \right\} \tag{42d}$$

$$Expected\ Balancing\ Market\ Revenue = \sum_{t=1}^{\mathscr{T}} \left\{ \sum_{e=1}^{N_e} \left\{ \sum_{k=1}^{N_k} \Phi_k (\lambda_{t,k}^{BL} + C_e^{ch}) (q_{t,e,k}^{ch,UP}) \right\} \right\} \tag{42e}$$

$$Expected\ Balancing\ Market\ Costs = \sum_{t=1}^{\mathscr{T}} \left\{ \sum_{e=1}^{N_e} \left\{ \sum_{k=1}^{N_k} \Phi_k (\lambda_{t,k}^{BL} + C_e^{ch}) (q_{t,e,k}^{ch,DN}) \right\} \right\} \tag{42f}$$

The firm maximises objective function (42) subject to several constraints on the hydrogen tank, the capacity of the electrolyser and some non-negativity constraints.

The Ideal Gas Law (43) models the firm's H_2 tank pressure for an storage system with volume V_e and temperature T^e. The pressure in the tank at each t is a function of the pressure at $t-1$, $\mathscr{P}_{t-1,e}$, the hydrogen in the tank, $Q_{t,e}$ and the electricity consumption, $p_{t,e}^{\text{DA}}$ net of the expected dispatch actions taken by the balancing market operator, $\sum_{k=1}^{N_k} \Phi_k (q_{t,e,k}^{ch,DN} - q_{t,e,k}^{ch,UP})$. We use the low heat value, LHV, to consider that not all energy injected is converted to hydrogen.

$$\mathscr{P}_{t,e} = \mathscr{P}_{t-1,e} - \frac{RT^e}{V_e M^{\text{H2}}} \left\{ Q_{t,e} - \frac{\eta_e \Delta t}{\text{LHV}} \left[p_{t,e}^{\text{DA}} + \sum_{k=1}^{N_k} \Phi_k (q_{t,e,k}^{ch,DN} - q_{t,e,k}^{ch,UP}) \right] \right\} \quad \forall t, \forall e \tag{43}$$

Constraint (44) defines the pressure bounds on the hydrogen tank:

$$\mathscr{P}_e^{\text{MIN}} \leq \mathscr{P}_{t,e} \leq \mathscr{P}_e^{\text{MAX}} \quad \forall t, \forall e \tag{44}$$

Equation (45) sets the initial tank pressure:

$$\mathscr{P}_{o,e} = \mathscr{P}_e^{\text{INI}} \quad \forall t, \forall e \tag{45}$$

Constraint (46) limits the power demand of the ESR:

$$P_e^{\text{MIN}} \leq \hat{p}_{t,e}^{\text{DA}} \leq P_e^{\text{MAX}} \quad \forall t, \forall e \tag{46}$$

Constraints (47) and (48) limit the upward and downward reserves offers, respectively:

$$0 \leq \hat{r}^{ch,UP}_{t,e} \leq R^{ch,UP}_{e} \quad \forall t, \forall e \tag{47}$$

$$0 \leq \hat{r}^{ch,DN}_{t,e} \leq R^{ch,DN}_{e} \quad \forall t, \forall e \tag{48}$$

Constraints (49) and (50) constrain the reserve offers to the power bid and rated capacity:

$$\hat{p}^{\text{DA}}_{t,e} + \hat{r}^{ch,DN}_{t,e} \leq P^{\text{MAX}}_{e} \quad \forall t, \forall e \tag{49}$$

$$\hat{r}^{ch,UP}_{t,e} - \hat{p}^{\text{DA}}_{t,e} \leq P^{\text{MIN}}_{e} \quad \forall t, \forall e \tag{50}$$

Constraint (51) defines the H_2 offer limited to the firm's hydrogen system capacity:

$$0 \leq \hat{Q}_{t,e} \leq \frac{\mathscr{P}^{\text{MAX}}_{e} V_e M^{\text{H2}}}{RT^{e}} \quad \forall t, \forall e \tag{51}$$

Finally, (52) enforce non-negative offers and bids:

$$\hat{b}^{ch}_{t,e},\ \hat{o}^{\text{H2}}_{t,e},\ \hat{o}^{ch,UP}_{t,e}, \hat{o}^{ch,DN}_{t,e} \geq 0 \quad \forall t, \forall e \tag{52}$$

In addition to the above constraints, the electrolyser firm's ULP also contains the KKT conditions of the DAM and BLM problems; these are shown in A.1 and A.2, respectively. Hence, the electrolyser firm's optimisation problem (which takes the form of an MPEC) is to maximise objective function (42), subject to constraints (43)-(52), the DAM KKT conditions (64)-(123) and subject to BLM KKT conditions (124)-(151).

2.2.2 Fuelcell Firm

Equation (53) is the fuel cell firm's objective function, it shows the firm's profits. The revenue comes from selling i) electricity, $p^{\text{DA}}_{t,f}$ less the discharge cost, C^{dis}_{f} (53a); ii) reserves, $r^{dis,\{UP,DN\}}_{t,f}$ (53c) and (53d); and iii) upward balancing energy, $q^{dis,UP}_{t,f,k}$ less the discharge cost (53e). The expenses are i) hydrogen procurement, $Q_{t,f}$, plus the charging cost, C^{ch}_{f} (53b) and ii) energy not produced (downward reserves deployment) less the avoided discharge cost (53f). Table 6 describes the variables in the ULP (53)-(63).

Table 6
Primal variables for the Fuel Cell firm ULP.

$\hat{b}^{\text{H2}}_{t,f}$	Hydrogen bid [€/kg]
$\hat{o}^{dis,UP}_{t,f}, \hat{o}^{dis,DN}_{t,f}$	Up/Down reserve capacity offer [€/MWh]
$\hat{o}^{dis}_{t,f}$	Energy offer [€/MWh]
$\hat{p}^{\text{DA}}_{t,f}$	Power offer [MW]
$\mathscr{P}_{t,f}$	Pressure of H_2 tank of fuelcell f [J/m^3]
$\hat{Q}_{t,f}$	Hydrogen gas bid [kg]
$\hat{r}^{dis,UP}_{t,f}, \hat{r}^{dis,DN}_{t,f}$	Up/Down dis. reserve capacity offer [MW]

$$\begin{aligned}\Pi_{\text{FC}} = \max_{\substack{\text{f}\\ \text{UL}}}\{&Electricity\ Revenue\\ &-Cost\ of\ hydrogen\\ &+Up\ Reserve\ Revenue\\ &+Down\ Reserve\ Revenue\\ &+Expected\ Balancing\ Market\ Revenue\\ &-Expected\ Balancing\ Market\ Costs\}\end{aligned} \tag{53}$$

where

$$Electricity\ Revenue = \sum_{t=1}^{\mathscr{T}}\left\{\sum_{f\in\Psi_n}(\lambda^{EN}_{t,n} - C^{dis}_f)p^{\text{DA}}_{t,f}\right\} \tag{53a}$$

$$Cost\ of\ hydrogen = \sum_{t=1}^{\mathscr{T}}\left\{\sum_{f}^{N_f}\left\{(\lambda^{\text{H2}}_t + C^{ch}_f)Q_{t,f}\right\}\right\} \tag{53b}$$

$$Up\ Reserve\ Revenue = \sum_{t=1}^{\mathscr{T}}\left\{\sum_{f}^{N_f}\left\{\lambda^{UP}_t r^{dis,UP}_{t,e}\right\}\right\} \tag{53c}$$

$$Down\ Reserve\ Revenue = \sum_{t=1}^{\mathscr{T}}\left\{\sum_{f}^{N_f}\left\{\lambda^{DN}_t r^{dis,DN}_{t,e}\right\}\right\} \tag{53d}$$

$$Expected\ Balancing\ Market\ Revenue = \sum_{t=1}^{\mathscr{T}}\left\{\sum_{f}^{N_f}\left\{\sum_{k=1}^{N_k}\Phi_k(\lambda^{BL}_{t,k} + C^{dis})(q^{dis,UP}_{t,e,k})\right\}\right\} \tag{53e}$$

$$Expected\ Balancing\ Market\ Costs = \sum_{t=1}^{\mathscr{T}}\left\{\sum_{f}^{N_f}\left\{\sum_{k=1}^{N_k}\Phi_k(\lambda^{BL}_{t,k} + C^{dis})(q^{dis,DN}_{t,e,k})\right\}\right\} \tag{53f}$$

The objective function (53) is maximised subject to several constraints on the hydrogen tank, the capacity of the fuelcell and non-negativity constraints.

The Ideal Gas Law (54) models the firm's H_2 tank pressure for an storage system with volume V_f and temperature T^f. The pressure in the tank at each t is a function of the pressure at $t-1$, $\mathscr{P}_{t-1,f}$, the hydrogen in the tank, $Q_{t,f}$ and the electricity production, $p^{\text{DA}}_{t,f}$ net of the expected dispatch actions taken by the balancing market operator, $\sum_{k=1}^{N_k} \Phi_k(q^{dis,UP}_{t,f,k} - q^{dis,DN}_{t,f,k})$. We use the low heat value, LHV, to consider that not all consumed hydrogen is converted to electricity.

$$\mathscr{P}_{t,f}=\mathscr{P}_{t-1,f}+\frac{RT^{\text{f}}}{V_f M^{\text{H2}}}\left\{Q_{t,f}-\frac{\Delta t}{\eta_f \text{LHV}}\left[p^{\text{DA}}_{t,f}+\sum_{k=1}^{N_k}\Phi_k(q^{dis,UP}_{t,f,k}-q^{dis,DN}_{t,f,k})\right]\right\} \quad \forall t, \forall f \tag{54}$$

Constraint (55) defines the pressure bounds on the fuelcell's hydrogen tank:

$$\mathscr{P}^{\text{MIN}}_f \leq \mathscr{P}_{t,f} \leq \mathscr{P}^{\text{MAX}}_f \quad \forall t, \forall f \tag{55}$$

Equation (56) sets the initial tank pressure:

$$\mathscr{P}_{o,f} = \mathscr{P}^{\text{INI}}_f \quad \forall t, \forall f \tag{56}$$

Constraint (57) limits the power offer according to the capacity of the fuelcell:

$$P^{\text{MIN}}_f \leq \hat{p}^{\text{DA}}_{t,f} \leq P^{\text{MAX}}_f \quad \forall t, \forall f \tag{57}$$

Constraints (58) and (59) limit the upward and downward reserves offers, respectively.

$$0 \leq \hat{r}^{dis,UP}_{t,f} \leq R^{dis,UP}_f \quad \forall t, \forall f \tag{58}$$

$$0 \leq \hat{r}^{dis,DN}_{t,f} \leq R^{dis,DN}_f \quad \forall t, \forall f \tag{59}$$

Constraints (60) and (61) limit the reserve offers to the power offer and rated capacity.

$$\hat{p}^{\text{DA}}_{t,f} + \hat{r}^{dis,UP}_{t,f} \leq P^{\text{MAX}}_f \quad \forall t, \forall f \tag{60}$$

$$r^{dis,DN}_{t,f} - \hat{p}^{\text{DA}}_{t,f} \leq P^{\text{MIN}}_f \quad \forall t, \forall f \tag{61}$$

Constraint (62) defines the H_2 bid limited to the firm's hydrogen system capacity:

$$0 \leq \hat{Q}_{t,f} \leq \frac{\mathscr{P}_f^{\text{MAX}} V_f M^{\text{H2}}}{RT^f} \qquad \forall t, \forall f \tag{62}$$

Lastly, (63) enforce non-negative offers/bids:

$$\hat{o}_{t,f}^{dis}, \hat{b}_{t,f}^{\text{H2}}, \hat{o}_{t,f}^{dis,UP}, \hat{o}_{t,f}^{dis,DN} \geq 0 \qquad \forall t, \forall f \tag{63}$$

In addition to the above constraints, the fuelcell firm's ULP also contains the KKT conditions of the DAM and BLM problems. The KKT conditions of the two LLPs are shown in A.1 and A.2. Hence, the optimisation problem (which takes the form of an MPEC) is to maximise objective function (53), subject to constraints (54)-(63), the DAM KKT conditions (64)-(123) and subject to BLM KKT conditions (124)-(151).

2.3 EPEC Problem

The previous section described the mathematical formulation. It comprises a Stackelberg modelling approach. Both strategic firms, described in Sections 2.2.1 and 2.2.2 respectively, solve a bi-level optimization problem and play a Nash game amongst each other. In each bi-level problem, the strategic firm, in the upper-level problem, anticipates the responses from the markets and other competitive firms, in the lower-level problems. The lower-level problems, described in 2.1.1 and 2.1.2, are replaced by their KKT conditions, shown in A.1 and A.2, then included as constraints to each upper-level problem yielding an MPEC. The optimization problems are transformed into mixed-integer optimization problems by linearizing the complementary conditions with the Fortuny-Amat McCarl method, described in B. The two coupled MPECs, one for each strategic firm, lead to an EPEC structure.

The objective is to find a Nash Equilibrium between the Electrolyser and Fuelcell firms. To search for an equilibrium solution, we implement the Gauss-Seidel Diagonalisation algorithm which iteratively solves each leader's MPEC problem by fixing the other leader's decisions, until successive decision variables converge. Determining the existence of global pure strategies to EPECs is an active research field thus stating existence and uniqueness is beyond the scope of this paper. However, this work yields informative insights about the market potential and interactions of merchant firms investing in salient technologies to decarbonize sectors of the economy.

3 Case Study

The case study is based on the IEEE 14-Bus System (Fig. 1). Wind power generation corresponds to scaled and randomized Irish data from 2019. Fig. 2 shows the aggregated demand, solar and wind

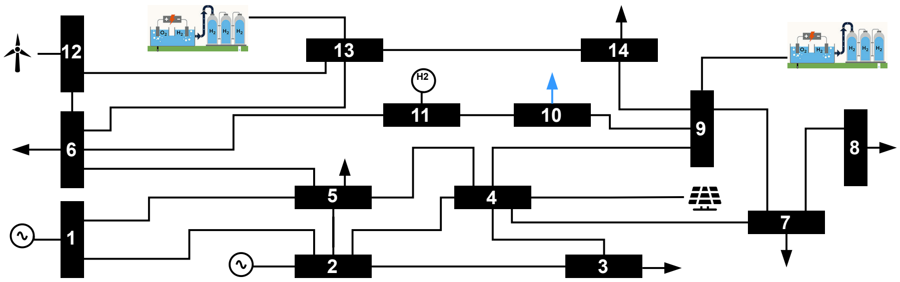


Figure 1: The FC and ESR are located at node 9 and 13, respectively. The SMR and FCFL are in node 11 and 10, respectively.

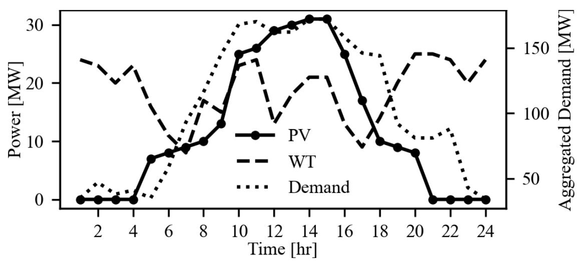


Figure 2: The aggregated demand and wind and solar power generation.

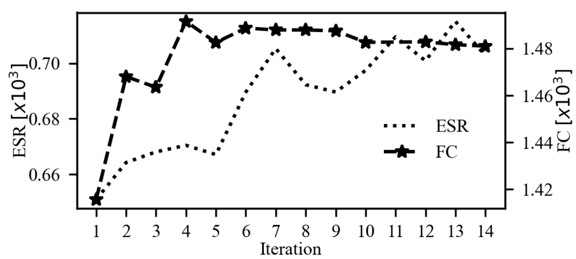


Figure 3: Convergence of the Diagonalization algorithm.

power generation. The reserve requirements, R_t^{UP} and R_t^{DN}, are 5 MW, this is equivalent to 20% of the maximum wind power forecast (Nasrolahpour et al., 2018). The uncertainty in the BLM is represented by two scenarios, $N_k = 2$, with equal probability, $\Phi_k = 0.5$.The grid comprises 6 electricity loads and 4 power plants. Loads bid at 95 €/MWh and those located at buses 6 and 8 offer reserves and balancing energy at 90 €/MWh. The competitive power firms are: CCGT with CCS, Gasoil, PV and wind generators at bus 1, 2, 4 and 12, respectively. The CCGT capacity is 30 MW and the Gasoil is 94 MW. The firms' energy offers are 55, 80, 13 and 13 €/MWh, respectively (Longoria et al., 2021). The firms' reserve offers are 10, 40, 6 and 6 €/MWh, respectively. Load and generators can offer up to 10% of their capacity, as per Voss et al. (2021).

The strategic fuelcell and electrolyser firms are in buses 9 and 13, respectively. Both have similar characteristics, i.e., the min and max tank pressure are 10 and 100 bars (Loisel et al., 2019), respectively. The electricity charging and discharging operational cost is 60 €/kW·yr and the charging and discharging of H_2 is 11 €/kg·yr (Grüger et al., 2018). The rated capacity is 1.5 MW (HDFEnergy, 2021).

A steam methane reformer (SMR) is in bus 11; the max throughput is 20 kg/hr and it offers H_2 at 3 €/kg (Grüger et al., 2018). At bus 10 there are 4 H_2 forklifts (FCFL) with a hydrogen capacity of 3 kg each (PlugPower, 2013). These represent a secondary H_2 demand (Loisel et al., 2019; Staffell et al., 2019). The refuelling hours are t= 7, 13 and 20.

4 Results and Discussion

In order to determine the impact of ancillary services markets, we solve the EPEC twice. First, we solve the EPEC (1)-(63) in its entirety. Second, we solve the EPEC described in Section 2 but without system services, i.e, without considering reserves provisioning and balancing energy. This is done by fixing reserves requirements in the DAM and all the variables in the BLM problem to be zero, which leads to balancing market prices equal to zero. Solving the EPEC twice allows us to determine the optimal decisions of the profit-maximising firms with and without participation in ancillary services. In the following, all profits, revenues and costs, refer to *expected* profits, revenues and costs.

4.1 Revenues and Costs

Fig. 4 shows the profit structure for both leader firms in the integrated markets. For the electrolyser firm, the costs of electricity and down balancing represent 90% and 10% of the operating expenses respectively. Sales of H_2, balancing energy, upward and downward reserves are 14.1%, 46.3%, 33.4% and 6.2% of the revenue,[1] respectively. For the fuelcell firm, the cost of H_2 and down balancing is 33% and 67% of the operating cost, respectively. The sale of electricity, balancing energy, upward and downward reserves represent 51%, 4%, 39% and 6% of the revenue, respectively.

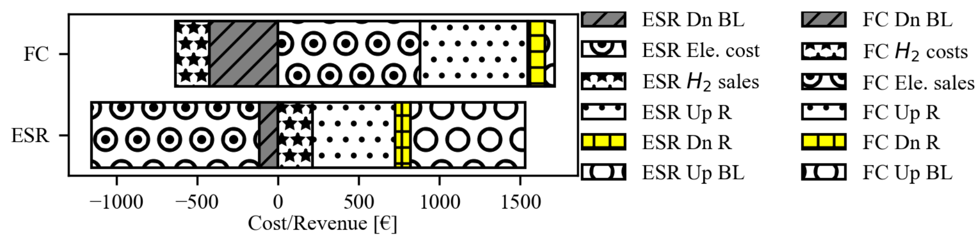


Figure 4: The firms' aggregated costs and revenues in the integrated markets.

Given the significant portion of costs and revenues that arise from the reserve and balancing markets, removing these markets from the analysis makes a significant difference. The daily profit of the leader firms is far higher under the integrated markets compared to the day ahead only market: the fuelcell's profit increases by 319% while the electrolyser's profit increases by two orders of magnitude. This suggests that reserve and balancing markets play a large role in determining the profitability of these firms and underlines the contribution of this research by considering integrated energy and ancillary service markets.

[1] This work does not consider the revenue from selling oxygen. However, the contribution could be modest in terms of the hydrogen production costs Olateju et al. (2016).

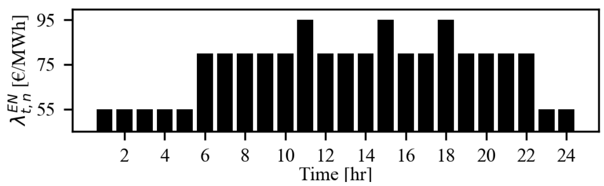


Figure 5: DAM's electricity clearing price in the integrated markets.

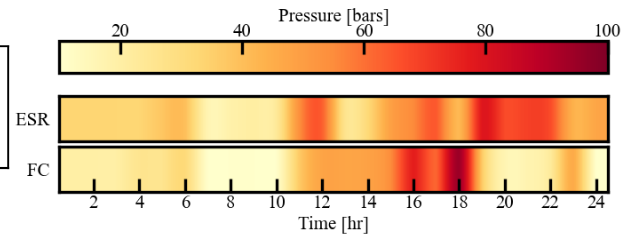


Figure 6: The H_2 tank pressure of the leader firms in the integrated markets.

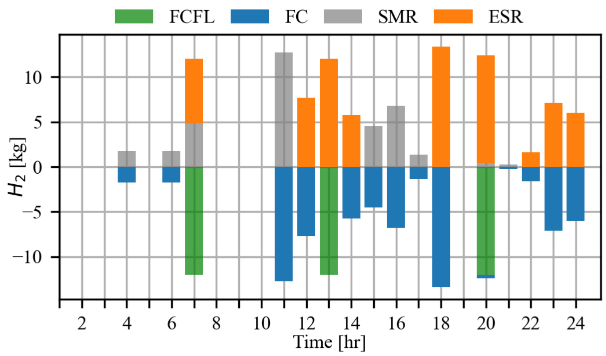
4.2 Strategic Trading

In the integrated market, the leader firms exploit arbitrage opportunities between the day ahead market and balancing market, primarily by procuring (selling) energy and selling (buying) it back as upward (downward) reserves. Securing a position in the reserve market enables dispatch, and hence profit-making, in the balancing market. In particular, for the electrolyser firm, this strategy covers its exposure in the DAM by totally or partially offsetting the electricity expense in the BLM but earning revenues from the offering of reserves.

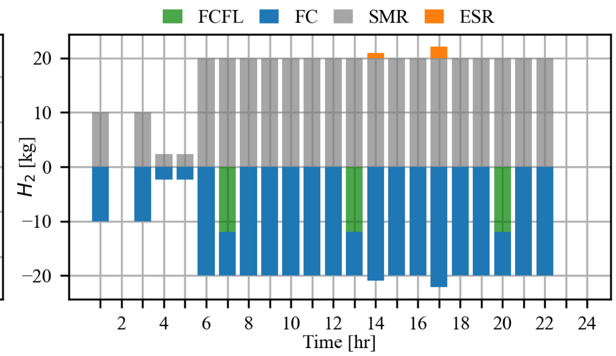
The electrolyser firm submits low offers to out-bid competitor firms. For example, when there is surplus power capacity, at t=1,17,19 and 24, the electrolyser's offer for upward reserve is 0 €/MW. This offer takes the rightmost position in the merit order curve and gains inframarginal rent at the reserve clearing price. Further profit is earned in the balancing market by being dispatched (via reduced electricity consumption).

Around 80% of the time, the fuelcell firm offers only a third of its capacity on the day ahead market, with the remaining capacity offered as upward reserves. The fuelcell firm manages its H_2 stock in such a way that there is pressure available to supply electricity during high priced hours. It does this by submitting offers similar to the marginal firm in the day ahead market. In this way, the electricity offered in the day ahead market is bought back in the balancing market at a lower price, thus still making a profit without consuming hydrogen from the tank. Whereas at t=19 (i.e., before the off-peak hours in Fig. 5) and t=24 the fuelcell firm uses the opposite strategy, i.e., submits a low offer analogous to the cheapest firm (i.e., $\hat{o}_{t,f}^{dis}$=13 €/MWh). By doing this, most of the energy offered in the DAM is committed and not dispatched down in the balancing market.

The fuelcell firm's stock of H_2, after providing the system operator with electricity at t=19 and 20, is just enough to sustain the minimum pressure in the tank as shown in Fig. 6. In the interim, before t=24, the firm returns to arbitrage trading between the day ahead market and the balancing market, while procuring enough H_2 from the electrolyser firm and the SMR to commit electricity in the day ahead market, ending with enough hydrogen stock to maintain pressure in the tank.



(a) Integrated markets.



(b) Energy and hydrogen market.

Figure 7: Hydrogen traded under two market configurations.

4.3 Hydrogen Price and Quantities

The average hydrogen clearing price is only marginally higher for the day ahead-only versus the integrated market analysis at 3.004 €/kg vs. 2.988 €/kg. However, as shown in Fig. 7, the total hydrogen traded is very different, driven particularly by the fuelcell's hydrogen demand. This suggests that hydrogen can play a role in balancing electricity, especially compared to the opportunities available to hydrogen in an energy-only market.

Overall, the possibility of revenue streams from AS means the total traded hydrogen is almost one-third of the quantity traded in the absence of the market for ancillary services. Thus without AS (Fig. 7b), the ESR's hydrogen production is driven out of market, being replaced by the SMR production supplying 330 kg of hydrogen more than in the case of integrated markets. Similarly, the FC's hydrogen demand grows 261 kg. In contrast, in the integrated market case, the market share of the ESR firm represents 68% of the total hydrogen demand; the SMR supplying the rest of the demand.

The shift from SMR to electrolyser production of H_2 enabled by trading in integrated markets, along with increased demand from the fuelcell, gives rise to a commensurate reduction in CO_2 emissions. The total emissions rate from the steam methane reforming process is 11.888 kg CO_2 equivalent per kg H_2 Spath and Mann (2001). The SMR plant's carbon footprint grows 962% for the energy only market.

4.4 Power Trading

In the DAM only, the fuelcell's revenue streams are entirely determined from day ahead power sales. Hence, the fuelcell's aggregated cleared power is 36% higher than in the integrated markets case. In contrast, the power procurement from the electrolyser firm is zero in the absence of the ancillary services market, suggesting that day ahead market price arbitrage opportunities are insufficient to enable electrolysers to make a profit. With ancillary services, the electrolyser firm can make a positive profit despite the power procurement costs. As shown in Fig. 8, the electrolyser clears relatively more power in the off-peak hours (i.e., before t=6 and after t=22) than during the more expensive time periods.

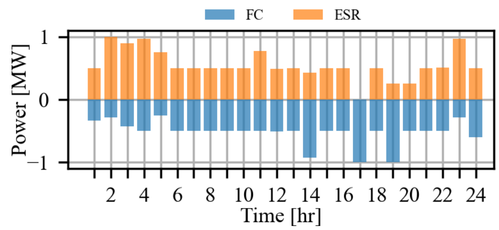


Figure 8: Cleared power in the integrated markets case. Power consumption is shown in the negative axis.

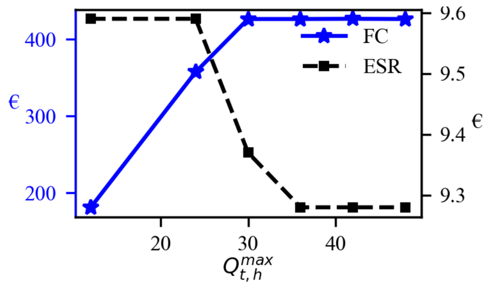
These changes in market activity change the capacity factor of both the fuelcell and electrolyser. The electrolyser increases its capacity factor from essentially zero to just over 11%, while the capacity of the fuelcell firm declines from 47.8% to 11%. The arbitrage opoprtunities afforded to the fuelcell allow it to reduce its dependance on hydrogen procurement for revenue, thus reducing the hydrogen required to generate electricity.

4.5 Sensitivity Analysis

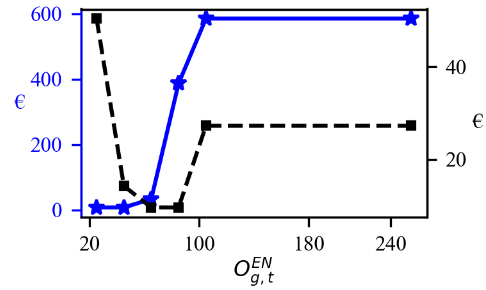
We performed a sensitivity analysis on exogenous model parameters related to hydrogen supply, demand and the power offer of the most expensive power firm. The analysis is performed for the energy and hydrogen market only because of the computational complexity of the integrated market model. Nevertheless, the results provide insights on the effect of exogenous market parameters on the overall performance of the upper level firms.

Fig. 9 shows the effect of the input parameters $Q^{max}_{t,h}$, $O^{EN}_{g,t}$, $O^{EN}_{t,h}$ and $Q^{max}_{t,d_{H2}}$ on the profit of the leader firms. As expected, Fig. 9a shows that a decline in the hydrogen supply capacity available in the market, *ceteris paribus*, means the electrolyser firm can increase its revenues from selling hydrogen whereas the fuelcell's profit is reduced. In contrast, Fig. 9c shows that when the exogenous marginal cost of the SMR firm or the demand of hydrogen rises, the electrolyser's profit also increases while the fuelcell's profit declines due to higher hydrogen clearing prices.

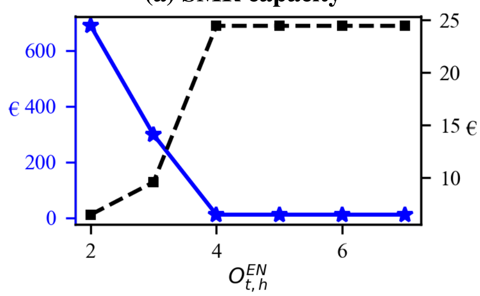
Fig. 9b shows the effect of the offer from the most expensive generator. The results show that a decrease of 30% on the generator's offer can increase the electrolyer's profit by 48%. In addition, the trend observed in the other plots changes as the offer of the marginal generator increases beyond a threshold offer. The revenues obtained by the fuelcell from selling electricity rise as a consequence of higher electricity clearing prices and so the fuelcell is willing to pay more for hydrogen. The electrolyser firm collects more revenue from higher hydrogen clearing prices without incurring an increased electricity cost by purchasing at the off-peak hours. The leader firms therefore both benefit from higher electricity generation costs.



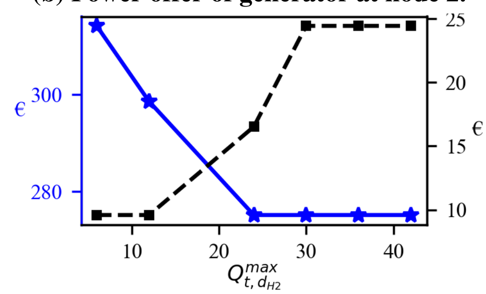
(a) SMR capacity



(b) Power offer of generator at node 2.



(c) SMR H_2 price.



(d) FCFL demand

Figure 9: Profit sensitivity to input parameters in the energy and hydrogen market.

5 Conclusion

This paper presented a novel market model for integrated day ahead, reserve and balancing markets in the electricity sector. The inclusion of a strategic hydrogen production firm, via electrolysis, and a strategic fuelcell that produces electricity from hydrogen, examined for the first time the impacts of strategic hydrogen producers and consumers in a competitive integrated electricity market. The impacts of the reserve and balancing market activity was also identified.

There are several main conclusions to be drawn. First, strategic trading by leader firms is impactful. The market equilibria alter significantly when the firms exploit arbitrage opportunities between day ahead and balancing markets. This underlines the importance of considering activity across all markets rather than focusing on individual markets only.

Second, hydrogen production and consumption can potentially play a significant role in electricity system balancing. Electrolyser firms play almost no role in day ahead-only markets, but can make a significant profit in integrated energy and reserves markets. Expanding the suite of ancillary service revenues available to hydrogen firms, such as providing frequency response, voltage control or inertia, may enhance their profitability further.

A third conclusion is the side benefit that ancillary service provision brings by allowing electrolysers to replace steam methane reformers as the main source of hydrogen. This facilitates the replacement of a high-carbon source of hydrogen by lower carbon electrolysers.

In general, this research highlights the importance of including ancillary service provision in market modelling by industry participants and policy makers, and the opportunities for hydrogen that can be exploited via ancillary service provision. Electricity markets should be structured such as to facilitate participation by hydrogen firms in a technology-neutral manner.

Future work will consider the impacts of financial incentives for green hydrogen on the market equilibria, as well as the impacts of strategic players amongst the electricity generators as well as the hydrogen players.

A KKT conditions of Lower-Level Problems

The 'perp' notation $0 \leq a \perp b \geq 0$, is equivalent to $a \geq 0$, $b \geq 0$ and $a.b = 0$.

A.1 Day-ahead market problem (1) - (30)

$$\partial_{p^{\mathrm{DA}}_{t,e}} : -\hat{b}^{ch}_{t,e} + \lambda^{EN}_{t,n} + \overline{\mu}^{\mathrm{DA}}_{t,e} - \underline{\mu}^{\mathrm{DA}}_{t,e} - \mu^{\mathrm{DA,UP}}_{t,e} = 0 \ \forall t, \forall e \in \Psi_n \tag{64}$$

$$\partial_{p^{\mathrm{DA}}_{t,f}} : \hat{o}^{dis}_{t,f} - \lambda^{EN}_{t,n} + \overline{\mu}^{\mathrm{DA}}_{t,f} - \underline{\mu}^{\mathrm{DA}}_{t,f} - \mu^{\mathrm{DA,DN}}_{t,f} = 0 \ \forall t, \forall f \in \Psi_n \tag{65}$$

$$\partial_{p_{d,t}} : -B^{EN}_{d,t} + \lambda^{EN}_{t,n} + \overline{\mu}_{d,t} - \underline{\mu}_{d,t} - \mu^{\mathrm{UP}}_{t,d} + \mu^{\mathrm{DN}}_{t,d} = 0 \ \forall t, \forall d \in \Psi_n \tag{66}$$

$$\partial_{p_{g,t}} : O^{EN}_{g,t} - \lambda^{EN}_{t,n} + \overline{\mu}_{g,t} - \underline{\mu}_{g,t} + \mu^{\mathrm{UP}}_{t,g} - \mu^{\mathrm{DN}}_{t,g} = 0 \ \forall t, \forall g \in \Psi_n \tag{67}$$

$$\partial_{Q_{t,f}} : -\hat{b}^{\mathrm{H2}}_{t,f} + \lambda^{\mathrm{H2}}_{t} + \overline{\mu}^{\mathrm{Q}}_{t,f} - \underline{\mu}^{\mathrm{Q}}_{t,f} = 0 \ \forall t, \forall f \tag{68}$$

$$\partial_{Q_{t,e}} : \hat{o}^{\mathrm{H2}}_{t,e} - \lambda^{\mathrm{H2}}_{t} + \overline{\mu}^{\mathrm{Q}}_{t,e} - \underline{\mu}^{\mathrm{Q}}_{t,e} = 0 \ \forall t, \forall e \tag{69}$$

$$\partial_{Q_{t,h}} : O^{EN}_{t,h} - \lambda^{\mathrm{H2}}_{t} + \overline{\mu}_{t,h} - \underline{\mu}_{t,h} = 0 \ \forall t, \forall h \tag{70}$$

$$\partial_{Q_{t,d_{\mathrm{H2}}}} : -B^{EN}_{t,d_{\mathrm{H2}}} + \lambda^{\mathrm{H2}}_{t} + \overline{\mu}_{t,d_{\mathrm{H2}}} - \underline{\mu}_{t,d_{\mathrm{H2}}} = 0 \ \forall t, \forall d_{\mathrm{H2}} \tag{71}$$

$$\partial_{r^{ch,UP}_{t,e}} : \hat{o}^{ch,UP}_{t,e} - \lambda^{UP}_{t} + \overline{\mu}^{\mathrm{ch,UP}}_{t,e} - \underline{\mu}^{\mathrm{ch,UP}}_{t,e} + \mu^{\mathrm{DA,UP}}_{t,e} = 0 \ \forall t, \forall e \tag{72}$$

$$\partial_{r^{ch,DN}_{t,e}} : \hat{o}^{ch,DN}_{t,e} - \lambda^{DN}_{t} + \overline{\mu}^{\mathrm{ch,DN}}_{t,e} - \underline{\mu}^{\mathrm{ch,DN}}_{t,e} = 0 \ \forall t, \forall e \tag{73}$$

$$\partial_{r^{dis,UP}_{t,f}} : \hat{o}^{dis,UP}_{t,f} - \lambda^{UP}_{t} + \overline{\mu}^{\mathrm{dis,UP}}_{t,f} - \underline{\mu}^{\mathrm{dis,UP}}_{t,f} = 0 \ \forall t, \forall f \tag{74}$$

$$\partial_{r^{dis,DN}_{t,f}} : \hat{o}^{dis,DN}_{t,f} - \lambda^{DN}_{t} + \overline{\mu}^{\mathrm{dis,DN}}_{t,f} - \underline{\mu}^{\mathrm{dis,DN}}_{t,f} + \mu^{\mathrm{DA,DN}}_{t,f} = 0 \forall t, \forall f \tag{75}$$

$$\partial_{r^{UP}_{g,t}} : O^{RS}_{g,t} - \lambda^{UP}_{t} + \overline{\mu}^{\mathrm{UP}}_{t,g} - \underline{\mu}^{\mathrm{UP}}_{t,g} + \mu^{\mathrm{UP}}_{t,g} = 0 \ \forall t, \forall g \tag{76}$$

$$\partial_{r^{DN}_{g,t}} : O^{RS}_{g,t} - \lambda^{DN}_{t} + \overline{\mu}^{\mathrm{DN}}_{t,g} - \underline{\mu}^{\mathrm{DN}}_{t,g} + \mu^{\mathrm{DN}}_{t,g} = 0 \ \forall t, \forall g \tag{77}$$

$$\partial_{r^{UP}_{d,t}} : B^{RS}_{d,t} - \lambda^{UP}_{t} + \overline{\mu}^{\mathrm{UP}}_{t,d} - \underline{\mu}^{\mathrm{UP}}_{t,d} + \mu^{\mathrm{UP}}_{t,d} = 0 \ \forall t, \forall d \tag{78}$$

$$\partial_{r^{DN}_{d,t}}: B^{RS}_{d,t} - \lambda^{DN}_t + \overline{\mu}^{\mathrm{DN}}_{t,d} - \underline{\mu}^{\mathrm{DN}}_{t,d} + \mu^{\mathrm{DN}}_{t,d} = 0\ \forall t, \forall d \tag{79}$$

$$\partial_\delta: \sum_{m\in\Theta_n} B_{nm}(\lambda^{EN}_{t,n} - \lambda^{EN}_{t,m} + \overline{\nu}_{t,n,m} - \overline{\nu}_{t,m,n} + \underline{\nu}_{t,m,n} - \underline{\nu}_{t,n,m})$$
$$+\overline{\xi}_{t,n} - \underline{\xi}_{t,n} + \xi_{t,n=1} = 0\ \forall t, \forall n \tag{80}$$

$$(2)-(3) \tag{81}$$

$$0 \le [F^{max}_{nm} - B_{nm}(\delta_{t,n} - \delta_{t,m})] \perp \overline{\nu}_{t,n,m} \ge 0\ \forall t, \forall n, \forall m \in \Theta_n \tag{82}$$

$$0 \le [B_{nm}(\delta_{t,n} - \delta_{t,m}) + F^{max}_{nm}] \perp \underline{\nu}_{t,n,m} \ge 0\ \forall t, \forall n, \forall m \in \Theta_n \tag{83}$$

$$0 \le (\pi - \delta_{t,n}) \perp \overline{\xi}_{t,n} \ge 0\ \forall t, \forall n \tag{84}$$

$$0 \le (\pi + \delta_{t,n}) \perp \underline{\xi}_{t,n} \ge 0\ \forall t, \forall n \tag{85}$$

$$0 \le p_{g,t} \perp \underline{\mu}_{g,t} \ge 0\ \forall t, \forall g \tag{86}$$

$$0 \le (P^{max}_g - p_{g,t}) \perp \overline{\mu}_{g,t} \ge 0\ \forall t, \forall g \tag{87}$$

$$0 \le p_{d,t} \perp \underline{\mu}_{d,t} \ge 0\ \forall t, \forall d \tag{88}$$

$$0 \le (P^{max}_d - p_{d,t}) \perp \overline{\mu}_{d,t} \ge 0\ \forall t, \forall d \tag{89}$$

$$0 \le Q_{t,h} \perp \underline{\mu}_{t,h} \ge 0\ \forall t, \forall h \tag{90}$$

$$0 \le (Q^{max}_{t,h} - Q_{t,h}) \perp \overline{\mu}_{t,h} \ge 0\ \forall t, \forall h \tag{91}$$

$$0 \le Q_{t,d_{\mathrm{H2}}} \perp \underline{\mu}_{t,d_{\mathrm{H2}}} \ge 0\ \forall t, \forall d_{\mathrm{H2}} \tag{92}$$

$$0 \le (Q^{max}_{t,d_{\mathrm{H2}}} - Q_{t,d_{\mathrm{H2}}}) \perp \overline{\mu}_{t,d_{\mathrm{H2}}} \ge 0\ \forall t, \forall d_{\mathrm{H2}} \tag{93}$$

$$0 \le p^{\mathrm{DA}}_{t,e} \perp \underline{\mu}^{\mathrm{DA}}_{t,e} \ge 0\ \forall t, \forall e \tag{94}$$

$$0 \le (\hat{p}^{\mathrm{DA}}_{t,e} - p^{\mathrm{DA}}_{t,e}) \perp \overline{\mu}^{\mathrm{DA}}_{t,e} \ge 0\ \forall t, \forall e \tag{95}$$

$$0 \le Q_{t,e} \perp \underline{\mu}^{Q}_{t,e} \ge 0\ \forall t, \forall e \tag{96}$$

$$0 \le (\hat{Q}_{t,e} - Q_{t,e}) \perp \overline{\mu}^{Q}_{t,e} \ge 0\ \forall t, \forall e \tag{97}$$

$$0 \le Q_{t,f} \perp \underline{\mu}^{Q}_{t,f} \ge 0\ \forall t, \forall f \tag{98}$$

$$0 \le (\hat{Q}_{t,f} - Q_{t,f}) \perp \overline{\mu}^{Q}_{t,f} \ge 0\ \forall t, \forall f \tag{99}$$

$$0 \le p^{\mathrm{DA}}_{t,f} \perp \underline{\mu}^{\mathrm{DA}}_{t,f} \ge 0\ \forall t, \forall f \tag{100}$$

$$0 \le (\hat{p}^{\mathrm{DA}}_{t,f} - p^{\mathrm{DA}}_{t,f}) \perp \overline{\mu}^{\mathrm{DA}}_{t,f} \ge 0\ \forall t, \forall f \tag{101}$$

$$0 \le r^{ch,UP}_{t,e} \perp \underline{\mu}^{\mathrm{ch,UP}}_{t,e} \ge 0\ \forall t, \forall e \tag{102}$$

$$0 \le (\hat{r}^{ch,UP}_{t,e} - r^{ch,UP}_{t,e}) \perp \overline{\mu}^{\mathrm{ch,UP}}_{t,e} \ge 0\ \forall t, \forall e \tag{103}$$

$$0 \le r^{ch,DN}_{t,e} \perp \underline{\mu}^{\mathrm{ch,DN}}_{t,e} \ge 0\ \forall t, \forall e \tag{104}$$

$$0 \le (\hat{r}^{ch,DN}_{t,e} - r^{ch,DN}_{t,e}) \perp \overline{\mu}^{\mathrm{ch,DN}}_{t,e} \ge 0\ \forall t, \forall e \tag{105}$$

$$0 \le r^{dis,UP}_{t,f} \perp \underline{\mu}^{\mathrm{dis,UP}}_{t,f} \ge 0\ \forall t, \forall f \tag{106}$$

$$0 \leq (\hat{r}^{dis,UP}_{t,f} - r^{dis,UP}_{t,f}) \perp \overline{\mu}^{\text{dis,UP}}_{t,f} \geq 0 \; \forall t, \forall f \tag{107}$$

$$0 \leq r^{dis,DN}_{t,f} \perp \underline{\mu}^{\text{dis,DN}}_{t,f} \geq 0 \; \forall t, \forall f \tag{108}$$

$$0 \leq (\hat{r}^{dis,DN}_{t,f} - r^{dis,DN}_{t,f}) \perp \overline{\mu}^{\text{dis,DN}}_{t,f} \geq 0 \; \forall t, \forall f \tag{109}$$

$$0 \leq (p^{\text{DA}}_{t,e} - r^{ch,UP}_{t,e}) \perp \mu^{\text{DA,UP}}_{t,e} \geq 0 \; \forall t, \forall e \tag{110}$$

$$0 \leq (p^{\text{DA}}_{t,f} - r^{dis,DN}_{t,f}) \perp \mu^{\text{DA,DN}}_{t,f} \geq 0 \; \forall t, \forall f \tag{111}$$

$$0 \leq r^{UP}_{g,t} \perp \underline{\mu}^{\text{UP}}_{t,g} \geq 0 \; \forall t, \forall g \tag{112}$$

$$0 \leq (R^{UP}_{g} - r^{UP}_{g,t}) \perp \overline{\mu}^{\text{UP}}_{t,g} \geq 0 \; \forall t, \forall g \tag{113}$$

$$0 \leq r^{DN}_{g,t} \perp \underline{\mu}^{\text{DN}}_{t,g} \geq 0 \; \forall t, \forall g \tag{114}$$

$$0 \leq (R^{DN}_{g} - r^{DN}_{g,t}) \perp \overline{\mu}^{\text{DN}}_{t,g} \geq 0 \; \forall t, \forall g \tag{115}$$

$$0 \leq (P^{max}_{g} - p_{g,t} - r^{UP}_{g,t}) \perp \mu^{\text{UP}}_{t,g} \geq 0 \; \forall t, \forall g \tag{116}$$

$$0 \leq (p_{g,t} - r^{DN}_{t,g}) \perp \mu^{\text{DN}}_{t,g} \geq 0 \; \forall t, \forall g \tag{117}$$

$$0 \leq r^{UP}_{d,t} \perp \underline{\mu}^{\text{UP}}_{t,d} \geq 0 \; \forall t, \forall d \tag{118}$$

$$0 \leq (R^{UP}_{d} - r^{UP}_{d,t}) \perp \overline{\mu}^{\text{UP}}_{t,d} \geq 0 \; \forall t, \forall d \tag{119}$$

$$0 \leq r^{DN}_{d,t} \perp \underline{\mu}^{\text{DN}}_{t,d} \geq 0 \; \forall t, \forall d \tag{120}$$

$$0 \leq (R^{DN}_{d} - r^{DN}_{d,t}) \perp \overline{\mu}^{\text{DN}}_{t,d} \geq 0 \; \forall t, \forall d \tag{121}$$

$$0 \leq (P^{max}_{d} - p_{d,t} - r^{DN}_{d,t}) \perp \mu^{\text{DN}}_{t,d} \geq 0 \; \forall t, \forall d \tag{122}$$

$$0 \leq (p_{d,t} - r^{UP}_{t,d}) \perp \mu^{\text{UP}}_{t,d} \geq 0 \; \forall t, \forall d \tag{123}$$

A.2 Balancing market problem (31) - (41)

$$\partial_{\ell_{d,t,k}} : \text{VOLL} - \lambda^{BL}_{t,k} + \overline{\rho}_{d,t,k} - \underline{\rho}_{d,t,k} = 0 \; \forall d, \forall t, \forall k \tag{124}$$

$$\partial_{q^{UP}_{g,t,k}} : O^{EN}_{g,t} - \lambda^{BL}_{t,k} + \overline{\rho}^{\text{UP}}_{g,t,k} - \underline{\rho}^{\text{UP}}_{g,t,k} = 0 \; \forall g, \forall t, \forall k \tag{125}$$

$$\partial_{q^{DN}_{g,t,k}} : -O^{EN}_{g,t} + \lambda^{BL}_{t,k} + \overline{\rho}^{\text{DN}}_{g,t,k} - \underline{\rho}^{\text{DN}}_{g,t,k} = 0 \; \forall g, \forall t, \forall k \tag{126}$$

$$\partial_{q^{dis,UP}_{f,t,k}} : \hat{o}^{dis}_{t,f} - \lambda^{BL}_{t,k} + \overline{\rho}^{\text{dis,UP}}_{f,t,k} - \underline{\rho}^{\text{dis,UP}}_{f,t,k} = 0 \; \forall f, \forall t, \forall k \tag{127}$$

$$\partial_{q^{dis,DN}_{f,t,k}} : -\hat{o}^{dis}_{t,f} + \lambda^{BL}_{t,k} + \overline{\rho}^{\text{dis,DN}}_{f,t,k} - \underline{\rho}^{\text{dis,DN}}_{f,t,k} = 0 \; \forall f, \forall t, \forall k \tag{128}$$

$$\partial_{q^{ch,UP}_{e,t,k}} : \hat{b}^{ch}_{t,e} - \lambda^{BL}_{t,k} + \overline{\rho}^{\text{ch,UP}}_{e,t,k} - \underline{\rho}^{\text{ch,UP}}_{e,t,k} = 0 \; \forall e, \forall t, \forall k \tag{129}$$

$$\partial_{q^{ch,DN}_{e,t,k}} : -\hat{b}^{ch}_{t,e} + \lambda^{BL}_{t,k} + \overline{\rho}^{\text{ch,DN}}_{e,t,k} - \underline{\rho}^{\text{ch,DN}}_{e,t,k} = 0 \; \forall e, \forall t, \forall k \tag{130}$$

$$\partial_{q^{UP}_{d,t,k}} : B^{EN}_{d,t} - \lambda^{BL}_{t,k} + \overline{\rho}^{\text{UP}}_{d,t,k} - \underline{\rho}^{\text{UP}}_{d,t,k} = 0 \; \forall d, \forall t, \forall k \tag{131}$$

$$\partial_{q^{DN}_{d,t,k}} : -B^{EN}_{d,t} + \lambda^{BL}_{t,k} + \overline{\rho}^{\text{DN}}_{d,t,k} - \underline{\rho}^{\text{DN}}_{d,t,k} = 0 \; \forall d, \forall t, \forall k \tag{132}$$

$$(32) \tag{133}$$

$$0 \leq q^{UP}_{g,t,k} \perp \underline{\rho}^{\mathrm{UP}}_{g,t,k} \geq 0 \; \forall g, \forall t, \forall k \tag{134}$$

$$0 \leq (r^{UP}_{g,t} - q^{UP}_{g,t,k}) \perp \overline{\rho}^{\mathrm{UP}}_{g,t,k} \geq 0 \; \forall g, \forall t, \forall k \tag{135}$$

$$0 \leq q^{DN}_{g,t,k} \perp \underline{\rho}^{\mathrm{DN}}_{g,t,k} \geq 0 \; \forall g, \forall t, \forall k \tag{136}$$

$$0 \leq (r^{DN}_{g,t} - q^{DN}_{g,t,k}) \perp \overline{\rho}^{\mathrm{DN}}_{g,t,k} \geq 0 \; \forall g, \forall t, \forall k \tag{137}$$

$$0 \leq q^{UP}_{d,t,k} \perp \underline{\rho}^{\mathrm{UP}}_{d,t,k} \geq 0 \; \forall d, \forall t, \forall k \tag{138}$$

$$0 \leq (r^{UP}_{d,t} - q^{UP}_{d,t,k}) \perp \overline{\rho}^{\mathrm{UP}}_{d,t,k} \geq 0 \; \forall d, \forall t, \forall k \tag{139}$$

$$0 \leq q^{DN}_{d,t,k} \perp \underline{\rho}^{\mathrm{DN}}_{d,t,k} \geq 0 \; \forall d, \forall t, \forall k \tag{140}$$

$$0 \leq (r^{DN}_{d,t} - q^{DN}_{d,t,k}) \perp \overline{\rho}^{\mathrm{DN}}_{d,t,k} \geq 0 \; \forall d, \forall t, \forall k \tag{141}$$

$$0 \leq q^{dis,UP}_{f,t,k} \perp \underline{\rho}^{\mathrm{dis,UP}}_{f,t,k} \geq 0 \; \forall f, \forall t, \forall k \tag{142}$$

$$0 \leq (r^{dis,UP}_{f,t} - q^{dis,UP}_{f,t,k}) \perp \overline{\rho}^{\mathrm{dis,UP}}_{f,t,k} \geq 0 \; \forall f, \forall t, \forall k \tag{143}$$

$$0 \leq q^{dis,DN}_{f,t,k} \perp \underline{\rho}^{\mathrm{dis,DN}}_{f,t,k} \geq 0 \; \forall f, \forall t, \forall k \tag{144}$$

$$0 \leq (r^{dis,DN}_{f,t} - q^{dis,DN}_{f,t,k}) \perp \overline{\rho}^{\mathrm{dis,DN}}_{f,t,k} \geq 0 \; \forall f, \forall t, \forall k \tag{145}$$

$$0 \leq q^{ch,UP}_{e,t,k} \perp \underline{\rho}^{\mathrm{ch,UP}}_{e,t,k} \geq 0 \; \forall e, \forall t, \forall k \tag{146}$$

$$0 \leq (r^{ch,UP}_{e,t} - q^{ch,UP}_{e,t,k}) \perp \overline{\rho}^{\mathrm{ch,UP}}_{e,t,k} \geq 0 \; \forall e, \forall t, \forall k \tag{147}$$

$$0 \leq q^{ch,DN}_{e,t,k} \perp \underline{\rho}^{\mathrm{ch,DN}}_{e,t,k} \geq 0 \; \forall e, \forall t, \forall k \tag{148}$$

$$0 \leq (r^{ch,DN}_{e,t} - q^{ch,DN}_{e,t,k}) \perp \overline{\rho}^{\mathrm{ch,DN}}_{e,t,k} \geq 0 \; \forall e, \forall t, \forall k \tag{149}$$

$$0 \leq \ell_{d,t,k} \perp \underline{\rho}_{d,t,k} \geq 0 \; \forall d, \forall t, \forall k \tag{150}$$

$$0 \leq (p_{d,t} - \ell_{d,t,k}) \perp \overline{\rho}_{d,t,k} \geq 0 \; \forall d, \forall t, \forall k \tag{151}$$

B Linearization

The KKT equivalent formulation of the LLPs introduce complementary conditions of the form $0 \leq a \perp b \geq 0$. These expressions can be linearized using the method in Fortuny-Amat and McCarl (1981): $a \geq 0, b \geq 0, a \leq uM, b \leq (1-u)M$. Where M is a large enough positive constant. This removes the non linearities without approximation but introduces a binary variable, u, for each complementary condition Nasrolahpour et al. (2018).

The strong duality theorem can be applied to the LLPs (1) and (31) to find a linear expression to the bilinear terms in the ULPs (42) and (53). Because of space limitations, this appendix only presents the linearization procedure applied to the electrolyser ULP however a similar approach applies to the fuelcell

ULP. The Strong duality of (1) is given by (152):

$$
\begin{aligned}
&-\sum_{t=1}^{\mathscr{T}}\Bigg\{\sum_{e=1}^{N_e}\hat{b}^{ch}_{t,e}p^{\mathrm{DA}}_{t,e}+\sum_{f=1}^{N_f}\hat{b}^{\mathrm{H2}}_{t,f}Q_{t,f}+\sum_{d_{\mathrm{H2}}=1}^{N_{d_{\mathrm{H2}}}}B^{EN}_{t,d_{\mathrm{H2}}}Q_{t,d_{\mathrm{H2}}}-\sum_{h=1}^{N_{\mathrm{H2}}}O^{EN}_{t,h}Q_{t,h}-\sum_{e=1}^{N_e}\hat{o}^{\mathrm{H2}}_{t,e}Q_{t,e}\\
&-\sum_{f=1}^{N_f}\hat{o}^{dis}_{t,f}p^{\mathrm{DA}}_{t,f}+\sum_{d=1}^{N_d}B^{EN}_{d,t}p_{d,t}-\sum_{g=1}^{N_g}O^{EN}_{g,t}p_{g,t}-\sum_{e=1}^{N_e}(\hat{o}^{ch,UP}_{t,e}r^{ch,UP}_{t,e}+\hat{o}^{ch,DN}_{t,e}r^{ch,DN}_{t,e})\\
&-\sum_{f=1}^{N_f}(\hat{o}^{dis,UP}_{t,f}r^{dis,UP}_{t,f}+\hat{o}^{dis,DN}_{t,f}r^{dis,DN}_{t,f})-\sum_{d=1}^{N_d}B^{RS}_{d,t}(r^{UP}_{d,t}+r^{DN}_{d,t})-\sum_{g=1}^{N_g}O^{RS}_{g,t}(r^{UP}_{g,t}+r^{DN}_{g,t})\Bigg\}=\\
&\sum_{t=1}^{\mathscr{T}}\Bigg\{-\sum_{n(m\in\Theta_n)}F^{max}_{nm}(\underline{\nu}_{t,n,m}+\overline{\nu}_{t,n,m})-\sum_{n}\pi(\overline{\xi}_{t,n}+\underline{\xi}_{t,n})+\lambda^{UP}_tR^{UP}_t+\lambda^{DN}_tR^{DN}_t-\sum_{g=1}^{N_g}(\overline{\mu}_{g,t}P^{max}_g\\
&+\overline{\mu}^{\mathrm{UP}}_{t,g}R^{UP}_g+\overline{\mu}^{\mathrm{DN}}_{t,g}R^{DN}_g+\mu^{\mathrm{UP}}_{t,g}P^{max}_g)-\sum_{d=1}^{N_d}(\overline{\mu}_{d,t}P^{max}_d+\overline{\mu}^{\mathrm{UP}}_{t,d}R^{UP}_d+\overline{\mu}^{\mathrm{DN}}_{t,d}R^{DN}_d\\
&+\mu^{\mathrm{DN}}_{t,d}P^{max}_d)-\sum_{h=1}^{N_{\mathrm{H2}}}\overline{\mu}_{t,h}Q^{max}_{t,h}-\sum_{d_{\mathrm{H2}}=1}^{N_{d_{\mathrm{H2}}}}\overline{\mu}_{t,d_{\mathrm{H2}}}Q^{max}_{t,d_{\mathrm{H2}}}-\sum_{e=1}^{N_e}(\overline{\mu}^{\mathrm{DA}}_{t,e}\hat{p}^{\mathrm{DA}}_{t,e}+\overline{\mu}^{\mathrm{Q}}_{t,e}\hat{Q}_{t,e}+\overline{\mu}^{\mathrm{ch,UP}}_{t,e}\hat{r}^{ch,UP}_{t,e}\\
&+\overline{\mu}^{\mathrm{ch,DN}}_{t,e}\hat{r}^{ch,DN}_{t,e})-\sum_{f=1}^{N_f}(\overline{\mu}^{\mathrm{DA}}_{t,f}\hat{p}^{\mathrm{DA}}_{t,f}+\overline{\mu}^{\mathrm{Q}}_{t,f}\hat{Q}_{t,f}+\overline{\mu}^{\mathrm{dis,UP}}_{t,f}\hat{r}^{dis,UP}_{t,f}+\overline{\mu}^{\mathrm{dis,DN}}_{t,f}\hat{r}^{dis,DN}_{t,f})\Bigg\}
\end{aligned}
\tag{152}
$$

from (95), (97), (103) and (105)

$$\overline{\mu}^{\mathrm{DA}}_{t,e}\hat{p}^{\mathrm{DA}}_{t,e}=\overline{\mu}^{\mathrm{DA}}_{t,e}p^{\mathrm{DA}}_{t,e} \tag{153}$$

$$\overline{\mu}^{Q}_{t,e}\hat{Q}_{t,e}=\overline{\mu}^{Q}_{t,e}Q_{t,e} \tag{154}$$

$$\overline{\mu}^{\mathrm{ch,UP}}_{t,e}\hat{r}^{ch,UP}_{t,e}=\overline{\mu}^{\mathrm{ch,UP}}_{t,e}r^{ch,UP}_{t,e} \tag{155}$$

$$\overline{\mu}^{\mathrm{ch,DN}}_{t,e}\hat{r}^{ch,DN}_{t,e}=\overline{\mu}^{\mathrm{ch,DN}}_{t,e}r^{ch,DN}_{t,e} \tag{156}$$

Eq. (152) can be simplified using (153)-(156):

$$
\begin{aligned}
&\sum_{t=1}^{\mathscr{T}}\Bigg\{\sum_{e=1}^{N_e}[(-\hat{b}^{ch}_{t,e}+\overline{\mu}^{\mathrm{DA}}_{t,e})p^{\mathrm{DA}}_{t,e}+(\hat{o}^{\mathrm{H2}}_{t,e}+\overline{\mu}^{Q}_{t,e})Q_{t,e}+(\hat{o}^{ch,UP}_{t,e}+\overline{\mu}^{\mathrm{ch,UP}}_{t,e})r^{ch,UP}_{t,e}+(\hat{o}^{ch,DN}_{t,e}+\overline{\mu}^{\mathrm{ch,DN}}_{t,e})r^{ch,DN}_{t,e}]\Bigg\}\\
&=\sum_{t=1}^{\mathscr{T}}\Bigg\{\sum_{f=1}^{N_f}[\hat{b}^{\mathrm{H2}}_{t,f}Q_{t,f}-\hat{o}^{dis}_{t,f}p^{\mathrm{DA}}_{t,f}-\hat{o}^{dis,UP}_{t,f}r^{dis,UP}_{t,f}-\hat{o}^{dis,DN}_{t,f}r^{dis,DN}_{t,f}-(\overline{\mu}^{\mathrm{DA}}_{t,f}\hat{p}^{\mathrm{DA}}_{t,f}+\overline{\mu}^{\mathrm{Q}}_{t,f}\hat{Q}_{t,f}+\overline{\mu}^{\mathrm{dis,UP}}_{t,f}\hat{r}^{dis,UP}_{t,f}\\
&+\overline{\mu}^{\mathrm{dis,DN}}_{t,f}\hat{r}^{dis,DN}_{t,f})]-\sum_{n(m\in\Theta_n)}F^{max}_{nm}(\underline{\nu}_{t,n,m}+\overline{\nu}_{t,n,m})-\sum_{n}\pi(\overline{\xi}_{t,n}+\underline{\xi}_{t,n})+\sum_{d_{\mathrm{H2}}=1}^{N_{d_{\mathrm{H2}}}}B^{EN}_{t,d_{\mathrm{H2}}}Q_{t,d_{\mathrm{H2}}}\\
&-\sum_{h=1}^{N_{\mathrm{H2}}}O^{EN}_{t,h}Q_{t,h}+\sum_{d=1}^{N_d}B^{EN}_{d,t}p_{d,t}-\sum_{g=1}^{N_g}O^{EN}_{g,t}p_{g,t}-\sum_{d=1}^{N_d}B^{RS}_{d,t}(r^{UP}_{d,t}+r^{DN}_{d,t})-\sum_{g=1}^{N_g}O^{RS}_{g,t}(r^{UP}_{g,t}+r^{DN}_{g,t})\\
&+\lambda^{UP}_tR^{UP}_t+\lambda^{DN}_tR^{DN}_t-\sum_{g=1}^{N_g}(\overline{\mu}_{g,t}P^{max}_g+\overline{\mu}^{\mathrm{UP}}_{t,g}R^{UP}_g+\overline{\mu}^{\mathrm{DN}}_{t,g}R^{DN}_g+\mu^{\mathrm{UP}}_{t,g}P^{max}_g)-\sum_{d=1}^{N_d}(\overline{\mu}_{d,t}P^{max}_d
\end{aligned}
$$

$$+\overline{\mu}^{\text{UP}}_{t,d}R^{UP}_d+\overline{\mu}^{\text{DN}}_{t,d}R^{DN}_d+\mu^{\text{DN}}_{t,d}P^{max}_d) - \sum_{h=1}^{N_{\text{H2}}} \overline{\mu}_{t,h}Q^{max}_{t,h} - \sum_{d_{\text{H2}}=1}^{N_{d_{\text{H2}}}} \overline{\mu}_{t,d_{\text{H2}}} Q^{max}_{t,d_{\text{H2}}} \Bigg\} \tag{157}$$

In (42) the terms involving FC firm variables are considered constants. Similarly, the terms related to the ESR firm's variables are constant in (53). Thus from the ESR firm's point of view (157) can be rearranged as: u! u! u!

$$\sum_{t=1}^{\mathscr{T}} \Bigg\{ \sum_{e=1}^{N_e}(-\hat{b}^{ch}_{t,e} + \overline{\mu}^{\text{DA}}_{t,e})p^{\text{DA}}_{t,e} + \sum_{e=1}^{N_e}(\hat{o}^{\text{H2}}_{t,e} + \overline{\mu}^{Q}_{t,e})Q_{t,e} + \sum_{e=1}^{N_e}[(\hat{o}^{ch,UP}_{t,e} + \overline{\mu}^{\text{ch,UP}}_{t,e})r^{ch,UP}_{t,e} + (\hat{o}^{ch,DN}_{t,e} + \overline{\mu}^{\text{ch,DN}}_{t,e})r^{ch,DN}_{t,e}]\Bigg\} = \mathfrak{F} + \mathfrak{L} \tag{158}$$

The LHS of (158) contains only the bilinear terms under the control of the electrolyser firm. On the RHS, $\mathfrak{F}$ represents variables under the control of the fuelcell firm and $\mathfrak{L}$ represents the linear terms.

A linear equivalent of the bilinear terms involving DAM variables in the objective functions (42) and (53) can be obtained from (157). Expressions for the products of DAM prices and energy/hydrogen/reserve quantity schedules can be derived from the KKT conditions of the DAM. The dual variables $\lambda^{EN}_{t,n}$, λ^{H2}_t, λ^{UP}_t and λ^{DN}_t are isolated from (64),(69),(72) and (73), respectively. Then we multiply both sides of these 4 expression by $p^{\text{DA}}_{t,e}$, $Q_{t,e}$, $r^{ch,UP}_{t,e}$ and $r^{ch,DN}_{t,e}$, respectively. Lastly, the products of $\mu/\underline{\mu}$ dual variables and quantities vanish according to (94), (96), (102) and (104).

Using the above steps, the sum of bilinear terms in (42), comprising DAM variables, equals the LHS of (158), i.e.,

$$- \lambda^{EN}_{t,n}p^{\text{DA}}_{t,e} + \lambda^{\text{H2}}_t Q_{t,e} + \lambda^{UP}_t r^{ch,UP}_{t,e} + \lambda^{DN}_t r^{ch,DN}_{t,e} = (-\hat{b}^{ch}_{t,e} + \overline{\mu}^{\text{DA}}_{t,e})p^{\text{DA}}_{t,e} + (\hat{o}^{\text{H2}}_{t,e} + \overline{\mu}^{Q}_{t,e})Q_{t,e} + (\hat{o}^{ch,UP}_{t,e} + \overline{\mu}^{\text{ch,UP}}_{t,e})r^{ch,UP}_{t,e} + (\hat{o}^{ch,DN}_{t,e} + \overline{\mu}^{\text{ch,DN}}_{t,e})r^{ch,DN}_{t,e} \tag{159}$$

Thus, in (42) these bilinear terms can be replaced with the RHS of (158); which is an expression comprising only linear and constant terms. With the same method described in this Appendix but using the BLM lower-level problem, the remaining bilinear terms in (42), involving products of balancing market variables, can be replaced by an affine expression.

The strong duality equality of each problem in (31) is given by (160):

$$\sum_{d=1}^{N_d} \left[\text{VOLL}\,\ell_{t,d,k} + B^{EN}_{d,t}(q^{UP}_{d,t,k} - q^{DN}_{d,t,k})\right] \tag{160}$$

$$+ \sum_{g=1}^{N_g} O^{EN}_{g,t}(q^{UP}_{g,t,k} - q^{DN}_{g,t,k}) + \sum_{e=1}^{N_e} \hat{b}^{ch}_{t,e}(q^{ch,UP}_{t,e,k} - q^{ch,DN}_{t,e,k}) +$$

$$\sum_{f=1}^{N_f} \hat{o}^{dis}_{t,f}(q^{dis,UP}_{t,f,k} q^{dis,DN}_{t,f,k}) = -\sum_{f=1}^{N_f}(\overline{\rho}^{\text{dis,UP}}_{f,t,k} r^{dis,UP}_{f,t} + \overline{\rho}^{\text{dis,DN}}_{f,t,k} r^{dis,DN}_{f,t}) - \sum_{e=1}^{N_e}(\overline{\rho}^{\text{ch,UP}}_{e,t,k} r^{ch,UP}_{e,t} + \overline{\rho}^{\text{ch,DN}}_{e,t,k} r^{ch,DN}_{e,t})$$
$$-\sum_{g=1}^{N_g}(\overline{\rho}^{\text{UP}}_{g,t,k} r^{UP}_{g,t} + \overline{\rho}^{\text{DN}}_{g,t,k} r^{DN}_{g,t}) - \sum_{d=1}^{N_d}(\overline{\rho}_{d,t,k} p_{d,t} + \overline{\rho}^{\text{UP}}_{d,t,k} r^{UP}_{d,t} + \overline{\rho}^{\text{DN}}_{d,t,k} r^{DN}_{d,t}) + \lambda^{BL}_{t,k} L_{t,k} \tag{161}$$

Similarly to the derivation of (153)-(156), the complementary conditions (147) and (149) can be used to eliminate variables in (160):

$$\sum_{e=1}^{N_e}[(\hat{b}^{ch}_{t,e} + \overline{\rho}^{\text{ch,UP}}_{e,t,k}) q^{ch,UP}_{t,e,k} - (\hat{b}^{ch}_{t,e} - \overline{\rho}^{\text{ch,DN}}_{e,t,k}) q^{ch,DN}_{t,e,k}] = \sum_{f=1}^{N_f}[\hat{o}^{dis}_{t,f}(q^{dis,DN}_{t,f,k} - q^{dis,UP}_{t,f,k}) - \overline{\rho}^{\text{dis,UP}}_{f,t,k} r^{dis,UP}_{f,t}$$
$$- \overline{\rho}^{\text{dis,DN}}_{f,t,k} r^{dis,DN}_{f,t}] - \sum_{d=1}^{N_d}\left[\text{VOLL}\,\ell_{t,d,k} + B^{EN}_{d,t}(q^{UP}_{d,t,k} - q^{DN}_{d,t,k})\right] - \sum_{g=1}^{N_g} O^{EN}_{g,t}(q^{UP}_{g,t,k} - q^{DN}_{g,t,k})$$
$$- \sum_{g=1}^{N_g}(\overline{\rho}^{\text{UP}}_{g,t,k} r^{UP}_{g,t} + \overline{\rho}^{\text{DN}}_{g,t,k} r^{DN}_{g,t}) - \sum_{d=1}^{N_d}(\overline{\rho}_{d,t,k} p_{d,t} + \overline{\rho}^{\text{UP}}_{d,t,k} r^{UP}_{d,t} + \overline{\rho}^{\text{DN}}_{d,t,k} r^{DN}_{d,t}) + \lambda^{BL}_{t,k} L_{t,k} \tag{162}$$

The next step is to isolate the balancing market shadow price, $\lambda^{BL}_{t,k}$, from (129) and (130). Then multiply the two expression by $q^{ch,UP}_{t,e,k}$ and $q^{ch,DN}_{t,e,k}$, respectively. Using, (146) and (148), the terms involving products of $\underline{\rho}$ dual variables and UP/DN balancing quantities vanish. Thus, $\lambda^{BL}_{t,k} q^{ch,UP}_{t,e,k} - \lambda^{BL}_{t,k} q^{ch,DN}_{t,e,k}$ replaces the two summands on the LHS of (162).All products of shadow prices with energy, hydrogen or reserve quantities, in (42), can be linearized using (157) and (162):

$$\max_{\substack{e\\ \text{UL}}} \sum_{t=1}^{\mathscr{T}} bigg\{-\sum_{n(m\in\Theta_n)} F^{max}_{nm}(\underline{\nu}_{t,n,m} + \overline{\nu}_{t,n,m}) - \sum_{n} \pi(\overline{\xi}_{t,n} + \underline{\xi}_{t,n}) + lambda^{UP}_t R^{UP}_t + \lambda^{DN}_t R^{DN}_t - \sum_{h=1}^{N_{\text{H2}}}(O^{EN}_{t,h} Q_{t,h}$$
$$+ \overline{\mu}_{t,h} Q^{max}_{t,h}) + \sum_{f}^{N_f}\{\hat{b}^{\text{H2}}_{t,f} Q_{t,f} - hato^{dis}_{t,f} p^{\text{DA}}_{t,f} - \hat{o}^{dis,UP}_{t,f} r^{dis,UP}_{t,f} - \hat{o}^{dis,DN}_{t,f} r^{dis,DN}_{t,f} - (\overline{\mu}^{\text{DA}}_{t,f} \hat{p}^{\text{DA}}_{t,f} + \overline{\mu}^{\text{Q}}_{t,f} \hat{Q}_{t,f}$$
$$+ \overline{\mu}^{\text{dis,UP}}_{t,f} \hat{r}^{dis,UP}_{t,f} + \overline{\mu}^{\text{dis,DN}}_{t,f} \hat{r}^{dis,DN}_{t,f}) + \sum_{k=1}^{N_k} \Phi_k[\hat{o}^{dis}_{t,f}(q^{dis,DN}_{t,f,k} - q^{dis,UP}_{t,f,k}) - \overline{\rho}^{\text{dis,UP}}_{f,t,k} r^{dis,UP}_{f,t} - \overline{\rho}^{\text{dis,DN}}_{f,t,k} r^{dis,DN}_{f,t}]\}$$
$$+ \sum_{d_{\text{H2}}=1}^{N_{d_{\text{H2}}}}(B^{EN}_{t,d_{\text{H2}}} Q_{t,d_{\text{H2}}} - \overline{\mu}_{t,d_{\text{H2}}} Q^{max}_{t,d_{\text{H2}}}) - \sum_{g=1}^{N_g}\Big\{O^{EN}_{g,t} p_{g,t} + O^{RS}_{g,t}(r^{UP}_{g,t} + r^{DN}_{g,t}) + \overline{\mu}_{g,t} P^{max}_{g}$$
$$+ \overline{\mu}^{\text{UP}}_{t,g} R^{UP}_{g} + \overline{\mu}^{\text{DN}}_{t,g} R^{DN}_{g} + \mu^{\text{UP}}_{t,g} P^{max}_{g} + \sum_{k=1}^{N_k} \Phi_k[O^{EN}_{g,t}(q^{UP}_{g,t,k} - q^{DN}_{g,t,k}) + \overline{\rho}^{\text{UP}}_{g,t,k} r^{UP}_{g,t} + \overline{\rho}^{\text{DN}}_{g,t,k} r^{DN}_{g,t}]\Big\}$$
$$+ \sum_{d=1}^{N_d}\Big\{B^{EN}_{d,t} p_{d,t} - B^{RS}_{d,t}(r^{UP}_{d,t} + r^{DN}_{d,t}) - \overline{\mu}_{d,t} P^{max}_{d} - \overline{\mu}^{\text{UP}}_{t,d} R^{UP}_{d} - \overline{\mu}^{\text{DN}}_{t,d} R^{DN}_{d} - \mu^{\text{DN}}_{t,d} P^{max}_{d}$$
$$- \sum_{k=1}^{N_k} \Phi_k\Big[\text{VOLL}\,\ell_{t,d,k} + B^{EN}_{d,t}(q^{UP}_{d,t,k} - q^{DN}_{d,t,k}) + \overline{\rho}_{d,t,k} p_{d,t} + \overline{\rho}^{\text{UP}}_{d,t,k} r^{UP}_{d,t} + \overline{\rho}^{\text{DN}}_{d,t,k} r^{DN}_{d,t}\Big]\Big\}$$

$$+ \sum_{k=1}^{N_k} \Phi_k \lambda_{t,k}^{BL} L_{t,k} - \sum_{e=1}^{N_e} \left[C_e^{dis} Q_{t,e} + C_e^{ch} p_{t,e}^{\mathrm{DA}} - \sum_{k=1}^{N_k} \Phi_k C_e^{ch} (q_{t,e,k}^{ch,UP} - q_{t,e,k}^{ch,DN}) \right] \Bigg\} \tag{163}$$

The binary expansion method Ruiz et al. (2012) is used to linearize the remaining bilinear terms: $\overline{\rho}_{f,t,k}^{\mathrm{dis,UP}} r_{f,t}^{dis,UP}$, $\overline{\rho}_{f,t,k}^{\mathrm{dis,DN}} r_{f,t}^{dis,DN}$, $\overline{\rho}_{g,t,k}^{\mathrm{UP}} r_{g,t}^{UP}$, $\overline{\rho}_{g,t,k}^{\mathrm{DN}} r_{g,t}^{DN}$, $\overline{\rho}_{d,t,k} p_{d,t}$, $\overline{\rho}_{d,t,k}^{\mathrm{UP}} r_{d,t}^{UP}$, $\overline{\rho}_{d,t,k}^{\mathrm{DN}} r_{d,t}^{DN}$. Further details can be found in Nasrolahpour et al. (2018).

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