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Cournot competition in an integer-constrained electricity market model

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Abstract

The costs associated with electricity generation include costs that are independent of their marginal output, including the cost of starting their units, and constraints such as minimum generation levels. Modelling these costs and constraints requires integer formulation of the units, and so they have typically been ignored in electricity market modelling and simulation to date. We develop a stochastic equilibrium model to include these costs and constraints in a Cournot model and solve it using the Gauss-Seidel diagonalization algorithm. We apply the model to the power system of the island of Ireland with varying levels of variable renewable power generation. We find that the impacts of integer modelling are non-trivial, and are heterogeneous across firms and wind levels. Furthermore, excluding integer modelling exaggerates the impact of price-making behaviour. We conclude that neglecting integer constraints in power system market models leads to inaccurate results, particularly at high penetrations of renewable energy sources.

OR in energy; Cournot modelling; Unit commitment;

1 Introduction

Electricity markets have been liberalised since the 1990s, and are typically operated under an auction-based framework to efficiently dispatch and compensate electricity generators. Under these auction frameworks, firms submit price-quantity pairs (Wolak, 2021), which can vary in their complexity from "plain vanilla" to more complex bids that allow firms to embed technical and economic constraints into their bids, such as limitations on how quickly a unit can change its output (ramp rates) as well as extra costs such as the cost of starting a unit. Market operators then dispatch generation and demand in a cost-minimising manner. In a competitive market, the dispatch equates to market clearing on the basis of short-run marginal costs.

In recent years, concerns over climate change have led to greatly increased levels of renewable generation in electricity systems worldwide IEA (2023), while the Ukrainian war and concerns over energy affordability and security have prompted further acceleration in renewable generation by European policy makers via the "Fit for 55" package Commission (2021). Furthermore, in an effort to reduce electricity bills to final consumers, attempts to reduce inframarginal rents to generators, including renewable generators, via instruments such as contracts for differences and forward contracting, have been proposed by the European Commission European Commission (2023b,a).

This increase in renewable generation poses new challenges for electricity market design and modelling. These challenges include price cannibalisation, optimal location of renewables, and network design. Newbery et al. (2018) and M. Lynch et al. (2021) provide summaries of the market design challenges at high penetrations of variable Renewable Energy Sources (v-RES). In general, however, the interaction between v-RES penetration and non-marginal thermal generator costs (including, for example, start up and no load costs) has received less attention. This most likely stems from the fact that such costs make up a small proportion of the total costs, both of the power system and of individual firms' cost functions. As such, these costs have generally been ignored in market design discussions: the literature that focuses on the suitability of short-run marginal cost pricing generally restricts its attention to the price cannibalisation problem. Furthermore, electricity market operation itself does not always allow for the incorporation of these costs into firms' bids, particularly in EU markets. While most electricity markets in the USA include start and no load costs in the bids submitted by generators, EU markets generally require participants to internalise such costs in their bids, although fairly complex bidding patterns are available in at least some markets Herrero et al. (2020).

Increased levels of v-RES have been shown to substantially change the optimal dispatch when start costs are considered, compared to when such costs are ignored Shortt et al. (2012). The incorporation of start costs and no-load costs into power system dispatch models makes generators' cost functions discontinuous and hence require integer (binary) decision variables. This is known as the unit commitment problem, and has been widely studied in the engineering literature. Tuohy et al. (2009) presented an early examination of the impact of renewable generation on the unit commitment problem, and there is a vast literature on the topic, with many recent papers focusing on new efficient solution techniques for the problem itself; see e.g., Wu et al. (2023); Ali et al. (2023); Zhang et al. (2023). However, the incorporation of start and no-load costs into electricity market models, particularly when the assumption of perfect competition is relaxed, has received little attention in the literature, and motivates this paper.

Unit commitment models generally assume a cost minimization approach, which aligns with perfect competition, but which cannot account for strategic decisions by firms with price-making behaviour. Such strategic behaviour, also known as market power, aligns with imperfect competition and occurs when market players alter their decisions so as to increase market prices. For example, a generating firm may withhold some generation in order to increase the market price and hence their overall profits. One recent exception is Kumar et al. (2023), which uses a monarch butterly algorithm to determine a profit-based unit commitment, but this paper does not take an equilibrium-based approach - many of the results associated with the price-making behaviour in game theoretic models are not captured. Previous work on start costs in oligopolistic electricity markets was performed by Reguant (2014), based on an *ex post* analysis of bids made in the Spanish electricity market. The start costs, and their subsequent impacts on bids and prices, is examined via an econometric analysis of historical bids, and are found to limit the ability of firms to change output levels over time and thus to exacerbate price volatility.

In contrast to unit commitment models, imperfect competition resulting in price making behaviour in electricity market models is generally modelled *ex ante* via oligopolistic game-theoretic models. For instance, Devine & Bertsch (2022) use a Mixed Complementarity Problem (MCP) to examine the impact that demand response has on price-making behaviour. MCPs allow one to model the constrained optimisation problems of oligopolists as Cournot players. Other examples of MCPs being used to model electricity markets include Devine & Bertsch (2018), Koschker & Möst (2016), Lise & Kruseman (2008), and Liu et al. (2007). Further examples of MCPs in the wider energy market modelling literature include Egging-Bratseth et al. (2020), Baltensperger et al. (2016), Egging (2013), and Gabriel et al. (2009). For a more comprehensive list of such papers, we refer the reader to Egging-Bratseth et al. (2020).

In addition, several other equilibrium models have been used to model price-making in power systems. Approaches include Betrand equilibrium (Lee & Baldick, 2003), Supply Function Equilibria (Ruiz et al., 2011; Pozo & Contreras, 2011), Stackelberg equilibrium (Fanzeres et al., 2019), Multi-leader-multi-follower equilibrium (Devine & Siddiqui, 2023; Wogrin, Centeno, & Barquin, 2013; Wogrin, Hobbs, et al., 2013), and Generalised Hierarchical equilibrium (Zerrahn & Huppmann, 2017; Huppmann & Egerer, 2015). For a comprehensive review of equilibrium models in electricity systems, we refer the reader to Pozo et al. (2017). None of the equilibrium models mentioned above incorporate discontinuous costs and hence unit-commitment constraints. Indeed, a recent review of European electricity market design in light of the Ukrainian crisis includes no mention of unit commitment or start costs Fabra (2023).

In recent years, equilibrium problems with integer variables have started to gain attention in the Operations Research and wider literature, particularly for applications in electricity systems. However, there are no tailored algorithms for solving such problems (De Santis et al., 2022). One heuristic that has been deployed is to relax integrality constraints and solve the resulting problem using Karush-Kuhn-Tucker conditions in order to obtain initial solutions. Integrality constraints are then re-added. This approach has been applied to stylized electricity markets in Gabriel, Siddiqui, et al. (2013); Gabriel, Conejo, et al. (2013) and Ruiz et al. (2012). This approach leads to integer-value MCPs, but the solutions do not necessarily represent equilibria. In Weinhold & Gabriel (2020) and Huppmann & Siddiqui (2018) relaxation of integrality is first applied. Then solutions that ensure incentive compatibility for each player are obtained. A similar two-stage approach can be found in Fomeni et al. (2019). They consider a Reformulation Linearization Technique (RLT) in the first phase and a Mixed-Integer-Linear-Program in the second phase.

Gabriel et al. (2021) reformulate a Discreetly Constrained MCP as a purely continuous problem. This allows them to utilise local Non-Linear Programming (NLP) solvers and hence to obtain locally optimal solutions. Looking to non-energy applications, De Santis et al. (2022) introduces a branch-and-bound algorithm to solve a Mixed-Integer Linear Complementarity Problem (MILCP).

Todd (2016) propose binary search and other algorithms to solve an integer constrained Cournot production problem. In contrast to the present work, Todd (2016) does not consider maximum production constraints.

This paper considers the impact of discontinuous costs (integer variables) in an imperfectly competitive market and furthermore, the extent to which the resulting equilibrium deviates from that which would prevail under continuous, non-integer, costs. This is, to the best of our knowledge, the first time this question has been explored in the literature. Furthermore, the impact of variability in the electricity supply via higher penetrations of v-RES is quantified. We thus provide insight in two main areas. The first is the consideration of market equilibria in a Cournot framework compared to a cost minimisation approach including unit commitment: we provide evidence on the impact of ignoring price-making behaviour modelled *á la* Cournot. The second main set of insights concerns the impacts of including discontinuous costs such as start costs and no-load costs, which require integer modelling, in firms' profit-maximising Cournot decisions, particularly as v-RES generation increases. Furthermore, we provide insight on whether, and to what extent, these two mechanisms compound: the impact of integer variables on market power exploitation, and vice versa, can be shown. Thus, we provide evidence for policy makers and market modellers alike on these impacts.

Furthermore, we make several methodological contributions. To the best our knowledge, this is the first work in the equilibrium with integers literature that solely uses the Gauss-Seidel diagonalization algorithm to find a Nash equilibrium. Previous works seek solution points via model reformulations and/or constraint relaxations. The Gauss-Seidel approach is commonly used to solve Equilibrium Problem with Equilibrium Constraints (Devine & Siddiqui, 2023; Pozo et al., 2017; Gabriel et al., 2012). It involves solving the individual constrained optimisation problem of each player individually and iteratively. When one player's problem is being solved, the decision variables for all other players are assumed fixed. The algorithm iteratively convergences to an equilibrium and stops once no player has an optimal deviation. An advantage of the algorithm is that it allows us to find a Nash equilibrium point in a computational tractable manner, without having to consider any reformulations or constraint relaxations. A disadvantage of the Gauss-Seidel algorithm is that the equilibrium point found is not guaranteed to be unique (Zerrahn & Huppmann, 2017; Gabriel et al., 2012).

To help improve computational efficiency for the models presented, we combine our solution techniques with a Rolling Horizon algorithm. In such an algorithm, instead of solving the entire optimisation/equilibrium problem once over all time periods, the problem is split into several smaller optimisation/equilibrium problems whose time sets are overlapping subsets of the overall time set (Devine et al., 2016) - see Section 2 for further details. Given the computational difficulties associated with solving equilibrium problem with integers (De Santis et al., 2022; Gabriel et al., 2021), this is a further contribution of the present work.

Furthermore, we present an advance on the literature by considering a real-world example: specifically, the all-island Irish electricity market comprising 58 generating units spread across 16 generating firms. This is due to the availability of a unique dataset produced and maintained system operators on the island of Ireland. The dataset includes the technical and economic features of every generator in the market. In this manner, we avoid having to estimate the fixed and marginal costs of each generator in the market via their bids and are instead able to simulate their optimal generation decisions based on their technical characteristics.

We introduce a game-theoretic unit-commitment model which allows for price-making behaviour amongst the generating firms. We consider $|F|$ generating firms, who each hold multiple generating units with discontinuous costs. Each firm seeks to maximise its expected profits over several timesteps. The first set of timesteps is split into two stages. In first stage, firms have perfect information on wind capacity factors. For the second stage, they have uncertain information. This uncertainty is modelled using $|S|$ scenarios, each of which have a probability associated with it. Thus, each firm's optimisation problem is a stochastic program. The overall model contains an inverse demand curve, and we assume each firm has price-making ability, modelled *á la* Cournot. Consequently, the solution technique seeks a Nash-Equilibrium.

To answer our research questions, we consider three further models, each of which are simplifications of the equilibrium with integers model described above. We consider an equilibrium model where all firms behave *á la* Cournot but none of them have discontinuous costs. This model does not require integers and hence is solved as an MCP. Next, we consider a cost minimisation model where all firms have unit commitment constraints and hence discontinuous costs, but none behave *á la* Cournot. Instead, for this model, all the firms are assumed to be price-takers and hence the market is perfectly competitive. This model is solved as a Mixed Integer Quadratically Constrained Program. Finally, we consider a model where all firms are price-takers and only have continuous decision variables, i.e., there is no unit commitment constraints. This model is solved using Quadratic Programming.

The results show considerable heterogeneity across firms and installed wind capacities, with integers and price-making behaviour giving rise to significantly different equilibrium solutions compared to the models that neglect either or both of these features. These findings generally, though not always, increase as wind generation increases. We therefore present evidence of the importance of considering market power and integer representations of generation constraints in order to ensure accurate results at high RES-E levels.

Furthermore, several works in the literature examine the impact of price-making behaviour by comparing the difference between solutions to models formulated with and without market power (Devine & Bertsch, 2022; Egging et al., 2017; Koschker & Möst, 2016). Our findings suggest that excluding integer variables from such studies may overestimate the impact of price-making behaviour.

The rest of this paper is structured as follows. Section 2 describes the methodology. Section 3 describes the dataset used. Section 4 outlines the results while Section 5 provides a discussion. Section 6 concludes our paper.

2 Methodology

In this section we describe the four models used in this work:

1. Cournot equilibrium with integers model
2. Cournot equilibrium without integers model
3. Cost minimisation with integers model
4. Cost minimisation without integers model

For the four models we consider $|F|$ generating firms, each of which have multiple generating units. Each firm decides how much electricity to generate from their units over $|P|$ (hourly) time steps. In the two equilibrium models, each firm f behaves as a price-making Cournot player. Hence, these models represent an oligopolistic market. In the two cost minimisation models, each firm exhibits price-taking behaviour and hence these models represent a perfectly competitive market.

Furthermore, each of the models take the form of a stochastic program containing two stages. In the first-stage firms have perfect foresight of renewable energy capacity factors and make 'here-and-now' generation decisions that they commit to. In the subsequent second-stage, firms have uncertain information regarding renewable energy capacity factors and thus make 'wait-and-see' generation decisions that are scenario dependent. There are $|S|$ scenarios in total with each having a different time series of renewable energy capacity factors.

Each of the four models are also solved using a Rolling Horizon algorithm. In such an algorithm, instead of solving the entire optimisation/equilibrium problem once over all time periods, the problem is split into several smaller optimisation/equilibrium problems whose time sets are overlapping subsets of the overall time set (Devine et al., 2016). Each of these smaller problems are known as a 'roll', with $|R|$ rolls being solved in total for each model.

A rolling horizon has a number of advantages. Firstly, it can improve computational efficiency (Devine et al., 2016). Secondly, it can reflect decision making in electricity markets more realistically as it allows parameters to be updated after each roll of the model (Devine et al., 2016, 2014; Tuohy et al., 2009). In this work, the uncertain information firms have about renewable energy capacity factors are updated after each roll of the optimisation/equilibrium model is solved. This reflects reality. For example, in the real-world, firms make generating decisions today with uncertain information for the next day's renewable capacity factors. However, as time goes on and the next day arrives, the firms have updated information for renewable capacity factors (for instance, through updated weather forecasts) and can thus adjust their generation decisions accordingly.

Tables 1 - 4 describe the indices and sets, binary variables, continuous variables, and parameters of the models, respectively. For each of the models, there are three time period subsets at each roll r: the fixed time period subset (P_r^{FIX}), followed by the first-stage time period subset (P_r^{FS}), followed by the second-stage time period subset (P_r^{SS}). The length of $|P_r^{FIX}|$ is LEN^{FS}. The decision variables at all time periods in this subset for roll r must equal the corresponding first-stage decision variables from the previous roll $r-1$. This ensures, if a firm decides to generate from a particular unit in the first-stage time periods of roll r then those decisions are accounted for in the subsequent roll $r+1$, again showing how the rolling horizon can reflect reality more accurately. Thus, the first-stage time period subset also has a cardinality of $|P_r^{FS}| = LEN^{FS}$. The length of the second-stage time period subset is $|P_r^{SS}| = LEN^{SS}$. The time period subsets change for each roll r and are defined as follows:

$$P_r^{FIX} = \{1+(r-1)\times LEN^{FS}, 2+(r-1)\times LEN^{FS}, ..., r\times LEN^{FS}\}, \forall r \in R, \quad (1a)$$

$$P_r^{FS} = \{1+r\times LEN^{FS}, 2+r\times LEN^{FS}, ..., (r+1)\times LEN^{FS}\}, \forall r \in R, \quad (1b)$$

$$P_r^{SS} = \{1+(r+1)\times LEN^{FS}, 2+(r+1)\times LEN^{FS}, ..., LEN^{SS}+(r+1)\times LEN^{FS}\}, \forall r \in R. \quad (1c)$$

Thus, the time periods for roll r are $P_r = P_r^{FIX} \cup P_r^{FS} \cup P_r^{SS}$ with length $|P_r| = 2 \times LEN^{FS} + LEN^{SS}$.

Throughout this paper the following conventions are used: lowercase letters indicate indices or primal variables while upper-case letters represent parameters or sets. Sections 2.1 - 2.4 describe each of the models enumerated above, respectively, while Section 2.5 presents the Rolling Horizon algorithm.

Table 1: Indices and sets.

$f \in F$	Generating firms
$t \in T$	Unit
$p \in P$	Time periods for entire model
$p \in P_r \subset P$	Time periods for roll r
$p \in P_r^{SS} \subset P_r$	Time periods in second stage for roll r
$p \in P_r^{FS} \subset P_r$	Time periods in first stage for roll r
$p \in P_r^{FIX} \subset P_r$	Time periods in fixed time period for roll r
$s \in S$	Scenarios
$r \in R$	Rolls
$k \in K$	Iterations for Gauss-Seidel diagonalization algorithm

Table 2: Binary decision variables.

$z^{\text{fs, on}}_{f,t,p}$	Binary variable indicating if firm f's unit p is online or offline at time p of first-stage
$z^{\text{on}}_{f,t,p,s}$	Binary variable indicating if firm f's unit p is online or offline at time p and scenario s of second-stage
$z^{\text{fs, su}}_{f,t,p}$	Binary variable indicating if firm f's unit p is starting up at time p of first-stage
$z^{\text{su}}_{f,t,p,s}$	Binary variable indicating if firm f's unit p is starting up at time p and scenario s of second-stage

Table 3: Continuous variables.

Decision variables

$gen^{fs}_{f,t,p}$	Generation from firm f's unit t in period p of the first-stage
$gen^{ss}_{f,t,p,s}$	Generation from firm f's unit t in period p and scenario s of the second-stage
Δg^{fs}_p	Consumer load shedding for period p of the first-stage (cost minimisation models only)
$\Delta g_{p,s}$	Consumer load shedding for period p and scenario s of the second-stage (cost minimisation models only)

Other variables

γ^{fs}_p	Electricity price for time period p of first-stage
$\gamma_{p,s}$	Electricity price for time period p and scenario s of second-stage
$\pi_{f,r}$	Firm f's expected profit in roll r
$\pi^{fs}_{f,r}$	Firm f's first-stage profit in roll r
$\pi^{ss}_{f,r,s}$	Firm f's profit in scenario s of second-stage in roll r
π^*_f	Firm f's optimal expected profit
$\pi_{f,r,k}$	Firm f's expected profit at iteration k of Gauss-Seidel diagonalization algorithm (roll r)
ψ_r	Overall system costs for roll r
ψ^{fs}_r	Overall system costs for first-stage of roll r
$\psi^{ss}_{r,s}$	Overall system costs for scenario s of the second-stage of roll r

Table 4: Parameters.

A_p	Demand curve intercept for timestep p
B	Demand curve slope
$C^{\mathrm{M}}_{f,t}$	Marginal generation cost curve intercept for firm f's unit t
$C^{\mathrm{Q}}_{f,t}$	Marginal generation cost curve slope for firm f's unit t
$C^{\mathrm{ON}}_{f,t}$	Online cost for firm f's unit t
$C^{\mathrm{SU}}_{f,t}$	Start-up cost for firm f's unit t
$GEN^{FS}_{p,-f}$	Combined generation for all firms except firm f at time p of first-stage
$GEN^{SS}_{p,-f,s}$	Combined generation for all firms except firm f at time p and scenario s of second-stage
LEN^{FS}	Length of fixed and first-stage time periods
LEN^{SS}	Length of second-stage time period
$MAX\_CAP_{f,t,p}$	Maximum generating capacity for firm f's unit t at time p
$MIN\_CAP_{f,t,p}$	Minimum generating capacity for firm f's unit t at time p
$GEN^{\mathrm{FIX}}_{f,t,p,r}$	Firm f's fixed generation for unit t in period p and roll r
$Z^{\mathrm{ON,\ FIX}}_{f,t,p,r}$	Firm f's fixed online decision for unit t in period p and roll r
$Z^{\mathrm{SU,\ FIX}}_{f,t,p,r}$	Firm f's fixed start-up decision for unit t in period p and roll r
$NORM_{t,p,s}$	Generating profile for technology t at time p and scenario s
$PROB_s$	Probability associated with scenario s
TOL	Convergence tolerance for Gauss-Seidel diagonalization algorithm

2.1 Cournot Equilibrium with Integers Model

In the Cournot Equilibrium with Integers Model, each firm f behaves as a Cournot player and chooses its generation levels for each unit t so as to maximise its profits over $|P_r|$ timesteps. They have knowledge of the inverse demand curve and how their generation levels can affect it. Thus, firm f knows that as it reduces its generation levels the market price increases. Consequently, we consider each firm f to be *price-making*. This is in contrast to the cost minimisation models to be presented.

In the Equilibrium with Integers Model, firm f also has unit commitment constraints and hence its optimisation problem contains binary variables in order to represent online and start-up costs. This is in contrast to the Equilibrium without Integers model (Section 2.2) where binary variables are excluded.

The Cournot Equilibrium with Integers Model contains $|F|$ optimisation problems, one for each firm. The model is solved using the Gauss-Seidel diagonalization algorithm (Section 2.1.1), which solves each the individual optimisation problems iteratively until a (Generalised) Nash Equilibrium is found. When firm f's optimisation is being solved the decisions variables of all other firms are assumed fixed.

Because the firm f's problem contains binary variables, the Lagrangrian of its objective function cannot be differentiated. Hence, Karush-Kuhn-Tucker decisions cannot be obtained, and the model cannot be solved using a Mixed Complementarity Problem. As mentioned previously, the model is solved using a Rolling Horizon algorithm (Section 2.5).

For each roll r, firm f seeks to maximise its profits by choosing when to have each of its units start-up ($z^{\text{fs, su}}_{f,t,p}$, $z^{\text{ss, su}}_{f,t,p,s}$), be online ($z^{\text{fs, on}}_{f,t,p}$, $z^{\text{ss, on}}_{f,t,p,s}$), and the generation levels for each unit ($gen^{fs}_{f,t,p}$, $gen^{ss}_{f,t,p,s}$). Firm f's problem is a stochastic program. In the both the first- and second-stage, its profits equal the revenue it gains from generation less the cost of generation and less the costs associated with starting-up and being online.

Its objective function at roll r is

$$\max \pi_{f,r} = \pi^{fs}_{f,r} + \sum_{s\in S} PROB_s \times \pi^{ss}_{f,s,r}, \tag{2a}$$

where $\pi^{fs}_{f,r}$ represents firm f's profits from the first stage:

$$\pi^{fs}_{f,r} = \sum_{p\in P^{FS}_r} \sum_{t\in T} \left((\gamma^{fs}_p - C^M_{f,t} - C^Q_{f,t} \times gen^{fs}_{f,t,p}) \times gen^{fs}_{f,t,p} - C^{\text{ON}}_t \times z^{\text{fs, on}}_{f,t,p} - C^{\text{SU}}_t \times z^{\text{fs, su}}_{f,t,p} \right), \tag{2b}$$

while $PROB_s$ is the probability associated with scenario s and $\pi^{ss}_{f,s,r}$ represents firm f's profit in the second stage for scenario s:

$$\pi^{ss}_{f,s,r} = \sum_{p\in P^{SS}_r} \sum_{t\in T} \left((\gamma_{p,s} - C^M_{f,t} - C^Q_{f,t} \times gen_{f,t,p,s}) \times gen_{f,t,p,s} - C^{\text{ON}}_t \times z^{\text{on}}_{f,t,p,s} - C^{\text{SU}}_t \times z^{\text{su}}_{f,t,p,s} \right). \tag{2c}$$

The constraints associated with the fixed time period (P^{FIX}_r) of firm f's problem for roll r are:

$$gen^{fs}_{f,t,p} = GEN^{\text{FIX}}_{f,t,p,r}, \ \forall t \in T, p \in P^{FIX}_r, \tag{2d}$$

$$z^{\text{fs,on}}_{f,t,p} = Z^{\text{FIX, ON}}_{f,t,p,r}, \ \forall t \in T, p \in P^{FIX}_r, \tag{2e}$$

$$z^{\text{fs,su}}_{f,t,p} = Z^{\text{FIX, SU}}_{f,t,p,r}, \ \forall t \in T, p \in P^{FIX}_r, \tag{2f}$$

Constraints (2d) - (2f) ensure that the first-stage decisions of roll $r-1$ are accounted for in roll r. For instance, if firm f commits a unit t to be online at the last time period of first-stage in roll $r-1$ and it is optimal for them to keep that unit online in the first hour of the first-stage of roll r, then firm f does not have to pay an online cost to do so. In the absence of constraints (2d) - (2f), firm f's objective function would include start-up costs for units that were already online from the previous roll.

The constraints associated with the first-stage (P_r^{FS}) of firm f's problem at roll r are

$$\gamma_p^{fs} = A_p - B \times (GEN_{p,-f}^{FS} + \sum_{t \in T} gen_{f,t,p}^{fs}), \; \forall p \in P_r^{FS}, \tag{2g}$$

$$gen_{f,t,p}^{fs} \leq z_{f,t,p}^{\text{fs, on}} \times MAX\_CAP_{f,t,p} \times \sum_{s \in S} PROB_s \times NORM_{t,p,s}, \; \forall t \in T, p \in P_r^{FS}, \tag{2h}$$

$$gen_{f,t,p,s}^{fs} \geq z_{f,t,p}^{\text{fs, on}} \times MIN\_CAP_{f,t,p}, \; \forall t \in T, p \in P_r^{FS}, \tag{2i}$$

$$z_{f,t,p}^{\text{fs, su}} = z_{f,t,p}^{\text{fs, on}} - z_{f,t,p-1}^{\text{fs, on}}, \; \forall t \in T, p \in P_r^{FS}. \tag{2j}$$

Equation (2g) represents the inverse demand curve for the first-stage. When solving firm f's optimisation problem, the generation levels of all other firms is assumed fixed. The parameter $GEN_{p,-f}^{FS}$ represents the combined generation levels of all firms, except firm f, for first-stage time periods. This parameter updates after each optimisation problem are solved in the Gauss-Seidel diagonalization algorithm (Section 2.1.1). Equation (2h) provides maximum capacity constraints. The capacity factors in this constraint are expected capacity factors ($\sum_{s \in S} PROB_s \times NORM_{t,p,s}$). Equation (2i) ensures that if unit t is online then it must generate at least to minimum capacity level. Equation (2j) represent unit commitment logic constraints for the first-stage.

The constraints associated with the second-stage (P_r^{SS}) of firm f's problem at roll r are

$$\gamma_{p,s} = A_p - B \times (GEN_{p,-f,s}^{SS} + \sum_{t \in T} gen_{f,t,p,s}), \; \forall s \in S, p \in P_r^{SS}, \tag{2k}$$

$$gen_{f,t,p,s} \leq z_{f,t,p,s}^{\text{on}} \times MAX\_CAP_{f,t,p} \times NORM_{t,p,s}, \; \forall t \in T, s \in S, p \in P_r^{SS}, \tag{2l}$$

$$gen_{f,t,p,s} \geq z_{f,t,p,s}^{\text{on}} \times MIN\_CAP_{f,t,p}, \; \forall t \in T, p \in P_r^{SS}, \tag{2m}$$

$$z^{\text{su}}_{f,t,p+1,s} = z^{\text{on}}_{f,t,p+1,s} - z^{\text{on}}_{f,t,p,s}, \ \forall t \in T, s \in S, p \in P^{SS}_r \tag{2n}$$

Constraints (2k) - (2n) follow from constraints (2g) - (2j), respectively. In contrast to the first-stage constraints, they are scenario dependent and hence contain a subscript s. In particular, the capacity factors in equation (2l) are not expected values. For the data used in this work, capacity factors for renewable energy vary across scenarios. The parameter $GEN^{SS}_{p,-f,s}$ represents the combined generation levels of all firms, except firm f, for scenario s of the second-stage. This parameter updates after each optimisation problem are solved in the Gauss-Seidel diagonalization algorithm (Section 2.1.1).

Constraint (2o) includes variables from both the first-stage and the second-stage. It ensures that if a unit t is committed to be on at the last time period of the first stage ($p = r \times LEN^{FS}$), then that must be accounted for in all scenarios of the first time period of the second stage.

$$z^{\text{su}}_{f,t,p+1,s} = z^{\text{on}}_{f,t,p+1,s} - z^{\text{fs, on}}_{f,t,p}, \ \forall t \in T, s \in S, p = r \times LEN^{FS}. \tag{2o}$$

Firm f's optimisation problem is to maximise objective function (2a) subject to constraints (2g) - (2o). It is a Mixed Integer Quadratically Constrained Program (MIQCP) with a convex objective function and linear constraints. Hence, when firm f's optimisation is being solved in the Gauss-Seidel diagonalization algorithm (Section 2.1.1), it can be solved using a standard MIQCP solver.

2.1.1 Diagonalization algorithm for Equilibrium with Integers

As firm f's optimisation problem (2) contains integer variables we cannot derive Karush-Kuhn-Tucker optimality conditions. Instead, at each roll r, we solve the equilibrium with integers model by implementing the following Gauss-Seidel diagonalization algorithm (Gabriel et al., 2012). The algorithm iteratively solves each firm f optimisation problem (2) by fixing all other firm decisions, until it converges to a point where no firm has an optimal deviation.

while $\sum_f |\pi_{f,r,k} - \pi_{f,r,k-1}| > TOL \; and \; k < |K|$ **do**

  for $f = 1,..,|F|$ **do**

    Assume firm$_{-f}$'s decision variables are fixed;

    $GEN^{FS}_{p,-f} \longleftarrow \sum_{\hat{f}\in F\setminus\{f\}} \sum_{t\in T} gen^{fs}_{\hat{f},t,p}, \quad \forall\, p \in P^{FS}_r$;

    $GEN^{SS}_{p,-f,s} \longleftarrow \sum_{\hat{f}\in F\setminus\{f\}} \sum_{t\in T} gen_{\hat{f},t,p,s}, \quad \forall\, s \in S, p \in P^{SS}_r$;

    Solve Firm f's MIQCP (2) ;

    $\pi_{f,r,k} \longleftarrow \pi^*_f$;

    $k \longleftarrow k+1$;

  end

end

Algorithm 1: Gauss-Seidel diagonalization algorithm.

The parameter TOL and the set K represent a pre-defined convergence tolerance and the set of iterations, respectively. The variable $\pi_{f,k}$ represents firm f's expected profits (objective function (2a)) at iteration k.

Algorithm 1 is based on Gauss-Seidel diagonalization, which is standard in the literature (Gabriel et al., 2012). As described in the literature, if Algorithm 1 converges, the point it converges to is guaranteed to be a Generalised Nash Equilibrium if each optimisation is solved to optimality in each iteration.

As the Equilibrium with Integer Model is solved using a Rolling Horizon, Algorithm 1 is solved $|R|$ times, once for each roll r.

2.2 Cournot Equilibrium without Integers model

The Cournot Equilibrium without Integers Model follows from the model presented in Section 2.1. As before each firm f is a Cournot player and chooses its generation level for each unit t so as to maximise profits. In contrast to the Cournot Equilibrium With Integers Model, start-up and online costs, and associated constraints, are excluded. Consequently, the Equilibrium Without Integers Model only contains continuous decision variables. As previously, it is solved using a Rolling Horizon Algorithm (Section 2.5) and for each firm's optimisation problem takes the form of a stochastic program.

Firm f's optimisation problem at roll r takes the following format:

$$\max \sum_{p\in P_r^{FS}}\sum_{t\in T}\left((\gamma_p^{fs} - C_{f,t}^{M} - C_{f,t}^{Q}\times gen_{f,t,p}^{fs})\times gen_{f,t,p}^{fs}\right) + \sum_{s\in S}\sum_{p\in P_r^{SS}}\sum_{t\in T} PROB_s \times \left((\gamma_{p,s} - C_{f,t}^{M} - C_{f,t}^{Q}\times gen_{f,t,p,s})\times gen_{f,t,p,s}\right), \tag{3a}$$

subject to:

$$gen_{f,t,p}^{fs} = GEN_{f,t,p,r}^{\text{FIX}},\ \forall t\in T, p\in P_r^{FIX}, \tag{3b}$$

$$\gamma_p^{fs} = A_p - B\times(\sum_{f\in F}\sum_{t\in T} gen_{f,t,p}^{fs}),\ \forall p\in P_r^{FS}, \tag{3c}$$

$$gen_{f,t,p}^{fs} \le MAX\_CAP_{f,t,p}\times\sum_{s\in S} PROB_s\times NORM_{t,p,s},\ \forall t\in T, p\in P_r^{FS}, \tag{3d}$$

$$\gamma_{p,s} = A_p - B\times(\sum_{f\in F}\sum_{t\in T} gen_{f,t,p,s}),\ \forall s\in S, p\in P_r^{SS}, \tag{3e}$$

$$gen_{f,t,p,s} \le MAX\_CAP_{f,t,p}\times NORM_{t,p,s},\ \forall t\in T, s\in S, p\in P_r^{SS}. \tag{3f}$$

Furthermore, firm f's generation levels are constrained to be non-negative. Because the Equilibrium without Integers model does not contain unit commitment constraints and hence integer variables, the Lagrangrian of firm f's optimisation problem can be differentiated with respect to each of firm f's primal and dual variables. Thus, in contrast to Section 2.1, the KKT conditions for each firm can be derived.

In contrast to Section 2.1, the generation levels of all firms except firm f in the inverse demand curves (equations (3c) and (3e)) are represented by decision variables rather than parameters. This is because all $|F|$ optimisation problems are solved simultaneously via a Mixed Complementarity Problem and thus the Gauss-Seidel diagonalization algorithm is not required.

As the optimisation problem of each firm f is convex, the KKT conditions are both necessary and sufficient for optimality and thus the solution obtained represents a (Generalised) Nash Equilibrium (Gabriel et al., 2012). In Section 4, the model is solved in GAMS using the PATH solver.

2.3 Cost minimisation with integers model

The cost minimisation with integers model follows from the model presented in Section 2.1. In comparison to that model, it contains start-up and online costs, as well as associated constraints. Hence it contains binary variables. In contrast however, the generating firms in the cost minimisation with integers model are not assumed to be price-making (Cournot) players. Instead, it is assumed that no firm has knowledge of the inverse demand curve, and all firms exhibit price-taking behaviour with all their generating units for all time periods. Consequently, the market model as described in this section is perfectly competitive.

Modelling a market via a cost minimisation problem, whereby a central planner (real or implicit) chooses the firms' generation levels so as to minimise overall system costs, the optimal results correspond to the perfectly competitive solution (Gabriel et al., 2012; Devine, 2012). Hence, there is no need to model each firm f's optimisation problem individually, as in the Equilibrium with Integers Model.

As in Sections 2.1 and 2.2, the cost minimisation with integers model is solved using a Rolling Horizon Algorithm (Section 2.5) and, for each roll r, it takes the form of a stochastic program. The (real or implicit) central planner seeks to minimise system costs by choosing when to have each generation unit start-up ($z^{\text{fs, su}}_{f,t,p}$, $z^{\text{ss, su}}_{f,t,p,s}$), be online ($z^{\text{fs, on}}_{f,t,p}$, $z^{\text{ss, on}}_{f,t,p,s}$), and the generation levels for each unit ($gen^{fs}_{f,t,p}$, $gen^{ss}_{f,t,p,s}$). In addition, the central planner also chooses the amount of electricity consumers shed (Δg^{fs}_{p}, $\Delta g_{p,s}$).

The problem's objective function is

$$\min \psi_r = \psi^{fs}_r + \sum_{s\in S} PROB_s \times \psi^{ss}_{s,r}, \tag{4a}$$

where ψ^{fs}_r are overall system costs for the first stage:

$$\psi^{fs}_r = \sum_{p\in P^{FS}_r} \left((\sum_{f\in F}\sum_{t\in T} C^M_{f,t} \times gen^{fs}_{f,t,p} + C^Q_{f,t} \times (gen^{fs}_{f,t,p})^2 + C^{\text{ON}}_t \times z^{\text{fs, on}}_{f,t,p} + C^{\text{SU}}_t \times z^{\text{fs, su}}_{f,t,p}) + \frac{B}{2} \times (\Delta g^{fs}_p)^2 \right), \tag{4b}$$

while $\psi^{ss}_{s,r}$ represents overall system costs for scenario s in the second stage :

$$\psi^{ss}_{f,s,r} = \sum_{p\in P^{SS}_r} \left((\sum_{f\in F}\sum_{t\in T} C^M_{f,t} \times gen_{f,t,p,s} + C^Q_{f,t} \times (gen_{f,t,p,s})^2 + C^{\text{ON}}_t \times z^{\text{on}}_{f,t,p,s} + C^{\text{SU}}_t \times z^{\text{su}}_{f,t,p,s}) + \frac{B}{2} \times (\Delta g_{p,s})^2 \right). \tag{4c}$$

The constraints of the cost minimisation with integers model at roll r are

$$\Delta g^{fs}_p + \sum_{f\in F}\sum_{t\in T} gen^{fs}_{f,t,p} = \frac{A_p}{B}, p \in P^{FS}_r, \tag{4d}$$

$$\Delta g_{p,s} + \sum_{f \in F} \sum_{t \in T} gen_{f,t,p,s} = \frac{A_p}{B},\ s \in S, p \in P_r^{SS}, \tag{4e}$$

$$\text{Constraints (2d) - (2f), (2h) - (2j), (2l) - (2n) } \forall f. \tag{4f}$$

Constraints (4d) and (4e) represent the demand balancing equations in the cost minimisation with integers model. The load shedding variables (Δg_p^{fs}, $\Delta g_{p,s}$) represent the difference between the consumers' demand, if the system price were zero, and the optimal generation levels of the firms. They are also constrained to be non-negative and do not appear in either of the model presented in Sections 2.1 and 2.2. The costs associated with these variables are $\frac{B}{2} \times (\Delta g_p^{fs})^2$ and $\frac{B}{2} \times (\Delta g_{p,s})^2$, respectively, and represent the Value of Lost Load (VoLL) for consumers. These terms follow from Devine & Bertsch (2018, 2022) and ensure system prices in this model are consistent with those described in Sections 2.1 and 2.2.

By assuming the system price for the first-stage (γ_p^{fs}) is the Lagrange Multiplier associated with constraint (4d) and by differentiating the Lagrangian of this model with respect to Δg_p^{fs}, leads us to the equation $\frac{\gamma_p^{fs}}{B} = \Delta g_p^{fs}$. Substituting this value for load shedding into constraint (4d) ensures that, at optimality,

$$\gamma_p^{fs} = A_p - B \times (\sum_{f \in F} \sum_{t \in T} gen_{f,t,p}^{fs}),\ \forall\ p \in P_r^{FS}, \tag{5a}$$

which is consistent with equation (2g). Similar logic ensures

$$\gamma_p = A_p - B \times (\sum_{f \in F} \sum_{t \in T} gen_{f,t,p,s}),\ \forall\ s \in S, p \in P_r^{FS}. \tag{5b}$$

The remaining constraints (equation (4f)) follow from those in the Cournot Equilibrium with Integers Model. In contrast to Section 2.1 however, at each roll r, the generation levels of all firms are determined by a single optimisation problem and hence the constraints of all firms are included in that problem.

The optimisation problem (4) has a strictly convex objective function and linear constraints. Thus, for each roll r, its optimal solutions can be obtained using a standard Mixed Integer Quadratically Constrained Program (MIQCP) solver.

2.4 Cost minimisation without integers model

The cost minimisation without integers model follows from the models presented in Sections 2.2 and 2.3. In comparison to the cost minimisation with integers model, the market considered is assumed to be perfectly competitive and thus all firms are assumed to be price-taking. Consequently, generation levels can be obtained by assuming there is a central planner (real or implied) who chooses the optimal levels so as to minimise overall system costs. In contrast to cost minimisation with integers but in comparison to the equilibrium without integers model, the cost minimisation without integers model excludes start-up and online costs, and associated constraints. Thus, it only contains continuous variables.

As in previous subsections, the cost minimisation without integers model is solved using a Rolling Algorithm (Section 2.5) and, for each roll r, it takes the form of a stochastic program. The central planner seeks to minimise system costs by choosing the generation levels for each unit ($gen^{fs}_{f,t,p}$, $gen^{ss}_{f,t,p,s}$) the amount of electricity consumers shed (Δg^{fs}_{p}, $\Delta g_{p,s}$). Its optimisation problem at roll r is

$$\begin{aligned}\min \psi_r = &\sum_{p \in P^{FS}_r} \Bigg(\Big(\sum_{f \in F} \sum_{t \in T} (C^M_{f,t} + C^Q_{f,t} \times gen^{fs}_{f,t,p}) \times gen^{fs}_{f,t,p} \Big) + \frac{B}{2} \times (\Delta g^{fs}_{p})^2 \Bigg) \\ &+ \sum_{p \in P^{SS}_r} \Bigg(\Big(\sum_{f \in F} \sum_{t \in T} (C^M_{f,t} + C^Q_{f,t} \times gen_{f,t,p,s}) \times gen_{f,t,p,s} \Big) + \frac{B}{2} \times (\Delta g_{p,s})^2 \Bigg). \end{aligned} \tag{6a}$$

subject to:

$$\text{Constraints (4d), (4e), (3b), (3d), (3f).} \tag{6b}$$

The optimisation problem (6) has a strictly convex objective function and linear constraints. Thus, as it only contains continuous decision variables, its optimal solutions can be obtained using a standard Quadratic Programming solver, from each roll r.

2.5 Rolling Horizon Algorithm

We solve each of the four models presented in Sections 2.1 - 2.4 using a Rolling Horizon algorithm. Instead of solving the entire optimisation/equilibrium problem once over all time periods, the problem is split into several smaller optimisation/equilibrium problem whose time sets are overlapping subsets of the overall time set. As mentioned previously, a Rolling Horizon algorithm improves computational efficiency and brings added realism to energy market modelling.

for $r = 1,...,|R|$ **do**

Define time period subsets using equation (1) ;

Solve equilibrium/optimisation problem ;

$GEN^{\text{FIX}}_{f,t,p,r+1} \longleftarrow gen^{fs}_{f,t,p}, \forall f \in F, t \in T, p \in P^{FS}_r;$

$Z^{\text{ON, FIX}}_{f,t,p,r+1} \longleftarrow z^{fs,on}_{f,t,p}, \forall f \in F, t \in T, p \in P^{FS}_r;$

$Z^{\text{SU, FIX}}_{f,t,p,r+1} \longleftarrow z^{fs,su}_{f,t,p}, \forall f \in F, t \in T, p \in P^{FS}_r;$

end

Algorithm 2: Rolling Horizon algorithm.

Algorithm 2 describes the Rolling Horizon algorithm. Firstly, at the start of each roll r the fixed, first-stage, and second-stage time period subsets are updated using equation (1). Secondly, the optimisation/equilibrium problem of interest is solved. Table 5 displays the solution for each of the model. Finally, the fixed decisions for roll $r+1$ are set at the first-stage decisions for roll r.

Table 5: Summary of solution techniques for model

Model	Solution technique
Cournot Equilibrium with integers	Algorithm 1
Cournot Equilibrium without integers	MCP (equations (2))
Cost minimisation with integers	MIQCP (equations (4))
Cost minimisation without integers	QP (equations (6))

3 Data

In this section we discuss the input parameters we apply to the models presented in Section 2. In Section 3.1 we describe the data for $|F| = 16$ generation firms while in Section 3.2 we discuss the wind capacity factor scenarios. All parameter values not explicitly stated in this section can be found can be found Online Appendix[1].

[1] `https://figshare.com/articles/dataset/Input_data_Devine_Lynch_2023_xlsx/24382018`

For each roll r, we assume the lengths of the fixed-stage, first-stage, and second-stage are each 12 hours. Thus, the number of hourly timesteps for each roll is 36. In total we consider $|r| = 15$ rolls. Hence, the model is solved over $|P| = 36 + 14 \times 12 = 204$ hourly time periods. The first roll considers 36 hours and at each of the subsequent 14 rolls, the model moves forward 12 hours.

While we choose 15 rolls, we do not present the results from the roll $r = 1$ in Section 4. This is to avoid start-of-model effects as the fixed decisions for the first roll are assumed to be zero. Hence, the results we present represent one week.

The demand curve intercept values (A_p) we assume are based off reference demand values taken from Devine & Bertsch (2022) and M. Á. Lynch et al. (2019), scaled to ensure a peak demand of 7500MW. These values can be found in our Online Appendix. The demand curve intercept value we choose is $B = -0.137$, which was obtained assuming an arc price elasticity of 0.11 from Di Cosmo & Hyland (2013), an average wholesale price from 2017-2021 of €67/MWh as reported by www.sem-o.com, and an average hourly demand over the same time period of 4456MW.

For the Gauss-Seidel diagonalisation algorithm, we set a convergence tolerance of $TOL = 10^{-4}$ with maximum number of iterations of $|K| = 10^2$.

3.1 Generator and power system data

The data for the thermal generation available is based on the electricity market of the island of Ireland. The regulatory bodies for the two power systems on the island of Ireland provide a validated set of technical and economic characteristics of every power plant on the system, and these data are publicly available (CRU and UREGNI, 2021). The installed capacity, excluding wind and solar generation, are shown by firm and by fuel type in Table 6. Exact values for each unit's maximum and minimum generating capacities can be found in the Online Appendix.

	Gas	Biomass / Peat	GasOil	Waste	Coal	Water	Oil	Waste wood	Total (MW)	Total (%)
Aughinish	162	-	-	-	-	-	-	-	162	2%
BGE	450	-	-	-	-	-	-	-	450	5%
BnM	-	118	116	-	-	-	-	-	234	3%
Contour	12	-	-	-	-	-	-	-	12	0%
Covanta	-	-	-	61	-	-	-	-	61	1%
Data and Power	116	-	-	-	-	-	-	-	116	1%
Energia Generation	745	-	-	-	-	-	-	-	745	9%
EP	350	-	258	-	476	-	-	-	1,084	12%
ESB	1,992	-	53	-	570	508	285	-	3,408	39%
Evermore Energy	-	-	-	-	-	-	-	18	18	0%
Grange	115	-	-	-	-	-	-	-	115	1%
Indaver	-	-	-	21	-	-	-	-	21	0%
iPower	-	-	58	-	-	-	-	-	58	1%
POWERNI	593	-	-	-	-	-	-	-	593	7%
SSE	464	-	208	-	-	-	592	-	1,264	14%
Tynagh	389	-	-	-	-	-	-	-	389	4%
Total (MW)	5,388	118	693	82	1,046	508	877	18		
Total (%)	62%	1%	8%	1%	12%	6%	10%	0%		

Table 6: Installed capacities by firm and fuel type, excluding wind capacity (MW)

The validated dataset includes quantities of fuel required for start-up and no load, as well as heat rates. In order to convert these into generator costs, we utilise fuel and carbon prices as per the recommended parameters from the European Commission Directorate-General for Climate Action, European Commission (2022). Start costs, online costs, and marginal intercept costs we assume can be found in the Online Appendix. In the models without integers, we increase marginal costs by 20% to account for the absence of start-up and online costs. Following Devine & Bertsch (2022), we assume a marginal slope cost of $C^{Q}_{f,t} = 0.000213, \forall f, t$.

In the numerical results presented in Section 4 we consider three different test cases for total installed wind capacity; 0GW, 4GW, and 8GW. In the absence of wind ownership data, we assume ownership is split evenly amongst the six largest firms (ESB, EP, SSE, Energia, POWERNI, and BGE).

3.2 Wind Capacity Factors

We consider $|S| = 3$ scenarios at each roll and obtain hourly wind capacity factor scenarios for $NORM_{t,p,s}$ by using the Simultaneous Backward Reduction algorithm (Dupačová et al., 2003; Heitsch & Römisch, 2003) to reduce down to three the six scenarios for south-west of Ireland used in M. Á. Lynch et al. (2019) and Bertsch et al. (2018). We then choose the 8.5 days (204 hours) that have median weighted average capacity factor, which works out at 33%. The probabilities ($PROB_s$) associated with scenarios $s = 1 - 3$ are 0.485, 0.4 and 0.115, respectively. The values for $NORM_{t,p,s}$ can be found in the Online Appendix.

4 Results

In this section we present the results obtained from the models. We solve the four models presented in Section 2 three times. That is, with installed wind capacities of 0GW, 4GW, and 8GW. Thus, we present 12 test cases. The results are obtained from the first-stage decisions (that is, the committed-to decisions) from rolls $r = 2 - 15$. Consequently, the results represent 168 hours (one week). As discussed previously, results for roll $r = 1$ are not included in order to avoid start-of-model effects.

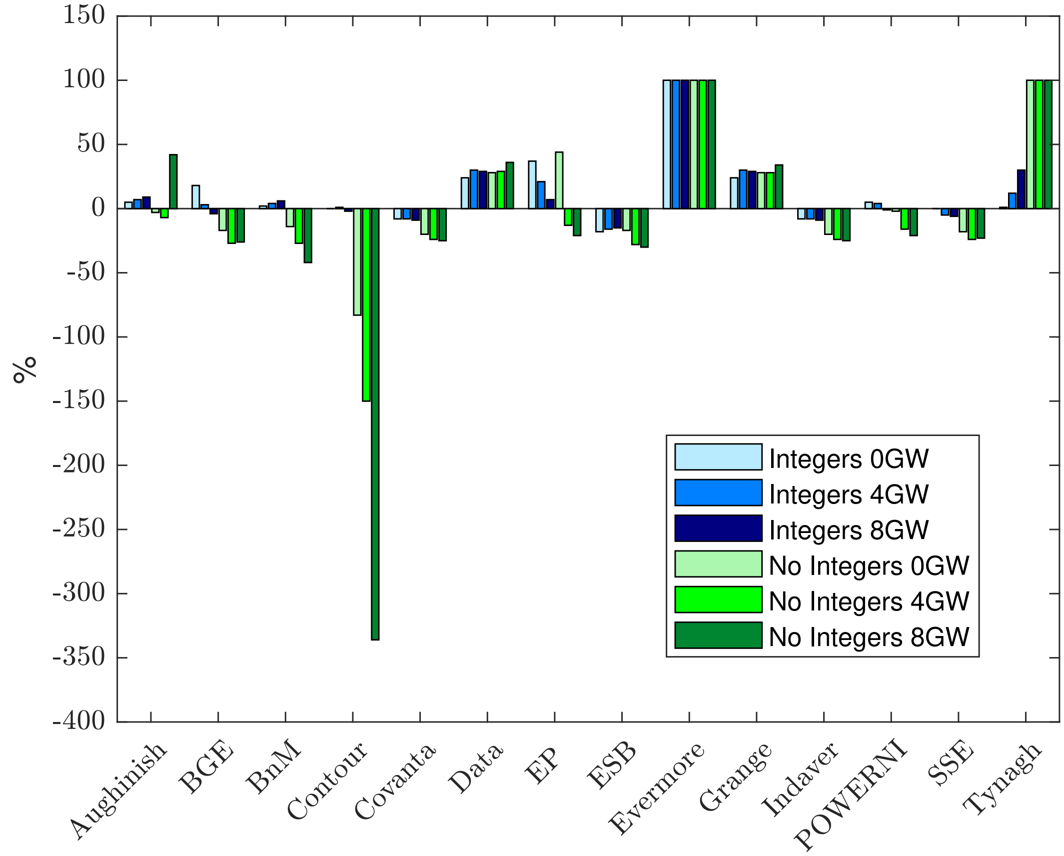


Figure 1: Decrease in firm profits from the introduction of integers

4.1 The impact of including integers with Cournot modelling

In this section, we analyse the impacts of including integer variables within a Cournot framework. Figure 1 shows the change in profits for each firm[2], in percentage terms, from the inclusion of integers. The results are graphed as the profits of each firm when integers are not modelled minus the profits of each firm when integers are included, divided by their profits without integers. In other words, Figure 1 shows the percentage decrease in profits from the inclusion of integers. We model three installed capacities of wind energy and assume the ownership of wind energy is distributed evenly amongst the firms. Table 11 in the Appendix gives the values that are graphed in Figure 1.

Several points are of note. First, there is a substantial change in profits for many firms from the inclusion of integers, for at least some test cases. Thus,

[2] We exclude Energia and iPower as the percentage change in their profits was too large to graph; however, the relative values for all firms are given in the Appendix.

this suggests that market modelling that excludes integers will not yield accurate results, particularly from the firm's point of view. Furthermore, this result is particularly strong under perfectly competitive modelling. We conclude that while the impact of integers that was observed in Shortt et al. (2012) is replicated here, this impact is dampened somewhat by the presence of price-making behaviour, modelled *á la* Cournot.

The second observation is the strong heterogeneity of results across firms (see Table 11). Aughinish, for example, sees a 5-9% increase in profits under Cournot modelling as a result of including integers, and a 3-7% decrease in profits under the assumption of perfect competition (cost minimisation) with 0 or 4GW of installed wind. Wind installations of 8GW, however, see a sharp increase in Aughinish profits, by 42%. Furthermore, the decrease in Aughinish's profits increases monotonically in installed wind energy under Cournot modelling, while under perfect competition there is no pattern. Grange, Data and Tynagh, on the other hand, see their profits increase under every test case. For Tynagh, the increase is monotonic in installed wind, while Data and Grange see the highest increase at 4GW of wind under Cournot modelling, and at 8GW under perfect competition.

In general, there is no consistent pattern to the impacts of the inclusion of integers from the firms' point of view. Across all firms, however, the impact of the inclusion of integers generally (though not always) increases in wind generation. This result aligns with that of Shortt et al. (2012). From a regulatory and policy perspective, neglecting the impact of discontinuous costs in market design may therefore lead to inaccurate results and conclusions. From a modelling perspective, there is no obvious heuristic to use to capture the impacts of discontinuous costs without explicitly including integers in the modelling.

A third insight is the fact that the magnitude of the impact of the inclusion of integers is much higher under perfect competition than under Cournot modelling. This result stems from the fact that in general, Cournot modelling results in higher equilibrium prices, and thus higher profits, than perfect competition. The percentage decrease in profits as a result of the inclusion of integers is therefore smaller in the presence of Cournot modelling, as profits are much higher to begin with. The absolute change in profits, however, shows different patterns of variation within and between firms (see Figure 2). In general, however, the impact of integers in absolute terms is proportional to the size of the firm, which is intuitive, but the impact varies by firm under different penetrations of wind energy. This underlines again the heterogeneity of the impact of integers at high RES-E.

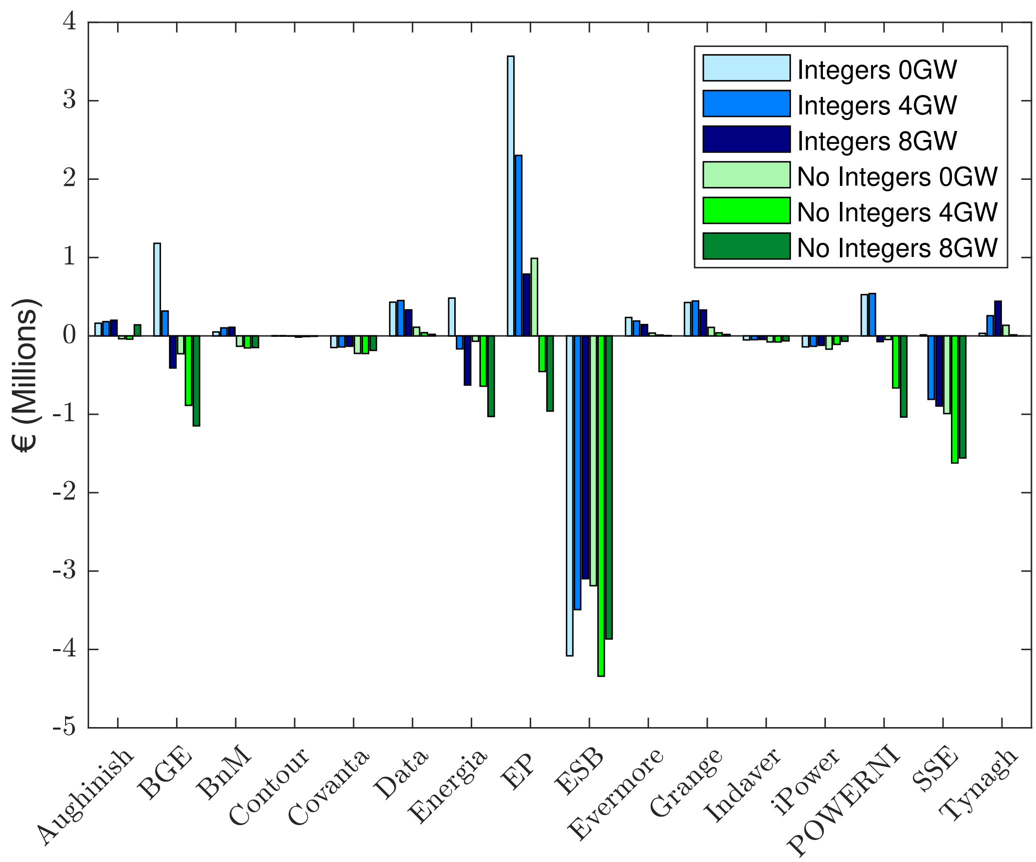


Figure 2: Change in firm profits from the introduction of integers

Table 7 shows the increase in consumer costs that arises as a result of including integers, and finds a similar result: consumer costs are lower under perfect competition, but including integer variables significantly increases consumer costs relative to a model that excludes integers. Under Cournot modelling, consumers already face high costs due to price mark-ups, and so there is a smaller proportional increase in consumer costs from the inclusion of integers.

	Cournot	Perfect competition
0GW	5%	17%
4GW	6%	22%
8GW	6%	23%

Table 7: Percentage increase in consumer costs as a result of including integers

In order to determine the drivers of these results, we consider the make-up of total costs that is accounted for by start costs, no load costs and incremental costs. This is shown in Table 8.

	Cournot			Perfect competition		
	0GW	4GW	8GW	0GW	4GW	8GW
% Incremental Costs	69%	67%	65%	66%	66%	65%
% Start Costs	1%	3%	5%	1%	4%	5%
% No load Costs	31%	30%	30%	33%	31%	30%

Table 8: The contribution of start, no load and incremental costs to total system costs.

The results under perfect competition are in line with those reported in Shortt et al. (2012). We note at this point that the total costs decrease as installed wind capacity increases, due to wind displacing conventional generation (and due to the fact that we consider operational costs only and do not consider investment costs). Thus, discontinuous costs make up a greater proportion of total costs at higher wind levels, but a lower absolute level of cost.

There are several insights from Table 8. The first has been mentioned already, namely that start costs are higher at higher wind levels under perfect competition compared to Cournot modelling. In addition, incremental costs and no-load costs decrease as wind increases under the perfect competition test case, although under Cournot modelling, no load costs make up an equal proportion of total costs at 4GW and 8GW of wind.

To examine these results further, Table 9 shows the number of times each of the firms incurred a start cost, under each test case (Indaver and iPower incurred no starts and so are omitted).

Considering first the total number of starts, there are more starts in general under perfect competition versus Cournot modelling, and more starts with wind

	Cournot			Perfect competition		
	0GW	4GW	8GW	0GW	4GW	8GW
Aughinish	0	0	0	0	0	8
BGE	7	8	5	7	7	6
BnM	7	7	17	28	22	18
Contour	0	11	9	14	14	14
Covanta	0	0	0	0	0	0
Data	0	12	16	14	13	11
Energia	7	5	2	7	2	0
EP	56	54	46	49	48	31
ESB	14	12	7	35	40	39
Evermore	0	0	0	0	0	0
Grange	0	5	7	7	7	5
POWERNI	7	15	17	7	19	18
SSE	49	34	35	35	49	69
Tynagh	0	6	7	0	0	0
Total	147	169	168	203	221	219

Table 9: Number of unit starts per firm

compared to no wind. There is no considerable increase in the total starts when wind moves from 4GW to 8GW, under either perfect competition or Cournot. Table 9 shows considerable variation in the number of starts both within and between firms, with ESB (the legacy monopolist and the firm with the greatest generation capacity) seeing considerably fewer starts under Cournot versus perfect competition, while the opposite pattern holds for most other firms. This could be because ESB starts their units once and is less likely to turn them off and restart them under Cournot, thereby avoiding incurring start costs, or it could be because ESB reduces starts because they are generally reducing their output under Cournot in an effort to increase prices.

In order to determine which case holds, Table 10 shows the number of hours each firm has a unit with non-zero output. Here we see that ESB has considerably fewer output hours under Cournot versus perfect competition, with other firms increasing their output. These results suggest that ESB is reducing power output in order to increase prices, with this reduction in output only partly compensated by other firms increasing their output. In other words, the reduction in starts by the monopolist is driven primarily by an attempt to reduce output and thus increase prices, rather than an attempt to avoid costly starts. Furthermore, wind curtailment does not vary significantly across the test cases (results not shown here), and so a reduction in thermal generation explains the reduction in total generation.

	Cournot			Perfect competition		
	0GW	4GW	8GW	0GW	4GW	8GW
Aughinish	336	336	336	336	336	211
BGE	224	263	224	119	248	214
BnM	504	413	349	406	288	223
Contour	168	153	121	168	60	37
Covanta	168	168	168	105	168	168
Data	257	281	226	126	100	49
Energia	28	190	173	14	172	168
EP	644	422	310	538	334	246
ESB	868	499	416	1443	1130	904
Evermore	133	0	0	140	0	0
Grange	168	146	115	168	50	19
Indaver	168	168	168	168	168	168
iPower	168	168	168	168	168	168
POWERNI	136	523	374	231	519	391
SSE	385	474	351	296	691	541
Tynagh	0	121	74	0	0	0

Table 10: Number of hours that each firm has a generation unit online.

4.2 The impact of Cournot behaviour with integer variables

We now consider the impact of price-making ability, modelled here *à la* Cournot, on market outcomes, and on whether these impacts vary depending on whether integers are considered in the modelling. Figure 3 shows the percentage change in profits, by firm, which arises from Cournot modelling (compared to perfect competition)[3].

Figure 3 shows that the introduction of Cournot modelling has a substantial impact on the profits of each firm (Table 12 in the Appendix shows the figures graphed in Figure 3)[4]. Unsurprisingly, Cournot modelling increases profits for every firm in all test cases.

Once again, we see heterogeneity in the results across firms. For some firms, there is a greater increase in profits when integers are included, while others see the opposite effect. Furthermore, some firms see their profits increase in wind generation, with and without integers (e.g., Aughinish), some see the opposite (e.g., PowerNI) and some see no pattern (e.g., SSE). Examining the absolute levels of profit earned by each firm, shown in Table 13 in the Appendix, the changes in profits in absolute terms is fairly constant within firms and within the test cases (with or without Cournot, with or without integers) as wind increases. In other words, increased wind leads to a relatively constant increase/decrease in

[3] Energia is omitted from the graph as the changes in profits without wind generation as a result of increasing integers lead to very large percentage changes.

[4] Tynagh is omitted as Tynagh's profits are 0 in the case of perfect competition with integers and are very low in the case of perfect competition without integers, leading to the percentage increase in profits to be infinite or unrealistically high.

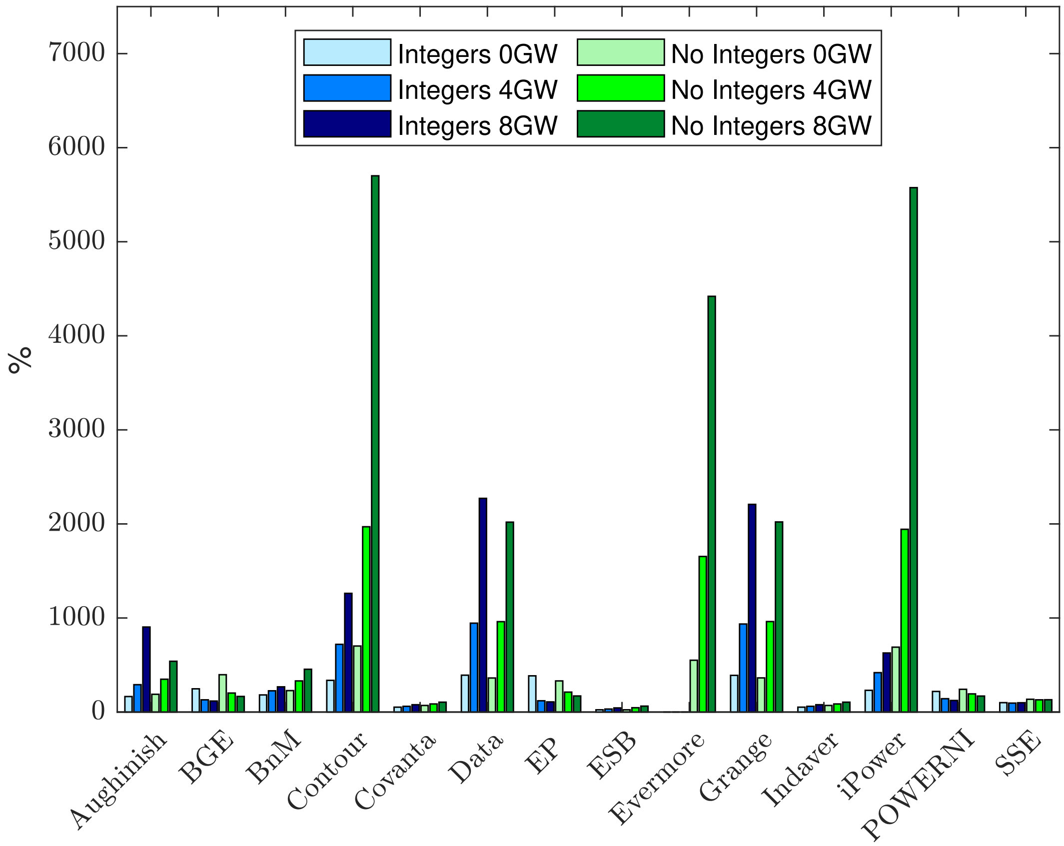


Figure 3: Change in profits from Cournot versus perfect competition

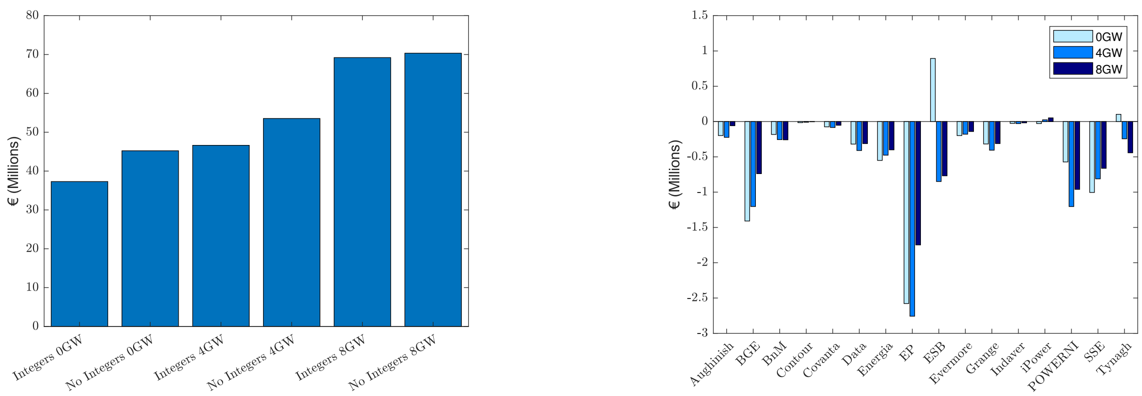
profits in absolute terms, for each firm, with the percentage increase or decrease being higher with integers than without (in general).

This broad pattern notwithstanding, there is heterogeneity in the changes in profits between firms as a result of Cournot modelling. In particular, the profit-maximising level of wind varies between firms. Some firms see an increase in profits as wind increases (e.g., Aughinish, Contour) while others see a decrease (e.g., BGE, EP). For ESB, there is no large increase in profits in percentage terms as a result of Cournot modelling.

The main result is that the impact of Cournot modelling is considerably higher, in percentage terms, when integers are included: in general, Cournot modelling leads to a higher percentage increase in profits with the inclusion of integers. For some firms, this result increases in wind generation, while for others the opposite holds. These results suggest again that ignoring the impact of integers can give misleading results, particularly under Cournot modelling.

Price-making behaviour impacts on market outcomes, but the percentage difference between perfect competition and Cournot modelling is magnified in the presence of integer modelling.

Several works in the literature examine the impact of price-making behaviour by solving a model that assumes such behaviour is present, then solving assuming it is not present, and then comparing the difference (Devine & Bertsch, 2022; Egging et al., 2017; Koschker & Möst, 2016). Figure 4a displays the increase in total costs to consumers as a result of moving from perfect competition to Cournot modelling, both in the presence and absence of integers.



(a) Impact on costs **(b) Impact on profits**

Figure 4: Impact of integers when moving from cost min to Cournot

Figure 4a indicates that the difference between consumer costs under perfect competition versus Cournot modelling is smaller when integers are considered compared to a case with no integers. This result holds both in absolute and percentage terms, and is stronger as wind increases. This suggests that excluding integer modelling may overestimate the impacts of price-making behaviour on consumer costs, to some degree. Given this, Figure 4b shows the impact of integers on the increase in profits when moving from perfect competition to Cournot, given in equation (7):

$$\sum_{r=2}^{15} (\pi_{f,r}^{*,\text{CournotIntegers}} - \pi_{f,r}^{*,\text{CostMinIntegers}}) - (\pi_{f,r}^{*,\text{CournotNoIntegers}} - \pi_{f,r}^{*,\text{CostMinNoIntegers}}) \quad (7)$$

Figure 4b shows that in general, the increase in profits under Cournot modelling versus perfect competition is greater, in absolute terms, when integers are not modelled. In other words, excluding integer modelling exaggerates the impacts of price-making behaviour modelled *à la* Cournot. It is noteworthy that one exception to this pattern is for the largest firm (ESB) with no wind generation: in this instance, the increase in profits under Cournot versus perfect competition is greater with integers compared to without integers. Furthermore, there is no obvious pattern observed across firms under different wind levels: the impact of integers on the difference between perfect competition and Cournot modelling peaks at 0, 4 and 8GW of wind for different firms.

5 Discussion

The results presented above indicate that integer modelling has a material impact on firm profits. This result holds when modelling markets as perfectly competitive and *à la* Cournot alike. Furthermore, the impact of integer modelling varies under each test case, and also varies between firms within test cases. Finally, for at least some results, the impact of integers increases in the presence of wind. Thus, while integer modelling may not be required at low levels of wind penetration, the delta between results with and without integers increases at higher wind generation levels. This suggests that market designs and models that fail to account for discontinuous costs may not allocate resources efficiently at higher levels of renewable generation penetration. The importance of this result is compounded by the cannibalization effect of renewable generation Cludius et al. (2014); Brown & Reichenberg (2021); Prol et al. (2020).

The heterogeneity of results across firms is of particular significance. For every metric considered here, the impact was heterogeneous across firms. This means that the impacts of integer modelling cannot be easily approximated by a general heuristic: the impacts are test case- and firm-specific.

These results also highlight the difference between market modelling, which considers firms as profit-maximising decision makers (imperfect competition), compared with power systems modelling, which typically takes a cost minimisation approach. While cost minimisation models, which correspond to perfect competition, did not deviate significantly from Cournot modelling on at least some metrics at low wind levels, this did not hold at higher levels of wind generation. Thus, the higher levels of wind generation projected for EU and other electricity markets, for example, necessitate consideration of discontinuous costs.

The above notwithstanding, Cournot modelling led to far higher equilibrium prices than perfect competition, which is in line with the literature. Integer modelling imposes new constraints on generators, either directly, via minimum generation levels, or by allowing the consideration of extra costs resulting from starting and stopping units. These extra constraints limit the decisions available to generators and therefore reduce their opportunities to increase prices by varying their output. This means that ignoring discontinuous costs will lead to an overestimation of the impacts of price-making behaviour, modelled *à la* Cournot.

The results suggest that strong market power mitigation measures continue to be required in order to protect consumers from anti-competitive pricing by generation firms. However, these measures should be complemented by the consideration of discontinuous costs, particularly at higher levels of renewable generation. Furthermore, the impact of integer modelling often varied for the largest firm (ESB) compared to other, smaller, market players. This suggests that the impact of integer modelling depends on the price-making ability of each firm.

This paper shows that it is possible to use the Gauss-Seidel algorithm (combined with a Rolling Horizon Algorithm) to solve a unit commitment problem where price-making behaviour is present. Moreover, we show this can be done for a non-trivial real-world example, namely, the all-island Irish power system. To the best of our knowledge, such contributions have not been previously seen in the literature.

Modelling equilibrium investment decisions may also be impacted by the consideration (or lack thereof) of discontinuous costs, particularly as renewable generators have fixed costs only (while thermal generators have both fixed and variable costs). The impacts of forward contracting on the energy market, and of different renewable subsidy mechanisms, may also yield different equilibria with and without integer modelling. We leave these considerations for future work.

6 Conclusion

This paper models output decisions by electricity generation firms, modelled under perfect competition and Cournot competition, with and without considering discontinuous costs such as start costs and no-load costs. The equilibrium prices and subsequent consumer costs are determined and compared at increasing levels of variable renewable generation. Modelling Cournot competition with discontinuous costs require integer variables. We solve such a model using the Gauss-Seidel diagonalization algorithm.

The results indicate that integer modelling yields different results for firms and consumers alike, and that the modelling impact of integers is heterogeneous across firms and under different levels of renewable generation. These results suggest that ignoring discontinuous costs, which is common in power and market modelling exercises, yields inaccurate results, and furthermore that there is no obvious heuristic that can readily be employed to estimate the impact of discontinuous costs in the absence of specific integer modelling. In addition, this work suggests that excluding integer variables may overestimate the impact of price-making behaviour.

Market power mitigation remains a priority for policymakers, but accurate modelling of energy markets is also of increasing importance as renewable generation increases. Possible extensions of this work include the impacts of forward contracting, renewable subsidisation, and investment decisions.

7 Appendix

	Cournot			Perfect Competition		
	0GW	4GW	8GW	0GW	4GW	8GW
Aughinish	5%	7%	9%	-3%	-7%	42%
BGE	18%	3%	-4%	-17%	-27%	-26%
BnM	2%	4%	6%	-14%	-27%	-42%
Contour	0%	1%	-2%	-83%	-150%	-336%
Covanta	-8%	-8%	-9%	-20%	-24%	-25%
Data	24%	30%	29%	28%	29%	36%
Energia	18%	-3%	-7%	-1415%	-23%	-24%
EP	37%	21%	7%	44%	-13%	-21%
ESB	-18%	-16%	-15%	-17%	-28%	-30%
Evermore	100%	100%	100%	100%	100%	100%
Grange	24%	30%	29%	28%	28%	34%
Indaver	-8%	-8%	-9%	-20%	-24%	-25%
iPower	-19%	-23%	-28%	-185%	-384%	-896%
POWERNI	5%	4%	-1%	-2%	-16%	-21%
SSE	0%	-5%	-6%	-18%	-24%	-23%
Tynagh	1%	12%	30%	100%	100%	100%

Table 11: Percentage decrease in profits as a result of adding integers

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	Integers			No integers		
	0GW	4GW	8GW	0GW	4GW	8GW
Aughinish	165%	293%	1077%	189%	350%	613%
BGE	247%	242%	247%	397%	394%	353%
BnM	182%	232%	316%	228%	334%	504%
Contour	337%	718%	1407%	702%	1973%	6168%
Covanta	52%	62%	91%	70%	86%	117%
Data	392%	945%	2521%	362%	964%	2251%
Energia	2918%	123%	123%	55632%	170%	157%
EP	385%	123%	128%	331%	205%	184%
ESB	24%	24%	33%	24%	40%	43%
Evermore	0%	0%	0%	551%	1659%	4832%
Grange	390%	928%	2443%	363%	965%	2254%
Indaver	52%	62%	91%	70%	86%	117%
iPower	231%	421%	720%	690%	1946%	6032%
POWERNI	219%	217%	224%	242%	280%	298%
SSE	100%	97%	123%	136%	133%	152%

Table 12: Change in profits by firm from the introduction of Cournot modelling (compared to perfect competition)

	Integers						No integers					
	Cournot			Perfect competition			Cournot			Perfect competition		
	0GW	4GW	8GW	0GW	4GW	8GW	0GW	4GW	8GW	0GW	4GW	8GW
Aughinish	2.95	2.51	1.94	1.11	0.64	0.19	3.11	2.69	2.14	1.07	0.60	0.33
BGE	5.33	9.47	11.93	1.54	4.13	5.49	6.51	9.79	11.52	1.31	3.24	4.34
BnM	2.96	2.38	1.84	1.05	0.73	0.50	3.01	2.48	1.95	0.92	0.58	0.35
Contour	0.15	0.12	0.09	0.03	0.01	0.01	0.15	0.12	0.09	0.02	0.01	0.00
Covanta	2.01	1.85	1.64	1.32	1.14	0.92	1.86	1.71	1.50	1.10	0.92	0.73
Data	1.38	1.06	0.83	0.28	0.10	0.03	1.81	1.51	1.16	0.39	0.14	0.05
Energia	2.21	6.39	9.88	0.07	3.47	5.23	2.69	6.23	9.25	0.00	2.83	4.20
EP	6.18	8.86	11.29	1.27	4.03	5.42	9.75	11.16	12.08	2.26	3.58	4.46
ESB	27.33	25.99	24.15	21.96	19.76	16.76	23.24	22.49	21.05	18.77	15.42	12.89
Evermore	0.00	0.00	0.00	0.00	0.00	0.00	0.24	0.19	0.14	0.04	0.01	0.00
Grange	1.37	1.05	0.82	0.28	0.10	0.04	1.80	1.50	1.15	0.39	0.14	0.05
Indaver	0.69	0.64	0.56	0.45	0.39	0.32	0.64	0.59	0.52	0.38	0.32	0.25
iPower	0.86	0.71	0.55	0.26	0.14	0.08	0.72	0.57	0.43	0.09	0.03	0.01
POWERNI	9.42	11.77	13.15	2.95	4.87	5.89	9.95	12.31	13.08	2.91	4.21	4.85
SSE	13.14	16.17	16.57	6.56	8.29	8.33	13.16	15.36	15.67	5.57	6.67	6.77
Tynagh	2.89	1.87	1.06	0.00	0.00	0.00	2.92	2.12	1.50	0.14	0.01	0.00

Table 13: Profits in M€for each firm with and without Cournot modelling and integers

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