THE ECONOMIC AND SOCIAL RESEARCH INSTITUTE

The 2 x 2 Contingency Table as a Test for Residual Autocorrelation

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In a recent note [7], attention was drawn to the possibility of using simple non-parametric tests for residual autocorrelation in least square regression. In that study, it was shown that Geary's τ test [4] and the Wald-Wolfcwitz runs test [10] gave substantially the same results as the more rigorous Durbin-Watson d test [2], which is, of course, rather more onerous to calculate.

Attention is now directed toward the possibilities of using a simple 2x2 contingency table to test for residual autccorrelation. The experiments have been carried out on the same **Irish family** expenditure data [1] as in [7].

Griliches et alia [5] used chi-squared in recent article to double-check on a misleading d - value - in that case, one extreme residual unduly influenced the d calculation. They have been followed by Thomas and Wallis [9] who, more recently, have used a modification of that approach to test for fourth-order residual autocorrelation. in a model using quarterly data. The comparison effected here, however, is concerned only with first-order autocorrelation.

Ninety functions were fitted to the data from [1] in order to ascertain the best fitting Engel function. The results are reported elsewhere [6.] The data, which is comprised of sixteen observations in each case, is actually cross-sectional, and not time-series. This does not affect the logic of the test but the method of application.

The sign of the ith residual is compared with the sign of the i + 1th, and the frequencies of the observed combinations of signs of successive residuals are arrayed in a 2x2 contingency table of the form

		Sign of the ith Residual		
		+	-	Total
Sign of the i + 1 th Residual	+	a_{11}	a_{12}	$_{11} + a_{12}$
	-	a_{21}	a_{22}	$a_{21} + a_{22}$
	Total	$a_{11}+a_{21}$	$a_{11}+a_{22}$	$N = \Sigma_i \Sigma_j a_{ij}$

and chi-squared is defined as

$$x = \frac{N\{ / a_{11}a_{22} - a_{21}a_{12}/ \quad \frac{N}{2} \}^2}{(a_{11} + a_{12})(a_{21} + a_{22})(a_{12} + a_{22})(a_{11} + a_{21})}$$

when Yates' correction is used.

If there is positive first-order autocorrelation of residuals, one would expect a_{11} and a_{22} to be significantly larger than a_{21} and a_{12}. Negative autocorrelation would require a_{21} and a_{12} to be significantly large. The null hypothesis of no autocorrelation is tested by applying the ordinary chi-squared test (with one degree of freedom) to the table. In this case, however, since N is comparatively small (N = 15), the individual cell entries are generally small, and the correctness of using chi-squared in such cases has been the subject of some controversy. The Fisher Exact Prbbability Test was used instead, utilizing the significance levels as tabulated by Finney [3] The detailed results are set out in Table 1, which also gives the significance appraisal for d, Geary's τ and the Wald-Wolfowitz n as reported in [7.]

(Insert Table 1)

It will be seen that the test does not show up well in comparison with the d test and the two non-parametric tests. What is surprising is how poorly the test compared with Geary's τ or the Wald-Wolfowitz u, since all three utilize the number of sign changes in various ways. Therefore, while the discrepancy between this chi-squared test and the Durbin-Watson could have been for the same reason that Griliches et alia (op. cit.) report-namely, the excessive influence that large residuals have on the value of d - the divergence between chi-squared and τ or u cannot be explained in this way.

Table 1. Significance Appraisal of Four Tests for Residual Autocorrelation.

Equation No.	Significance Appraisal				Equation No.	Significance Appraisal			
	d	u	τ	Fisher Exact		d	u	τ	Fisher Exact
1					46	ϕ			
2	ϕ	*	*		47	ϕ	**	**	
3					48				
4					49	**	*	*	
5					50	**	*	*	
6		*	*		51	*	*	*	
7	ϕ	*	*		52			*	
8					53				
9					54	**	*	*	
10		*	*		55	**	*	*	
11					56	*			
12					57				
13					58	*			
14	*	**	*		59	ϕ			
15					60				
16	ϕ				61	ϕ	**	**	*
17	**		*		62	ϕ			
18					63	ϕ	**	**	*
19	ϕ				64				
20	**	**	**	*	65	*	**	**	*
21	ϕ		*		66	**	**	**	*
22	*		*		67				
23					68	ϕ	*	*	
24					69				
25	**	**	**	*	70	**	**	**	*
26	*				71				
27					72				
28					73				
29	ϕ				74				
30					75				
31					76				
32					77				
33					78				
34					79				
35		*	*		80		*	*	
36		*			81				
37					82				
38					83				
39					84	ϕ			
40	ϕ	*	*		85				
41	*	*	*		86				
42	**				87				
43	ϕ	**	*		88				
44	**				89				
45	**	**	*		90				

Notes

** indicates significance at the 1 per cent level
* indicates significance at the 5 per cent level
ϕ indicates inconclusive d - test.

From the results of this short study, it appears that the 2x2 table does not offer a sufficiently sensitive alternative to any of the other three tests used. It is possible, however, that where N is larger, it might be more sensitive.

October 1969.

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