Analysing long-term interactions between demand response and different electricity markets using a stochastic market equilibrium model

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Abstract: Power systems based on renewable energy sources (RES) are characterised by increasingly distributed, volatile and uncertain supply leading to growing requirements for flexibility. In this paper, we explore the role of demand response (DR) as a source of flexibility that is considered to become increasingly important in future. The majority of research in this context has focussed on the operation of power systems in energy only markets, mostly using deterministic optimisation models. In contrast, we explore the impact of DR on generator investments and profits from different markets, on costs for different consumers from different markets, and on CO2 emissions under consideration of the uncertainties associated with the RES generation. We also analyse the effect of the presence of a feed-in premium (FIP) for RES generation on these impacts. We therefore develop a novel stochastic mixed complementarity model in this paper that considers both operational and investment decisions, that considers interactions between an energy market, a capacity market and a feed-in premium and that takes into account the stochasticity of electricity generation by RES. We use a Benders decomposition algorithm to reduce the computational expenses of the model and apply the model to a case study based on the future Irish power system. We find that DR particularly increases renewable generator profits. While DR may reduce consumer costs from the energy market, these savings may be (over)compensated by increasing costs from the capacity market and the feed-in premium. This result highlights the importance of considering such interactions between different markets.

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1. Introduction

The rapid expansion of electricity generation from renewable energy sources (RES-E) leads to an increasingly distributed, volatile and uncertain energy supply. The increasing volatility comes along with growing flexibility requirements. Meeting these increasing requirements by conventional supply side or storage technologies alone will ultimately be inefficient or not possible at all. Demand response (DR) is another source providing the required flexibility, which is expected to become increasingly important (Albadi and El-Saadany, 2008; Palensky and Dietrich, 2011; De Jonghe et al., 2012; Siano, 2014).

In terms of quantifying impacts and benefits of DR, many researchers have focussed on power system operations in energy only markets so far (e.g. Su and Kirschen, 2009; Hu et al., 2016), often using deterministic models. However, since DR changes the profile of the load to be covered by the power system, one can also expect that it has an impact on the optimal power plant portfolio and, hence, on generation investments. In countries where other markets or payment mechanisms exist in addition to the energy market (e.g., a capacity market rewarding dispatchable capacity or a feed-in premium (FIP) rewarding RES-E generation) this means that the interactions between the different markets need to be considered when analysing the impact of DR on investment decisions. Moreover, since RES-E generation is not just volatile but also uncertain, one can further expect that this uncertainty, which is often ignored (O’Connell et al., 2014), affects the impact of DR on power system operation and investments. In such an environment, we are therefore particularly interested in exploring how DR affects generator investments and profits from different markets, how it affects the costs for different consumers from different markets, and how it affects CO₂ emissions. Moreover, we explore how the availability of a FIP for RES-E generation changes each of these impacts. Finally, we analyse what the benefits of a stochastic optimisation approach are over a deterministic one.

To analyse these questions, we present a stochastic Mixed Complementarity Problem (s-MCP) specifically developed for the analysis in this paper. In contrast to previous works, the MCP considers both operational and investment decisions, interactions between an energy market, a capacity market and a feed-in premium while the model also accounts for the stochasticity of RES. The MCP models the optimisation problems of different electricity market players and thus, also allows different generation firms and consumer groups to be distinguished. The individual optimisation problems of each player are solved simultaneously and in equilibrium. MCPs allow both primal variables (e.g., power generation) and dual variables (e.g., prices) to be constrained together (Gabriel et al., 2012). They have been used to model various types of energy markets (Hobbs, 2001; Gabriel et al., 2009; Huppmann, 2013; Egging, 2013; Lynch and Devine, 2017; Devine et al., 2016; Devine and Bertsch, 2017; Nolan et al., 2017). For instance, Ehrenmann and Smeers (2011) consider capacity expansion using a stochastic equilibrium model, however, they do not explicitly consider DR. De Jonghe et al. (2012) analyse the impact of DR on the optimal generation portfolio using a MCP but they do not consider uncertainty of RES-E supply, capacity markets or the effects of a FIP. Moreover, they do not distinguish between different consumer groups and focus on one RES-E source (wind) only.

The players we consider on the supply side include power generating firms with different generation
portfolios, i.e. specialised baseload, midload and peakload firms as well as a specialised RES firm and an integrated firm with generation capacity across all considered technologies. RES supply is considered as uncertain in the model. All firms maximise their profits by optimising the dispatch of their existing assets and making investments into new conventional and/or renewable generation under consideration of the RES uncertainty. Firms earn revenues from an energy market and a quantity-based capacity market. Firms with RES-E generation earn additional revenues from the FIP in those cases where we consider this is available.

On the demand side, we consider a number of different consumer groups, including commercial/industrial as well as residential consumers. We also distinguish between traditional consumers and prosumers, i.e. consumers that have their own generation from solar PV modules. Since the PV supply is also uncertain and the different RES supply profiles (e.g., wind and PV) and their uncertainties are correlated spatially and temporally, we consider this correlation when generating input data for the model. All consumers in our model choose, within given constraints, how much of their load to shift in order to meet their demand at minimum costs. We hypothesise that there will be an aggregator who acts on behalf of the consumers (Good et al., 2017; Cesena et al., 2015; Burger et al., 2016) with the objective of minimising their energy supply costs since, in reality, most consumers would not want to decide themselves whether or not and when to shift any load. In order to manage the computational complexity of the model, we use a Benders decomposition approach of the MCP based on Gabriel and Fuller (2010) and Egging (2013). We apply this model to a case study based on data for the Irish power system in 2025, which has a high penetration of wind power and a significant presence of smart meters (Commission for Energy Regulation, 2014) allowing consumers or aggregators on their behalf to react quickly to short-term changes in the wholesale market.

The remainder of this paper is structured as follows: In section 2, we introduce the mathematical model. In section 3, we describe the data of our case study and the approach for generating consistent ensembles of correlated wind and PV data. In section 4, we present our findings. In section 5, we discuss and interpret the results before concluding the paper in section 6. In Appendix A, we present the firms’ and the consumers’ Karush-Kuhn-Tucker (KKT) conditions; information about the Benders decomposition is provided in Appendix B.

2. Methodology

In this section, the methodology is described. We develop a stochastic MCP to represent an electricity market with two types of players: generation firms and electricity consumer groups.

Firms receive revenues from energy and capacity markets as well as a FIP and seek to maximise their profits. They may hold multiple generating units with the technologies considered being baseload, mid merit, peakload and wind. In Section 3, we consider different wind regions of Ireland with varying, but correlated, hourly capacity factors. Wind for the different regions are represented in the model as different technologies, i.e., wind from one region is considered as a different technology from wind in another region. Firms are distinguished by the initial generating portfolio they hold but may also invest in additional capacity in any
Table 1: Indices and sets.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f \in F$</td>
<td>Generating firms</td>
</tr>
<tr>
<td>$t \in T$</td>
<td>Generating technologies</td>
</tr>
<tr>
<td>$p \in P$</td>
<td>Time periods</td>
</tr>
<tr>
<td>$k \in K$</td>
<td>Consumers groups</td>
</tr>
<tr>
<td>$s \in S$</td>
<td>Scenarios</td>
</tr>
<tr>
<td>$h \in H$</td>
<td>Time steps in storage/load shifting period</td>
</tr>
<tr>
<td>$P' \subseteq P$</td>
<td>Index representing starting points for storage period</td>
</tr>
<tr>
<td>$i \in I$</td>
<td>Iterations of Benders Decomposition algorithm</td>
</tr>
</tbody>
</table>

Note: sets contain a finite amount of non-zero natural numbers while $|H|$ represents the cardinality of $H$.

Table 2: Variables.

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firms’ primal variables</td>
<td></td>
</tr>
<tr>
<td>$\text{gen}_{f,t,p,s}$</td>
<td>Generation from firm $f$ with technology $t$ in period $p$ and scenario $s$</td>
</tr>
<tr>
<td>$\text{cap}^\text{bid}_{f,t}$</td>
<td>Capacity bid of firm $f$ with technology $t$</td>
</tr>
<tr>
<td>$\text{inv}_{f,t}$</td>
<td>Investment in new generation capacity for firm $f$ with technology $t$</td>
</tr>
<tr>
<td>$\text{exit}_{f,t}$</td>
<td>Decommissioning of old generation capacity for firm $f$ with technology $t$</td>
</tr>
<tr>
<td>Consumers’ primal variables</td>
<td></td>
</tr>
<tr>
<td>$g^\text{ls}_{k,p,s}$</td>
<td>Load shedding from consumer group $k$ in period $p$ and scenario $s$</td>
</tr>
<tr>
<td>$g^\text{up}_{k,p,s}$</td>
<td>Electricity stored for later time point from consumer group $k$ in period $p$ and scenario $s$</td>
</tr>
<tr>
<td>$g^\text{down}_{k,p,s}$</td>
<td>Electricity used from storage from consumer group $k$ in period $p$ and scenario $s$</td>
</tr>
<tr>
<td>$g^\text{micro}_{k,p,s}$</td>
<td>Micro generation from consumer group $k$ in period $p$ and scenario $s$</td>
</tr>
<tr>
<td>$g^\text{pv}_{k,p,s}$</td>
<td>PV generation from consumer group $k$ in period $p$ and scenario $s$</td>
</tr>
<tr>
<td>Dual variables</td>
<td></td>
</tr>
<tr>
<td>$\gamma_{p,s}$</td>
<td>System price for time period $p$ and scenario $s$</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Unit capacity price</td>
</tr>
<tr>
<td>$\lambda^#_{\cdot}$</td>
<td>Lagrange multipliers associated with constraint $#$ of the firms’ problem</td>
</tr>
<tr>
<td>$\mu^#_{\cdot}$</td>
<td>Lagrange multipliers associated with constraint $#$ of the firms’ problem consumers’ problem</td>
</tr>
</tbody>
</table>

Note: ‘.’ is used as a place-holder as the subscripts for both Lagrange multipliers vary depending on the constraint.
### Table 3: Parameters.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$PR_s$</td>
<td>Probability associated with scenario $s$</td>
</tr>
<tr>
<td>$MTC_t$</td>
<td>Maintenance cost form generating technology $t$</td>
</tr>
<tr>
<td>$CAP_{f,t}$</td>
<td>Initial generating capacity for firm $f$ with technology $t$</td>
</tr>
<tr>
<td>$D_{REF}^{k,p}$</td>
<td>Reference demand for consumer group $k$ in period $p$ and scenario $s$</td>
</tr>
<tr>
<td>$LOSS_k$</td>
<td>Storage loss factor for consumer group $k$</td>
</tr>
<tr>
<td>$C_{k}^{LS,MAX}$</td>
<td>Maximum load shedding for consumer group $k$ in any time period or scenario</td>
</tr>
<tr>
<td>$INT_{k}^{STOR}$</td>
<td>Electrical storage/ load shifting capacity for consumer group $k$</td>
</tr>
<tr>
<td>$INT_{k}^{MICRO}$</td>
<td>Micro generation capacity for consumer group $k$</td>
</tr>
<tr>
<td>$INT_{k}^{PV}$</td>
<td>PV generating capacity for consumer group $k$</td>
</tr>
<tr>
<td>$FAC_{k}^{STOR}$</td>
<td>Percentage of electrical storage capacity electricity consumer group $k$ can use in each period and scenario</td>
</tr>
<tr>
<td>$NORM_{PV}^{p,s}$</td>
<td>PV generating profile for period $p$ and scenario $s$</td>
</tr>
<tr>
<td>$NORM_{f,t,p,s}^{PV}$</td>
<td>Generating profile for firm $f$ with technology $t$ in period $p$ and scenario $s$</td>
</tr>
<tr>
<td>$TARGET$</td>
<td>Capacity target for overall market</td>
</tr>
<tr>
<td>$X_{t}$</td>
<td>Feed-In premium for technology $t$</td>
</tr>
<tr>
<td>$X_{PV}$</td>
<td>Feed-In premium for PV</td>
</tr>
<tr>
<td>$DR_{t}$</td>
<td>De-rating factor for technology $t$</td>
</tr>
<tr>
<td>$A_{t}$</td>
<td>Intercept associated with marginal cost functions</td>
</tr>
<tr>
<td>$B_{t}$</td>
<td>Slope associated with marginal cost functions</td>
</tr>
<tr>
<td>$C_{k,p}^{PV}$</td>
<td>Marginal cost of using PV generation for consumer group $k$ in period $p$</td>
</tr>
<tr>
<td>$IC_{t}^{GEN}$</td>
<td>Investment in generating technology $t$ cost</td>
</tr>
</tbody>
</table>

### Table 4: Functions.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{t}^{GEN}()$</td>
<td>Marginal cost function for technology $t$</td>
</tr>
<tr>
<td>$C_{k,p}^{LS}()$</td>
<td>Load shedding operational cost for consumer group $k$ in period $p$</td>
</tr>
<tr>
<td>$C_{k,p}^{MICRO}()$</td>
<td>Operational cost of using micro generation for consumer group $k$ in period $p$</td>
</tr>
</tbody>
</table>
of these technologies. In this paper, all firms are modelled as price-takers, i.e., we assume that no firm may 
exert market power.

The capacity payment mechanism is a quantity based mechanism. Under such a framework, a policy 
maker, regulator or Transmission System Operator (TSO), exogenously selects a quantity of capacity required 
for a given period (e.g., one year). An auction is held wherein generators submit capacity bids and the total 
amount sold equal the MW target. Generation firms compete in the auction to hold the options and therefore 
gain a fixed sum of money per the time period considered to compensate them for each installed unit of 
capacity.

Consumers seek to minimise the cost of meeting their demand. They may generally do so by utilising a 
range of possible demand-side flexibility measures, such as load shedding, load shifting or electrical storage, 
PV generation or thermal micro generation. We do not model individual consumers but rather consider 
different consumer groups whose decisions represent the aggregate actions of consumers in these groups. 
Consumer groups are distinguished by different levels of demand-side flexibility capability and whether they 
represent industrial/commercial or residential consumers.

The stochasticity of the model arises from the uncertainty surrounding wind and PV power. Thus, each 
scenario in our model represents different RES generation profiles, i.e. varying levels of wind and solar power 
availability at each point in time. Each of the generation firms and consumer groups considered have separate 
optimisation problems that are connected through market clearing conditions. The stochastic MCP is made 
up of these market clearing conditions along with the KKT conditions for optimality from each of the players. 
Thus, the MCP solves the optimisation problem of each player simultaneously and in equilibrium. The KKT 
conditions are presented in Appendix A.

Throughout this section the following conventions are used: lower-case Roman letters indicate indices or 
primal variables, upper-case Roman letters represent parameters (i.e., data, functions), while Greek letters 
indicate prices or dual variables. The variables in parentheses alongside each constraint in this section are 
the Lagrange multipliers associated with those constraints.

2.1. Firm $f$ ’s problem

Firm $f$ maximises its expected profits (revenues less cost) by choosing the amount of generation ($gen_{f,t,p,s}$), 
capacity bid ($cap^{bid}_{f,t}$), investment in new capacity ($inv_{f,t}$) and decommissioning of existing capacity ($exit_{f,t}$). 
It considers revenues received from a capacity and an energy market as well as a FIP for RES generation. 
Its costs include generation costs and investment costs in addition to costs incurred for maintaining all of its

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1 The exceptions to these conventions can be found in Appendix B where variables with and index $i$ or $M$ represent parameters 
derived from variables.
Firm $f$’s optimisation problem is:

$$\max_{gen_{f,t,p,s},\text{cap}^{\text{bid}}_{f,t}} \sum_{t,p,s} \left( PR_s \times gen_{f,t,p,s} \times (\gamma_{p,s} + X_t - C_{t}^{\text{GEN}}(gen_{f,t,p,s})) \right) -$$

$$\sum_{t} \left( IC_t^{\text{GEN}} \times inv_{f,t} + (inv_{f,t} + CAP_{f,t} - exit_{f,t}) \times MTC_t \right) +$$

$$\sum_{t} DR_t \times \kappa \times \text{cap}^{\text{bid}}_{f,t},$$

subject to:

$$gen_{f,t,p,s} \leq (CAP_{f,t} + inv_{f,t} - exit_{f,t}) \times NORM_{f,t,p,s}^{G}, \forall t, p, s, (\lambda_{f,t,p,s}^{1}),$$

$$\text{cap}^{\text{bid}}_{f,t} \leq CAP_{f,t} + inv_{f,t} - exit_{f,t}, \forall t, (\lambda_{f,t}^{2}),$$

where $t$ represents the different generating technologies, $p$ represents timesteps and $s$ represents different RES scenarios of the stochastic optimisation problem. The parameters $IC_t$ and $MTC_t$ are cost of investment and maintenance for technology $t$ respectively. The marginal cost of generating with technology $t$ is

$$C_{t}^{\text{GEN}}(x) = A_t^{\text{GEN}} + B_t^{\text{GEN}} \times x,$$

which means the overall cost of generating with technology $t$ is quadratic.

The parameters $X_t$, $PR_s$ and $DR_t$ represent the feed-in premium associated with technology $t$, the probability associated with scenario $s$ and the de-rating factor associated with technology $t$ respectively. In this work a de-rating factor reflects the proportion of its overall capacity a technology can provide to meet the capacity target. In Section 3 we assume $DR_t = 1$ for conventional technologies i.e., baseload, mid merit and peakload, and $DR_t = 0$ for RES generation. This is because we assume conventional technologies are always available for generation while RES generation, at times, is not available at all for generation.

The parameter $CAP_{f,t}$ is the initial installed capacity firm $f$ has for technology $t$ while $NORM_{f,t,p,s}^{G}$ represents the availability for technology $t$ in timestep $p$ and scenario $s$ for firm $f$. As with the de-rating factors, we assume $NORM_{f,t,p,s}^{G} = 1 \forall f, t, p, s$ for conventional generation. For RES generation, $NORM_{f,t,p,s}^{G}$ takes a value between zero and one depending on the timestep and scenario representing the intermittency and uncertainty of RES generation (which is basically generation from wind power in this paper as we consider solar PV power only as part of some of the consumers’ problems).

Constraint (1b) ensures that firm $f$ with technology $t$ cannot generate more than its capacity times its capacity factor, while constraint (1c) ensures it cannot make a capacity bid greater than the installed capacity. Each of firm $f$’s primal (decision) variables are also constrained to be non-negative.

The energy price at each period and scenario is $\gamma_{p,s}$. As each firm is modelled as a price-taker, generation decision variables cannot affect this price. Consequently, $\gamma_{p,s}$ is exogenous to firm $f$’s problem. However, it is a variable of the overall problem determined via the market clearing condition (6a). The capacity price
paid for each unit of installed capacity is $\kappa$. It is also exogenous to firm $f$’s problem but is a variable of the overall problem, determined via the market clearing condition (6b). Note that RES generators are not able to participate in the capacity market in this paper. Firm $f$’s problem is convex, assuming all values for $B^{\text{GEN}}_t$ are non-negative.

2.2. Consumer group $k$’s problem

Consumer group $k$ seeks to minimise the cost of meeting their expected demand. In our model they are generally able to choose from a range of demand side flexibility measures:

1. **Load shedding**: consumer group $k$ may reduce their demand at any period $p$ or scenario $s$. The decision variable that represents the amount they do so by is $g^{\text{ls}}_{k,p,s}$.

2. **Load shifting / electrical storage**: consumer group $k$ may also shift some of their demand and obtain less from the grid. In the model we consider shifting in a similar way as electrical storage where consumers may obtain electricity from electrical storage. Storing/shifting electricity and using electricity to meet demand in period $p$ and scenario $s$ are represented by the decision variables $g^{\text{up}}_{k,p,s}$ and $g^{\text{down}}_{k,p,s}$, respectively.

3. **Thermal micro generation**: instead of obtaining electricity from the grid, consumers may obtain electricity from their own micro-generation unit. The decision variable that represents the amount they do so by is $g^{\text{micro}}_{k,p,s}$.

4. **Photovoltaic (PV) generation**: instead of obtaining electricity from the grid, consumers may also obtain electricity from their own PV-generation unit. The decision variable that represents the amount they do so by is $g^{\text{pv}}_{k,p,s}$. In contrast to micro-generation, the availability of PV power varies across timesteps and scenarios similar to that of wind power (see section 2.1).

Consumer group $k$’s optimisation problem is:

$$
\min_{g^{\text{ls}}_{k,p,s}, g^{\text{up}}_{k,p,s}, g^{\text{down}}_{k,p,s}, g^{\text{micro}}_{k,p,s}, g^{\text{pv}}_{k,p,s}} \sum_{s,p} PR_s \left( \gamma_{p,s} \times \left( D^{\text{REF}}_{k,p} - g^{\text{ls}}_{k,p,s} + g^{\text{up}}_{k,p,s} - (1 - LOSS_k)g^{\text{down}}_{k,p,s} - g^{\text{micro}}_{k,p,s} - g^{\text{pv}}_{k,p,s} \right) ight)
$$

$$
- X^{\text{PV}} \times g^{\text{pv}}_{k,p,s} + g^{\text{ls}}_{k,p,s} \times C^{\text{LS}}_{k,p} + g^{\text{micro}}_{k,p,s} \times C^{\text{MICRO}}_{k,p} + g^{\text{pv}}_{k,p,s} \times C^{\text{PV}}_{k,p}
$$

subject to
\[ g_{k,p,s}^{\text{ls}} \leq C_{k}^{\text{LS,MAX}}, \forall p,s, (\mu_{k,p,s}^{1}), \quad (3b) \]
\[ g_{k,p,s}^{\text{up}} \leq FAC_{k}^{\text{STOR}} \times INT_{k}^{\text{STOR}}, \forall p,s, (\mu_{k,p,s}^{2}), \quad (3c) \]
\[ g_{k,p,s}^{\text{down}} \leq FAC_{k}^{\text{STOR}} \times INT_{k}^{\text{STOR}}, \forall p,s, (\mu_{k,p,s}^{3}), \quad (3d) \]
\[ g_{k,p,s}^{\text{micro}} \leq INT_{k}^{\text{MICRO}}, \forall p,s, (\mu_{k,p,s}^{4}), \quad (3e) \]
\[ g_{k,p,s}^{\text{pv}} \leq NORM_{p,s}^{PV} \times INT_{k}^{PV}, \forall p,s, (\mu_{k,p,s}^{5}), \quad (3f) \]
\[ \sum_{e=p'}^{p'+h-1} (g_{k,e,s}^{\text{up}} - g_{k,e,s}^{\text{down}}) \leq INT_{k}^{\text{STOR}}, \forall s,p',h, (\mu_{k,p',h,s}^{6}), \quad (3g) \]
\[ \sum_{e=p'}^{p'+h-1} (g_{k,e,s}^{\text{down}} - g_{k,e,s}^{\text{up}}) \leq 0, \forall s,p',h, (\mu_{k,p',h,s}^{7}), \quad (3h) \]
\[ g_{k,p,s}^{\text{ls}} + (1 - LOSS_{k}) g_{k,p,s}^{\text{down}} + g_{k,p,s}^{\text{micro}} + g_{k,p,s}^{\text{pv}} \leq D_{k,p}^{\text{REF}} + g_{k,p,s}^{\text{up}}, \forall p,s, (\mu_{k,p,s}^{8}), \quad (3i) \]

where \( D_{k,p}^{\text{REF}} \) represents consumer group \( k \)'s reference demand for period \( p \). This is the demand consumer group \( k \) would have in the absence of any demand side flexibility measures. The parameter \( LOSS_{k} \) is a percentage that represents the loss factor associated with storing electricity for consumer group \( k \) (which we assume equals zero for load shifting), while \( X^{PV} \) is the FIP associated with PV generation.

The marginal cost functions associated with load shedding and micro generation are [Devine and Bertsch 2017]:

\[ C_{k,p}^{\text{LS}}(x) = A_{k,p}^{\text{LS}} + B_{k,p}^{\text{LS}}x, \quad (4) \]
\[ C_{k,p}^{\text{MICRO}}(x) = A_{k,p}^{\text{MICRO}} + B_{k,p}^{\text{MICRO}}x, \quad (5) \]

respectively while \( C_{k,p}^{\text{PV}} \) represents the marginal costs of PV generation.

The parameter \( G_{k}^{\text{LS,MAX}} \) is the maximum amount of electricity that consumer group \( k \) can shed their load by in each period and scenario while \( INT_{k}^{\text{MICRO}} \) and \( INT_{k}^{\text{PV}} \) represent the micro generation and PV generation capacity, respectively, consumer group \( k \) holds. Furthermore, the parameter \( INT_{k}^{\text{STOR}} \) represents the amount of electricity consumer group \( k \) may keep in storage while \( FAC_{k}^{\text{STOR}} \) represents the percentage that consumer group \( k \) can use of their storage in each period and scenario.

The parameter \( NORM_{p,s}^{PV} \) represents the generating profile of PV power for time period \( p \) and scenario \( s \). Similar to \( NORM_{f,t,p,s}^{G} \) for wind power, it takes a value between zero and one depending on the timestep and scenario and thus represents the uncertainty and intermittency of PV power.

Constraints (3b) - (3i) constrain the amount electricity consumer group \( k \) can generate or store in time period \( p \) and scenario \( s \). Constraint (3g) ensures consumer group \( k \) cannot, over a \(|H|\)-timestep period, store more electricity than its storage capacity (or cannot increase load for shifting by more than a predefined limit). The index \( h \) presents timesteps in a \(|H|\)-timestep period while the index \( p' \) represents starting points for the storage period, i.e., \( p' \in P' = \{1, |H| + 1, 2|H| + 1, \ldots \} \subseteq P \). Constraint (3h) ensures consumer group \( k \) cannot, over the same \(|H|\)-step time period, use more electricity for meeting demand than what
has already been stored (or cannot decrease load for shifting by more than a predefined limit). Constraints (3g) and (3h) also ensure that any electricity stored in a $|H|$-timestep period cannot be used in any other $|H|$-timestep period. In reality, if a consumer stores electricity, they will be able to use it in any time period in the future. However, because we set $|H| = 48$ in Section 2.4 we believe this simplification is reasonable given the daily trough and peak structure of electricity demand. In addition, this simplification reduces the size and complexity of the model and hence increases computational efficiency.

Constraint (3i) ensures any electricity consumer group $k$ generate on their own, either from PV, storage, or micro-generation, must be less than their reference demand plus any increased consumption due to storing electricity. In other words, consumer group $k$’s own generation cannot be used to meet other consumers’ demand in this paper. However, for the chosen specification of the case study in this paper, this constraint will not be binding. All of consumer group $k$’s primal (decision) variables are also constrained to be non-negative. As with firm $f$’s problem, $\gamma_{p,s}$ is exogenous to consumer $k$’s problem but is a variable of the overall problem, determined via the market clearing condition (6a).

Consumer group $k$’s problem is convex, assuming all values for $B_{k,p}^{LS}$ and $B_{k,p}^{MICRO}$ are non-negative.

2.3. Market clearing conditions

The $|F| + |K|$ optimisation problems are connected via the following market clearing conditions:

$$
\sum_{f,t} \text{gen}_{f,t,p,s} = \sum_{k} \left( (D_{k,p}^{\text{REF}} - g_{k,p,s}^{\text{le}} + g_{k,p,s}^{\text{up}} - LOSS_{k,p,s}^{\text{down}} - g_{k,p,s}^{\text{micro}} - g_{k,p,s}^{\text{pv}}) \right), \forall p,s, (\gamma_{p,s}), \quad (6a)
$$

$$
\sum_{f,t} DR_{t} \times \text{cap}_{f,t}^{\text{bid}} = \text{TARGET}, \quad (\kappa), \quad (6b)
$$

Market clearing condition (6a) ensures that the total amount of electricity generated by the firms must equal the sum of the consumers’ demand. Consumers’ demand consists of their reference demand plus any electricity they shift/store less any electricity they shed or generate themselves. Market clearing condition (6b) ensures that the capacity bids of the firms must equal the capacity target level. The variables $\gamma_{p,s}$ and $\kappa$ are the prices and free Lagrange multiplier associated with conditions (6a) and (6b) respectively.

As each of the optimisation problems are convex, the KKT conditions are both necessary and sufficient for optimality for each type of player (Gabriel et al., 2012). Thus, the stochastic MCP consists of conditions (A.1) - (A.19) (see Appendix A) in addition to the market clearing conditions (6).

2.4. Solving the problem

For computational efficiency, the MCP is solved using a Benders Decomposition. The Benders Decomposition algorithm used in this paper is described in detail in Appendix B. It follows those used to solve MCPs in Egging (2013) and Gabriel and Fuller (2010); the overall scheme is presented in Appendix B.1. The decomposition involves two steps:
1. The MCP is solved for a selection of 24 days in hourly resolution. These days represent 8-day periods in winter, spring/autumn and summer, resulting in a total of $24 \times 8 \times 3 = 576$ hourly time steps. We choose 8-day periods, instead of 7-day periods (weeks), to ensure an even number of days in each period. This is because we choose $H = 48$ timesteps for each storage/load shifting period and thus require an even number of days to ensure there is equal amount of timesteps in each storage/load shifting period. The revenues and costs from each hour in the objective function of each player are multiplied by weighting factors of 11.375, 22.750 or 11.625 depending on whether that hour represents a week in winter, spring/autumn or summer, respectively. This ensures that the 576 hourly time steps represent a full year of 8784 hours. To determine the weighting factors we divided the whole year into seasons and assigned 91 days (13 weeks) to winter, spring and autumn respectively and 93 days to summer. To reduce the computational intensity of the model, spring and autumn are represented by the same week explaining the higher weighting factor of the time steps representing these seasons.

For the first step, a Benders Decomposition algorithm is used to solve the MCP. Benders Decomposition is a solution algorithm that has been shown to solve stochastic MCPs in a computational efficient manner (Egging, 2013; Gabriel and Fuller, 2010). Instead of solving for all variables simultaneously it divides the MCP into multiple smaller MCPs and solves the overall problem iteratively. At each iteration, a first-stage master problem MCP is solved for a subset of variables. The values of the remaining variables are determined by second-stage sub-problem MCPs.

In the model presented above, the different timesteps and scenarios are connected through the investment, exit and capacity bid variables. If these primal variables are fixed to specific values, the model could be solved for each period and scenario separately. Hence these variables are the first-stage variables and are also known as the complicating variables.

The master problem MCP of this work is presented in Appendix B.2. Its initial constraints are those of the overall problem that involves these variables. Constraints not involving any of these variables are not included in the master problem MCP. At each iteration, new constraints, known as Benders cuts, are added to the master problem MCP using the results from the preceding sub-problems. Consequently, the solution obtained is improved at each iteration. The algorithm stops when the first-stage variables produced by the master problem stop changing. A metric derived in Gabriel and Fuller (2010) is used to measure this convergence; see Appendix B.3.

The sub-problem MCPs can be represented by the overall MCP presented in Section 2. However, each sub-problem is solved with investment, exit and capacity bid variables fixed at the values determined by the master problem MCP of the same iteration. The second-stage variables are all scenario specific and, in this work, are the firms’ generation variables in addition to each of the consumers’ variables.

2. The optimal investment, exit and capacity bid variables from the first step are fixed as parameters in the second step. The MCP is then solved 93 times, each time representing a different 48-hour period in
a 366-day year. As before, we choose a 366-day year to ensure an even number of days and thus ensure there is an equal amount of timesteps in each 48-hour storage/load shifting period. As inter-temporal constraints do not exceed beyond 48 hours, splitting the MCP into smaller problems is equivalent to solving a single MCP with 8784 timesteps, assuming investment and exit decisions are fixed. However, splitting the problem up into multiple smaller problems is more computationally efficient (Devine et al., 2016). In addition, we believe it is reasonable to assume that investment/exit decisions are taken at times separate to generation, storage/shifting and load shedding decisions.

The outputs of the model are the optimal investment, exit and capacity bid decisions from the first step and the optimal generation, load shedding and load shifting / storage decisions and resulting prices from the second step. From this model output, it is straightforward to calculate consumer costs, generators profits and CO$_2$ emissions as well as RES share and curtailment.

3. Case study data

We apply the model described in section 2 to a case study based on the future Irish power system. For this purpose, we mainly use data for 2025 from EirGrid (2016). We also assume a significant presence of smart meters (Commission for Energy Regulation, 2014), allowing aggregators (Good et al., 2017; Ceseña et al., 2015; Burger et al., 2016) to respond to short-term changes in the market on behalf of the consumers with the objective of minimising their energy supply costs. In order to explore the impact of DR on generator investment decisions and profits, on consumer costs and on emissions and to understand how these impacts are affected by the presence of a FIP, we run the market model four times (with vs. without the possibility of load shifting combined with the presence vs. absence of a FIP) and compare the results (see section 4).

On the basis of EirGrid (2016), we assume a high penetration of RES-E in the system we study, namely 4800 MW of installed wind capacity and 50 MW of solar PV. For our case study, we assume that the installed wind capacity is owned and operated by firms on the supply side, while the installed PV capacity is installed on the demand side (where 30 MW are owned by households and 20 MW by industrial consumers). In terms of additional investments in renewable capacity, we only consider investments on the supply side (i.e. wind) in this paper and keep the installed PV capacity constant. Since the considered sources of renewable electricity are both volatile and uncertain, and, moreover, their uncertainties are correlated spatially and temporally, it is important to consider these correlations when generating input data for the stochastic market model. We therefore present our approach for providing normalised, hourly wind ($NORM^G_{f,t,p,s}$) and PV ($NORM_{p,*}$) capacity factor time series in section 3.1 below. Subsequently, we describe the conventional supply side data used in our model in section 3.2 and the demand side data in section 3.3.

3.1. Renewable power generation data

Data from the MERRA2 reanalysis (Bosilovich et al., 2016) were used to generate wind and PV data. MERRA2 data have a grid spacing of around 55km on each horizontal level. The following hourly data were
downloaded for years 1981 to 2015 inclusive, i.e. for a period of 35 years:

- SWGDN: surface incoming shortwave flux. Time-averaged hourly data (W/m²).
- T2M: 2-metre air temperature. Instantaneous hourly data (K).
- U2M/U10M/U50M: eastward wind at 2/10/50 metres above ground. Instantaneous hourly data (m/s).
- V2M/V10M/V50M: northward wind at 2/10/50 metres above ground. Instantaneous hourly data (m/s).

Using these historical wind and PV data as a basis for our analysis ensures that the spatial and temporal correlations from reality between wind and PV are preserved, which is an important requirement. The data were retrieved at eight different MERRA2 grid points around Ireland (see Figure 1). These eight locations correspond to the centres of the NUTS3 statistical regions of the Republic of Ireland and for each centre of a region, the closest available MERRA2 grid point was chosen.

![Figure 1: MERRA2 grid point locations](image)

**3.1.1. Wind data calculations**

MERRA2 wind speeds are available at heights above ground of 2 m, 10 m and 50 m. Recent wind farms have hub heights of around 100 m, whereas hub heights of older wind farms are lower. In this study, a hub height of 85 m was used at all eight locations, as a representative value.

MERRA 2 wind data were converted to wind power following the method used in [Cradden et al. 2017](#) and [Cannon et al. 2015](#). First, a logarithmic wind profile was used to extrapolate wind speeds at 85 m above ground from the input wind speeds at 2 m, 10 m and 50 m. Wind speed was then converted to wind capacity factors $NORM^G_{j,t,p,s}$ (where capacity factor = actual wind generation/installed rated capacity) using the
Office of Gas and Electricity Markets (Ofgem) 2013 wind power curve shown in Figure 2. Wind speeds below 3 m/s or above 30 m/s were assigned zero capacity factor.

![Wind speed to capacity factor curve](Ofgem, 2013)

3.1.2. PV data calculations

The MERRA2 variables SWGDN and T2M were used as input data to calculate PV generation profiles. These variables are converted into power generation profiles using a physical PV model (Ritzenhoff 1992; Ruppert et al. 2016; Schwarz et al. 2018a,b). This model calculates electricity generation of solar PV modules based on incident light, module efficiency and its orientation described by longitude, latitude, tilt and azimuth of the modules. The model considers direct and diffuse solar radiation and the ambient temperature to determine accurate module efficiencies. Furthermore, the albedo effect and average losses (e.g., cable or inverter losses and losses from shadowing or module mismatching) are taken into account. For reasons of simplicity, we assume a south orientation of the modules only. The inclination angle is set to 39° based on Bertsch et al. (2017). This model calculates normalised hourly PV generation values $NORM^{PV}$.  

3.1.3. Time series clustering of wind and PV

Despite the fact that we apply Benders Decomposition, it would be computationally intractable to consider the RES data of all 35 years in our stochastic market model. Therefore, the task in this section is to take the hourly bivariate time series of 35 years of (normalised) wind and PV, and to cluster these into a set of relatively few representative years.

For this purpose, and this purpose only, the hourly data for wind and PV were equally weighted across all eight stations and summed to give a total RES value including wind and PV for each hour from 1981 to 2015. Consequently, the data initially consisted of 306,792 hourly observations. This was trimmed down to 306,600 with the removal of the 29th February in leap years to ensure the same number of data points in each year.

We use the R package tsclust (Montero et al. 2014) to cluster similar years of the time series. The

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2See: http://www.jstatsoft.org/v62/i01/.
package works by creating a distances matrix of years, and subsequently running a clustering method from the distance matrix. The creation of the distance matrix is a key step, and we explored a number of different versions. We chose the periodogram approach (Caiado et al., 2006) as, in the exploratory data analysis, we noticed that the periodicity across year, month and day, was one of the most variable features of the data set.

Once the distances matrix between years was created, we used a standard hierarchical complete-linkage cluster method (Mardia et al., 1980) to judge the relatedness of the different time series years. We use the standard cluster-persistence approach to determine the optimal number of clusters via a dendrogram.

The resulting dendrogram of the clustering is shown in Figure 3. The most persistent solution was that of two clusters. We identified 1991, 2003/4, 2013, and 2015 in one cluster, with the remaining years in the other. A four cluster solution was the next most persistent, but for simplicity we chose the two cluster version. A plot of the time series contrasting the two sets of years shows that the first cluster tends to be more regular (i.e. shows less yearly seasonality) compared with cluster 2.

![Dendrogram of clustered wind and PV years.](image)

The clustering itself was based on identifying structural (dis)similarities between the 35 profiles. To proceed with the model runs, we then chose representative years from each cluster. In a first step, we find the yearly time series within each cluster that are the median Euclidean distance away from the time-wise overall median. This method identified years 2004 and 1983 respectively, which also correspond to the median capacity factors within each cluster. In a second step, to ensure that the set of representative profiles covers a broad range of RES availability, we chose further years representative of the lower and upper tercile of the total RES (wind and PV) capacity factor for each cluster. This leaves us with a total of six representative time series, which we consider as scenarios in the stochastic market model. We determine their respective probabilities according to the relative historical frequency of occurrence of the terciles. The chosen years and
probabilities of occurrence are summarised in Table 5.

Table 5: Representative years chosen for RES scenarios and corresponding probabilities of occurrence.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of occurrence</td>
<td>0.086</td>
<td>0.029</td>
<td>0.029</td>
<td>0.086</td>
<td>0.486</td>
<td>0.286</td>
</tr>
</tbody>
</table>

3.1.4. Use of regional RES generation data in stochastic market model

The procedure described in the previous section leaves us with six hourly time-series (scenarios) each for wind and PV for each of the eight NUTS3 regions in the Republic of Ireland. For computational regions, we aggregate these eight NUTS3 regions to three larger regions (see Table 6). In the stochastic market model, we consider the wind generation profiles of these three regions as different wind technologies (Wind 1-3). Since our analysis shows that the regional variations between RES generation profiles is much lower for PV as opposed to wind and the installed capacity of solar PV is much lower, we aggregate the regional PV generation profiles to just one region for Ireland. For the electricity generation of both wind and PV, we consider a FIP payment of \( X_t = X_{PV} = 23 \, \text{€/MWh}_{el} \) which we obtain from Devine et al. (2017). We do not consider a FIP for any other technology. For both wind and solar we assume a marginal generating cost of zero.

Table 6: Aggregation of NUTS3 regions for consideration of RES technologies in market model.

<table>
<thead>
<tr>
<th>Aggregated Region</th>
<th>Associated NUTS3 statistical regions</th>
<th>Technology name in model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Region 1 (North-West)</td>
<td>Border, West</td>
<td>Wind 1</td>
</tr>
<tr>
<td>Region 2 (South-West)</td>
<td>Mid-West, South-West</td>
<td>Wind 2</td>
</tr>
<tr>
<td>Region 3 (South-East)</td>
<td>Dublin, South-East, Midland, Mid-East</td>
<td>Wind 3</td>
</tr>
</tbody>
</table>

3.2. Conventional power generation data

On the supply side, we consider five power generating firms with different generation portfolios, i.e. a specialised baseload, mid merit and peakload firm each as well as a specialised RES firm and an integrated firm with generation capacity across all considered technologies. The maximum capacity values for each technology are broadly based on EirGrid (2016). The breakdown of initially installed capacity by technology and firm is summarised in Table 7. As part of the study, we look at the firms’ investment decisions in new generating capacity. Initially, however, we assume that the firms do not have any of these new technologies in their portfolio (see Table 7).

As described in section 2.1, our model considers quadratic cost functions for the conventional generators, i.e. the marginal costs at the intercept increase with the power output of each generator according to the marginal cost slope \( B_t^{GEN} = 0.000213 \), which we assume based on Grigg (1996).

To calculate marginal power generation costs at the intercept, we assume power plant efficiencies of 30% for existing baseload generators, 50% for existing mid merit generators and 32% for existing peakload
Table 7: Initial power generation portfolio by firm $(CAP_{t,t})$.

<table>
<thead>
<tr>
<th>Technology</th>
<th>firm 1</th>
<th>firm 2</th>
<th>firm 3</th>
<th>firm 4</th>
<th>firm 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Existing baseload (MW)</td>
<td>1947</td>
<td>1940</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Existing mid merit (MW)</td>
<td>512</td>
<td>-</td>
<td>404</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Existing peakload (MW)</td>
<td>270</td>
<td>-</td>
<td>-</td>
<td>234</td>
<td>-</td>
</tr>
<tr>
<td>New baseload (MW)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>New mid merit (MW)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>New peakload (MW)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Wind 1 (MW)</td>
<td>775</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>775</td>
</tr>
<tr>
<td>Wind 2 (MW)</td>
<td>964</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>964</td>
</tr>
<tr>
<td>Wind 3 (MW)</td>
<td>661</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>661</td>
</tr>
</tbody>
</table>

generators. Note that in an Irish context, these efficiencies may lead to existing baseload generation having higher variable power generation costs than existing mid merit generators. To calculate marginal costs of the new investment technologies at the intercept, we assume efficiencies of 45% for baseload, 60% for mid merit and 40% for peakload and specific investment costs as in Table 8. The marginal costs in Table 8 were calculated using coal, gas and CO$_2$ prices of the corresponding futures markets for 2017 as obtained from the European Energy Exchange (www.eex.com). For this purpose, we used the average market results of the futures markets for 2017 as traded during 2016. We use coal and CO$_2$ prices to calculate variable generation costs of baseload generation, while gas and CO$_2$ prices are used for determining the variable costs of mid merit and peakload generation, i.e. mid merit generators are assumed to be combined cycle gas turbines (CCGT), while peakload generators are assumed to be open cycle gas turbines.

Table 8: Summary of techno-economic input data of considered supply side technologies.

<table>
<thead>
<tr>
<th>Technology</th>
<th>Annuity of specific invest $(IC^\text{GEN}_t)$ (€/MW y)</th>
<th>Fixed O&amp; M costs $(MTC^\text{t}_t)$ (€/MW y)</th>
<th>Marginal power gen. costs at intercept $(A^\text{GEN}_{t,t})$ (€/MWhel)</th>
<th>Spec. CO$_2$ emissions $(t \text{CO}_2/\text{MWhel})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Existing baseload</td>
<td>-</td>
<td>41,667</td>
<td>48.87</td>
<td>1.17</td>
</tr>
<tr>
<td>Existing mid merit</td>
<td>-</td>
<td>27,778</td>
<td>41.10</td>
<td>0.36</td>
</tr>
<tr>
<td>Existing peakload</td>
<td>-</td>
<td>23,148</td>
<td>63.38</td>
<td>0.56</td>
</tr>
<tr>
<td>New baseload</td>
<td>110,769</td>
<td>41,667</td>
<td>31.58</td>
<td>0.78</td>
</tr>
<tr>
<td>New mid merit</td>
<td>67,268</td>
<td>27,778</td>
<td>34.00</td>
<td>0.30</td>
</tr>
<tr>
<td>New peakload</td>
<td>40,363</td>
<td>23,148</td>
<td>50.50</td>
<td>0.45</td>
</tr>
<tr>
<td>Wind</td>
<td>103,014</td>
<td>28,000</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

3.3. Demand side data

We consider four different consumer groups on the demand side. These include commercial/industrial as well as residential consumers and within each of these, we distinguish between traditional consumers and
prosumers, i.e. consumers that have their own generation from solar PV modules. While the model described in section 2 is also capable of considering prosumers with thermal micro generation, we restrict our analysis in the case study to prosumers with PV generation. Figure 4 shows the reference demand profiles of the industrial and residential consumer groups on a typical day, which reveals that the residential demand profile is less flat (the peak is more pronounced) than the industrial one. Based on EirGrid (2016), we consider an overall annual electricity demand of 33.6 TWh and a peak demand of 5655 MW. Similar to Nolan et al. (2017), we calculate the capacity target as 1.2 times the system peak demand, i.e. $TARGET = 1.2 \times 5655$ MW = 6786 MW.

![Figure 4: Reference demand of industrial and residential consumers on a typical day](image)

In this study, we use the reference demand shown in Figure 4 as a baseline and explore the impact of shifting a share of the overall demand in time. The assumptions for load shifting are based on Gils (2014), who reports shiftable load values per hour for Ireland of approximately 10% of the peak demand for industrial and residential consumers. In addition to load shifting, we allow consumers to shed parts of their load. We use values for sheddable load based on Gils (2014) and parameters for the marginal load shedding cost functions from Devine and Bertsch (2017). Devine and Bertsch (2017) use value of lost load (VOLL) estimates by Leahy and Tol (2011) for different types of consumers in Ireland to derive load shedding cost functions for industrial and residential consumers.

4. Results

Below, we now present the results of our study focussing on the impact of DR on generators’ investment decisions and profits from different markets, on costs to different consumers from different markets and on CO$_2$ emissions. When presenting the results, we focus on the impact of load shifting within demand response. While we also allow consumers to shed parts of their load as described in the previous section, they do not choose to do so in the considered case study. This can be explained as follows: we model all generators as price-takers in this paper and consider a quantity-based capacity market. Together, this leads to avoiding very high price peaks, which is why consumers prefer not to shed any load for the parameters of the case
study. Section 4.1 presents the results when a FIP is available before section 4.2 summarises the results in absence of a FIP. Subsequently, section 4.3 presents our findings in relation to the value of using a stochastic optimisation approach.

4.1. Findings in the presence of a feed-in premium

In the case, where a FIP is available, Figure 5 shows that generators invest in almost 1500 MW new mid merit capacity, which is mainly driven by the quantity target in the capacity market. There are no exits of existing capacity. Moreover, generators invest in over 1000 MW new wind capacity, which contributes to maximising their profits because of revenues from the energy market and the FIP. When load shifting is possible, the investment in wind is 100 MW higher than in the case without shifting. This suggests that load shifting helps integrate higher amounts of wind into the system. Other investment decisions are not affected by load shifting in this case.

![Figure 5: Existing capacity and new investments (in presence of FIP)](image)

Figure 6 (a) shows that all firms' profits increase when load shifting is possible. The increase of the total profits across all firms corresponds to approximately 4%. Firms 1 and 5 benefit most, which can be mainly explained by increasing profits of wind generation. In part, these are a result of higher offpeak prices. In addition, load shifting leads to lower wind curtailment and higher wind investment, both of which increase the quantity of wind generation in the market. This higher quantity leads to higher profits from the energy market and FIP revenues. Since firm 1 is the only firm to invest in new wind generation, we can conclude that the profit increase of firm 5 is a result of reduced wind curtailment and higher offpeak prices only. Moreover, firms 1 and 2 can increase their capacity market revenues. Figure 6 (b) reveals that this increase is mainly earned by their baseload generation. While the installed baseload capacity does not change, there is a slight increase in the capacity price. This, in turn, is a direct impact of decreasing profits of the baseload generators in the energy market (Figure 6 (b)) when shifting is possible. Existing baseload generators have the highest variable power generation costs in our case study based on the Irish power system with the exception of peak load generators, where the latter are rarely needed to clear the energy market. Therefore, load shifting has a negative impact on existing baseload generation through slightly lower peak prices and through reducing...
the volume they need to generate (see Figure 9 (a) below). Since we assume a competitive, quantity-based capacity market, this leads to a capacity price increase.

The expected price effects in the energy market referred to above are shown in Figure 7. The solid black line shows the price duration curve (energy market prices for each of the 8,760 hours of the year sorted in decreasing order) in the presence of a FIP when no shifting is possible. As shown by the dashed black line, load shifting leads to higher prices during low-price hours (right side of the diagram) and slightly lower prices during high-price hours (left side of the diagram). In addition, Figure 7 shows an asymmetry between price increase in offpeak and price decrease in peak hours. However, it is this price increase in offpeak hours (which is a result of the load increase in these hours resulting from load shifting) that contributes to the increased profits of renewable power generation.

Figure 8 (a) shows how load shifting changes the costs to consumers from different markets. In general, consumers are exposed to three different cost categories: energy costs, capacity costs and FIP costs. As expected, the energy costs decrease from 1,156 Million € (without shifting) to 1,138 Million € (with shifting), which corresponds to a change of -1.5%. At the same time, however, capacity costs and FIP costs increase and, together, slightly overcompensate the savings on energy costs. Altogether, this leads to an unexpected consumer cost increase of 0.1%. The capacity cost increase can be explained by an increase in the capacity price resulting from decreasing profits in the energy market of the marginal generator required to meet the capacity quantity target (see Figure 6 (b) above). The increase in FIP costs can be explained by an increase
in wind generation when shifting is possible. This increase results from (i) a reduced curtailment of wind generation with shifting and (ii) from an increased investment in wind (see Figure 5).

Figure 8 (b) shows the breakdown of the relative savings by consumer group. While all consumer groups are similarly exposed to the increase in capacity and FIP costs, the savings on costs from the energy market differ between the consumer groups. While industrial consumers only save around 0.5% on energy costs, residential consumers save around 4%. This can be explained by the different load profiles of these consumers. Industrial consumers have a rather flat load profile throughout the day, whereas the load of the residential consumers correlates more strongly with the system level load and, hence, the prices in the energy market. In other words, because of the asymmetry between price reduction of shifting during peak hours and price increase during offpeak hours (see Figure 7), residential consumers have a higher benefit from shifting load from peak to offpeak hours. Another observation from Figure 8 (b) is that prosumers (those consumers with their own PV generation) have slightly higher benefits from shifting. Load shifting helps these consumers increase their self-consumption which leads to additional savings. Altogether, the effects with respect to the different cost categories lead to around 0.8% cost increase for industrial consumers, while residential consumers save around 1.2%.

Figure 9 shows the impact of load shifting on power generation by technology (a) and on emissions by technology (b). While load shifting can increase the total wind generation in the system by 4%, baseload generation is reduced by 25%. Again, the increase in wind generation is a combination of a higher investment in and a lower curtailment of wind power. Furthermore, the power generation from new and existing mid merit generators is increased by 2-3% under load shifting. These shifts in power generation between technologies are also reflected in the impact of load shifting on emissions by technology. While emissions of new and existing mid merit generators increase slightly, emissions from existing baseload generation are reduced by 25% resulting in a total emission reduction effect of load shifting of 11%.
4.2. Findings in the absence of a feed-in premium

We now turn to the results when no FIP is available to the RES generators. Figure 10 shows that, as opposed to the case where a FIP is available, generators do not invest in new wind generation capacity at all. Moreover, generators invest in more than 1600 MW new mid merit capacity (slightly more than in the case where a FIP is available), while around 200 MW of existing baseload capacity are decommissioned. It is interesting to observe that, when there is no FIP, the investment in new mid merit and the decommissioning of existing baseload increase by 18 MW when shifting is possible compared to the case without shifting. This suggests that load shifting changes the residual demand profile in a way that makes the expensive existing baseload generation less profitable and new mid merit generation more profitable (which is supported by the solid and dashed red lines in Figure 7).

Figure 11 (a) shows that the profits of all firms increase when load shifting is possible similar to the case when a FIP is available. The increase of the total profits across all firms corresponds to approximately 7% in this case. Again, firms 1 and 5 benefit most because of increasing revenues of their wind generators (see Figure 11 (b)). Since there is no FIP and no investment in new wind capacity, the increased revenues can be explained exclusively by lower curtailment and higher offpeak prices (see the red lines in Figure 7) in the
energy market when shifting is possible. Moreover, the profits of firms 3 and 4 increase slightly, which is a result of the slightly higher investment in and increased profits from new mid merit when shifting is possible (which is supported by Figure 11(b)). The profits of firm 2 also increase as a result of investing in new mid merit. However, the positive impact on the firm’s profits is partly compensated by decreased revenues from the capacity market. While the capacity price is the same with and without shifting in the case without a FIP, this revenue decrease is a result of the decommissioning of existing baseload capacity by firm 2.

Comparing the profit changes between the cases with and without a FIP, one finds that the revenue increase in the energy market resulting from shifting is higher when no FIP is available. This can be explained by a higher increase of offpeak prices by shifting in the absence of a FIP (see difference between solid and dashed red lines as opposed to the difference between the black lines in Figure 7). Nevertheless, the overall impact on profits is larger in presence of a FIP because of the FIP-related revenues. We also find that the presence of a FIP increases the total profits of all firms by around 25%.

Figure 12(a) illustrates the impact of load shifting on costs to consumers. As mentioned in the context of the generators’ profits, the capacity price does not change between shifting and no shifting so that the costs from the capacity market stay constant. Interestingly, however, the costs from the energy market increase slightly when shifting is possible as opposed to the case with a FIP where load shifting lead to slightly decreased costs from the energy market. This is a result of the increase in offpeak prices when shifting is possible (see red lines in Figure 7) and the asymmetry between price increase and offpeak hours and decrease in peak hours.

Comparing the costs to all consumers between the case without a FIP (Figure 12(a)) and the case where a FIP is available (Figure 8(a)), shows that the costs from the energy market are around 200 Million €lower when a FIP is available. This is mainly driven by higher levels of wind generation which leads to lower prices in the energy market (see black lines compared to red lines in Figure 7). However, this price effect of wind generation is overcompensated by higher capacity market costs and, most importantly, the costs of the FIP. Comparing the total costs to consumers between the cases with and without a FIP shows that consumers have to pay around 200 Million €more in the case with a FIP.
Figure 11: Generator profits (in absence of FIP)

Figure 12 (b) shows the breakdown of the effects of load shifting by consumer group. Similar to the case with a FIP, Figure 12 (b) reveals that industrial consumers’ costs increase when shifting is possible, while we note small savings for residential consumers. Again, this can be explained by the different load profiles of the different consumer groups. The cost increase for industrial consumers is almost the same as in the case with a FIP. For residential consumers, however, the savings of shifting are much lower when no FIP is available. Finally, we observe that savings of residential consumers with their own PV generation are slightly higher (and the cost increase of industrial consumers with PV is slightly lower), which, again, shows that shifting can help increase self-consumption.

In the absence of a FIP, Figure 13 illustrates how load shifting affects the power generation by technology (a) and emissions by technology (b). In this case, load shifting can increase the wind generation in the system by 2%, while the generation of existing baseload capacity is decreased by 29%. Generation of existing as well as new mid merit capacity increases by 5% each. Since there is no investment in new wind, the increase in wind generation is a result of reduced curtailment only. The shifts in power generation by technology are reflected in the emissions by technology in a similar way. Emissions from baseload generation decrease by 29% while emissions from existing and new mid merit generation increase by 5% each. Overall, load shifting results in an emission reduction of 10% in this case.

Comparing the total CO$_2$ emissions between the cases with (Figure 9 (b)) and without a FIP (Figure 13 (b)) shows that emissions in the presence of a FIP are slightly less than 1 Million t CO$_2$ lower than those
in the absence of a FIP. However, as mentioned above, this emission reduction comes at a cost increase to consumers of almost 200 Million €.

4.3. Value of stochastic solution and expected value of perfect information for the considered case study

In order to demonstrate the value of employing a stochastic optimisation approach, we calculate well-known metrics in stochastic programming: the Value of Stochastic Solution (VSS) and the Expected Value of Perfect Information (EVPI) (see Birge and Louveaux, 2011 for details). We calculate these values separately for firms and consumers.

The VSS measures the cost of considering expected values in a deterministic optimisation instead of adopting a stochastic approach. For the generation firms, its value was found to be -€127,330,403, which is 21% of the corresponding costs associated with the stochastic optimisation, i.e., the firms’ profits are 21% higher when considering expected values compared to the stochastic solution. While this suggests firms do better in the presence of expected values, the VSS for the consumers is €25,323,723, which corresponds to 2% of the costs of the stochastic solution, i.e., their costs would increase when considering expected values.
In particular, there would be a strong increase in consumer costs from the capacity market. This can be explained as follows: using expected values leads firms to naively believe that there is no uncertainty in the capacity factor for wind. As a result, we observe higher investments in new wind power. This leads to lower investments in new mid merit generation and some investments in new peak load generation. However, because of the higher penetration of wind power, the thermal generation is less profitable in the energy market, which drives up the price in the capacity market and therefore leads to a cost increase for consumers.

The EVPI measures the costs of not having perfect information or, in other words, the amount players would be willing to pay for perfect information. For the firms, the EVPIs value was found to be €48,449,717 which corresponds to 8% of the profits associated with the stochastic solution, where, as expected, the firms’ profits would increase by 8% if they had perfect information. Consequently, the EVPI for the consumers was found to be -€49,136,878, which corresponds to 3% of the costs associated with the stochastic solution, where their costs would be 3% higher if all players had perfect information. Again, this result is largely driven by the effects in the capacity market. Altogether, these findings underline the importance of employing a stochastic optimisation approach, in particular from a consumer perspective.

5. Discussion

In relation to our research objectives set out in section 1 we can summarise five main findings of the research presented in this paper.

First, load shifting provides incentives for increased investments in low marginal cost generation capacities. In the case where a FIP is available, this low marginal cost generation is wind power, in the case without a FIP, it is new mid merit generation. Moreover, load shifting increases generators’ profits, particularly of RES generators, because it increases offpeak prices. Note that we do not observe any investment in new baseload generators despite the fact that their marginal power generation costs are the lowest of the thermal technologies. This suggests that the level of wind penetration reduces the capacity factors of thermal generation to an extent where the investment costs of baseload generation are simply too high to make this a profitable investment.

Second, in contrast to conventional wisdom, we find that load shifting does not decrease costs to consumers per se. On the one hand, while costs from the energy market may decrease, this decrease may be (over)compensated by increasing costs from the capacity market and FIP. On the other hand, it is important to distinguish between different consumer groups. While we generally observe a slight cost decrease for residential consumers, costs for industrial consumers increase slightly because of their different load profiles.

Third, we find that load shifting helps reduce CO₂ emissions in our case study. This reduction results from different effects. On the one hand, shifting helps integrate RES generation by reducing curtailment and, in the case where a FIP is available, by increasing investment in wind generation. On the other hand, the existing baseload generators have the highest marginal costs (except peak generators which are hardly needed) and the highest specific CO₂ emissions in our case study. Load shifting therefore reduces the volume that
needs to be supplied by these generators and replaces this by cheaper existing and new mid merit generation. However, we acknowledge that the latter emission reduction effect largely depends on the underlying power system.

*Fourth*, the presence of a FIP has a strong impact on how load shifting changes profits, consumer costs and emissions. While the absolute impact of load shifting on consumer costs and generator profits is slightly higher in the absence of a FIP compared to the case where a FIP is available, the absolute impact of shifting on emissions remains unchanged.

*Fifth*, the calculated VSS and EVPI provide proof that it is advisable to use a stochastic optimisation approach in the considered case study. Incorporating the stochasticity of renewable generation into the model more realistically replicates the uncertainty that the market participants face when making decisions. The results obtained therefore replicate real-world markets and decisions more convincingly.

Concerning the impact of DR on consumer costs, our estimated savings for residential consumers of 1-4% correspond to €3-7 Million across all private households in Ireland for one year. Assuming approximately 1.5 Million households, we find annual savings of €2-5/household. Note that these estimates do not consider any costs for metering/controlling infrastructure. Of course, the exact results are largely driven by the model input data and need to be interpreted with some caution. However, another important reason for these results is that our analysis does not just focus on the energy market. In contrast, we consider interactions between the energy market, the capacity market and a FIP payment mechanism. And, methodologically, our results demonstrate the importance of considering these interactions since savings in one market can be (over)compensated by cost increases from other markets.

Comparing our results to other findings internationally as reported in the literature we find that [Feuerriegel and Neumann (2016)](FeuerriegelandNeumann2016), for instance, report similar savings of approximately 3% for Germany. Note that their estimate does not include infrastructure investments either and [Feuerriegel et al. (2016)](Feuerriegeletal2016) conclude that those capital expenditures can hardly be recovered from the savings. In a study conducted in the US, [Alcott (2011)](Alcott2011) finds an average welfare increase for residential consumers of around US$ 10/year, which corresponds to around 1-2% of their electricity costs. He similarly concludes that this is insufficient to justify any investment in metering infrastructure.

However, as shown in this paper there are benefits of DR in terms of RES integration and emission reduction. In addition, [Zarnikau and Hallett (2008)](ZarnikauandHallett2008) and [Walawalkar et al. (2008)](Walawalkaretal2008) argue that DR limits the possibility of generators to exert market power, while [Zarnikau (2010)](Zarnikau2010) more widely discusses the benefits of DR in terms of generally increasing market efficiency and competitiveness.

Should policy makers wish to avail of these potential benefits, our results and those of others show that they should be aware that there is little economic incentive for consumers to participate in DR programmes or even invest in new metering infrastructure. We acknowledge that the value of load shifting can be expected to be higher in scarcity situations, in particular in the presence of market power. With a quantity-based capacity market in place (as in Ireland in the future), however, scarcity situations can be expected to be
rather rare. Other mechanisms would therefore be required to incentivise consumers to participate in DR programmes and realise any potential benefits.

Another interesting finding from our study is that the presence of a FIP (which we assume at a level of 23 \( \text{€/MWh} \) on the basis of Devine et al. (2017)) increases both generator profits and consumer costs by around \( \text{€150-200 Million} \). At the same time, \( \text{CO}_2 \) emissions are reduced by slightly less than 1 Million tonnes only. Beyond the impact of demand response, this is an interesting finding for policy makers, raising the question whether higher levels of emission reduction could be achieved at the same costs through mechanisms other than a FIP for RES-E.

Critically reflecting our approach, we wish to acknowledge that we modelled all firms as price-takers in this paper. Considering price-making behaviour would lead to increased electricity prices, particularly during peak times. Therefore, price-making behaviour can have an impact on investment and operating decisions of all market participants and would also increase the value of demand response. Moreover, while the input data are based on actual data of the Irish power system, the case study has a stylised character. In addition, we did not consider any investment decisions by the consumers (i.e., in solar PV generation or battery storage) nor did we consider any behavioural aspects related to consumer decisions. Finally, the model in this paper considers investment decisions in a one-period framework; extending the model to a multi-period game would be very interesting, for instance to yield insights in relation to the timing of investments.

6. Conclusions

In this work we develop a novel stochastic, game-theoretic equilibrium model that allows us to investigate how demand response affects generator investment decisions and profits, how it affects the costs for different consumers, and how it affects \( \text{CO}_2 \) emissions. While existing work usually focusses on the effects in energy markets, the model developed also allows us to focus on the interactions between the energy market, a quantity-based capacity market and a feed-in premium for renewable electricity generation. We also analyse how the presence of a feed-in premium changes the impact of demand response and what the benefits of adopting a stochastic optimisation approach are.

For our case study based on the future Irish power system, we find that demand response increases profits particularly of renewable generators. We also find that demand response may decrease consumer costs from the energy market but that, in the presence of a feed-in premium for renewables, these savings are (over)compensated by increasing costs from the capacity market and the feed-in premium. This finding underlines the importance of considering interactions between the different markets. Altogether, the savings of residential consumers amount to 1–4\%, while the costs of industrial consumers increase by approximately 1\%. Consumers with their own solar PV generation generally have slightly higher benefits. By reducing curtailment of renewable generation and replacing emission-intensive generation with less emission-intensive generation, demand response helps reduce \( \text{CO}_2 \) emissions by 10-11\%. However, the small savings of residential consumers do not provide any economic incentive for investing in new metering infrastructure. While the
exact values depend largely on the model input data, these findings demonstrate that policy makers need to think about additional/alternative incentive mechanisms should they wish to harness any potential benefits of demand response.

While the presence of a feed-in premium for renewable generation would increase investment in wind power and therefore reduce CO₂ emissions by almost 1 Million tonnes per year, it would increase the annual costs to consumers by almost €200 Million. This suggests that there might be alternatives to achieve more emission reduction at the same costs.

Future research in this area should take into account the effects of price-making behaviour. Moreover, investment decisions by consumers, e.g., in solar PV generation or battery storage, should be considered. Last but not least, the extension of the model from a one-period to a multi-period game would reveal interesting insights in terms of the timing of investments.

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Appendix A. Karush-Kuhn-Tucker conditions

This appendix presents the Karush-Kuhn-Tucker (KKT) conditions for optimality for the two types of players modelled in this work. These conditions, along with the market clearing conditions [6], make up the mixed complementarity problem. The ‘perp’ notation $0 \leq a \perp b \geq 0$ is equivalent to $a \geq 0$, $b \geq 0$ and $a.b = 0$.

Appendix A.1. Firms’ KKT conditions

The firms’ KKT conditions are

$$0 \leq gen_{f,t,p,s} \perp -PR_s(\gamma_{p,s} + X_t - \frac{\partial C_{f,t}^{\text{GEN}}}{\partial gen_{f,t,p,s}}) + \lambda_{f,t,p,s}^1 \geq 0, \ \forall f,t,p,s,$$  \hspace{1cm} (A.1)

$$0 \leq inv_{f,t} \perp IC_t^{\text{GEN}} + MTC_t^{\text{GEN}} - \sum_{p,s} NORM_{f,t,p,s}^{G} \lambda_{f,t,p,s}^1 - \lambda_{f,t}^2 \geq 0, \ \forall f,t,$$  \hspace{1cm} (A.2)

$$0 \leq exit_{f,t} \perp -MTC_t^{\text{GEN}} + \sum_{p,s} NORM_{f,t,p,s}^{G} \lambda_{f,t,p,s}^1 + \lambda_{f,t}^2 \geq 0, \ \forall f,t,$$  \hspace{1cm} (A.3)

$$0 \leq cap_{f,t}^{\text{bid}} \perp DR_t \times \kappa + \lambda_{f,t}^2 \geq 0, \ \forall f,t,$$  \hspace{1cm} (A.4)

$$0 \leq \lambda_{f,t,p,s}^1 \perp -gen_{f,t,p,s} + (CAP_{f,t} + inv_{f,t} - exit_{f,t}) \times NORM_{f,t,p,s}^{G} \geq 0, \ \forall f,t,p,s,$$  \hspace{1cm} (A.5)

$$0 \leq \lambda_{f,t}^2 \perp -cap_{f,t}^{\text{bid}} + CAP_{f,t} + inv_{f,t} - exit_{f,t} \geq 0, \ \forall f,t.$$  \hspace{1cm} (A.6)
Appendix A.2. Consumers’ KKT conditions

The consumers’ KKT conditions are

\[ 0 \leq g_{ls}^{k,p,s} \perp - PR_s(\gamma_{p,s} - \frac{\partial C_{LS}^{k,p}}{\partial g_{ls}^{k,p,s}}) + \mu_{k,p,s}^{1} + \mu_{k,p,s}^{8} \geq 0, \forall k,p,s, \]  
(A.7)

\[ 0 \leq g_{up}^{k,p,s} \perp PR_s(\gamma_{p,s} + \mu_{k,p,s}^{2} + \sum_{e=p}^{|H|} (\mu_{k,p,e,s}^{6} - \mu_{k,p,e,s}^{7}) - \mu_{k,p,s}^{8} \geq 0, \forall k,p,s, \]  
(A.8)

\[ 0 \leq g_{down}^{k,p,s} \perp - PR_s(\gamma_{p,s} + \mu_{k,p,s}^{3} - \sum_{e=p}^{\hat{p}+1} (\mu_{k,p,e,s}^{6} - \mu_{k,p,e,s}^{7}) + (1 - LOSS_k)\mu_{k,p,s}^{8} \geq 0, \forall k,p,s, \]  
(A.9)

where

\[ \hat{p} = \max\{p' | \hat{p} \leq p\}, \]

\[ 0 \leq g_{micro}^{k,p,s} \perp - PR_s(\gamma_{p,s} - \frac{\partial C_{MICRO}^{k,p}}{\partial g_{micro}^{k,p,s}}) + \mu_{k,p,s}^{1} + \mu_{k,p,s}^{8} \geq 0, \forall k,p,s, \]  
(A.10)

\[ 0 \leq g_{pv}^{k,p,s} \perp - PR_s(\gamma_{p,s} + X_{pv} - C_{pv}^{k,p} + \mu_{k,p,s}^{5} + \mu_{k,p,s}^{8} \geq 0, \forall k,p,s, \]  
(A.11)

\[ 0 \leq \mu_{k,p,s}^{1} \perp - g_{ls}^{k,p,s} + C_{LS,MAX}^{k} \geq 0 \forall k,p,s, \]  
(A.12)

\[ 0 \leq \mu_{k,p,s}^{2} \perp - g_{up}^{k,p,s} + FAC_{k} \times INT_{STOR}^{k} \geq 0 \forall k,p,s, \]  
(A.13)

\[ 0 \leq \mu_{k,p,s}^{3} \perp - g_{down}^{k,p,s} + FAC_{k} \times INT_{STOR}^{k} \geq 0 \forall k,p,s, \]  
(A.14)

\[ 0 \leq \mu_{k,p,s}^{4} \perp - g_{micro}^{k,p,s} + INT_{MICRO}^{k} \geq 0 \forall k,p,s, \]  
(A.15)

\[ 0 \leq \mu_{k,p,s}^{5} \perp - g_{pv}^{k,p,s} + INT_{PV}^{k} \times NORM_{PV}^{p,s} \geq 0 \forall k,p,s, \]  
(A.16)

\[ 0 \leq \mu_{k,p',h,s}^{6} \perp - \sum_{e=p'}^{\hat{p}+1} (g_{k,e,s}^{up} - g_{k,e,s}^{down}) + INT_{STOR}^{k} \geq 0 \forall k,p',s,h, \]  
(A.17)

\[ 0 \leq \mu_{k,p',h,s}^{7} \perp - \sum_{e=p'}^{\hat{p}+1} (g_{k,e,s}^{up} - g_{k,e,s}^{down}) \geq 0 \forall k,p',s,h, \]  
(A.18)

\[ 0 \leq \mu_{k,p,s}^{8} \perp - g_{ls}^{k,p,s} - (1 - LOSS_k)g_{down}^{k,p,s} - g_{micro}^{k,p,s} - g_{pv}^{k,p,s} + D_{REF}^{k,p} + g_{up}^{k,p,s} \geq 0 \forall k,p,s. \]  
(A.19)

Appendix B. Benders Decomposition

In this appendix we detail the pseudo-code for the Benders Decomposition algorithm, its associated master problem and, in addition, the algorithm’s convergence metric. The sub-problems solved can be represented by the MCP presented in Section 2, but with the first-stage variables fixed, as explained in Section 2.4. At each iteration, that MCP is solved twelve times, each time representing a different 48-hour period and sub-problem. As explained in Section 2.4, these twelve 48-hour periods represent a week in winter, spring/autumn and summer.
The revenues and costs are weighted such that they represent a full year. Furthermore, each sub-problem is solved with the first-stage decision variables fixed at the values determined by the master problem of the same iteration.

Appendix B.1. Algorithm

In this appendix we present the pseudo-code for the Benders Decomposition algorithm.

Read dataset;
Define Master Problem MCP;
Define sub-problem MCP;
for \(i = 1 \rightarrow 1000\) do
  if \(i = 1\) then
    Set all Master Problem variables to zero;
  end
  else
    Solve Master Problem;
    if Solution is found then
      BREAK
    end
    else
      Set all Master Problem variables to zero;
      Remove benders cuts that have not been binding for at least 10 iterations;
    end
  end
Solve sub-problems ;
if \(i > 1\) then
  Calculate convergence metric \(TOL^i\);
end
if \(TOL^i > -10^{-4}\) then
  BREAK ;
end
end
OutputData;

Algorithm 1: Benders Decomposition Algorithm

Appendix B.2. KKT conditions for master problem of \(i\)th iteration

In this appendix we present the KKT conditions for the master problem MCP, solved at the \(i\)th iteration of the Benders Decomposition algorithm. All variables with an index \(i\) in these conditions are exogenous to
the master problem and are determined by the sub-problems solved at the $i$th iteration. For example, $\lambda^{1,i}_{f,t,p,s}$ is equal to the value of $\lambda^{1,i}_{f,t,p,s}$ obtained at the $i$th iteration.

Equations (B.1) - (B.4) correspond to equations (A.2) - (A.4) and equation (A.6) respectively, while equation (B.5) corresponds to market clearing condition (6b). Equation (B.6) is the convexity constraint (Egging [2013]), while equation (B.7) represents the Benders cuts.

The KKT conditions for the MCP master problem are:

\[
\begin{align*}
0 \leq \text{inv}_{f,t} & \perp IC^\text{GEN}_t + MTC^\text{GEN}_t - \sum_{i \leq i} \theta^i \left( \sum_{p,s} NORM^G_{f,t,p,s} \lambda^{1,i}_{f,t,p,s} \right) - \lambda^2_{f,t} \geq 0, \forall f,t, \quad \text{(B.1)} \\
0 \leq \text{exit}_{f,t} & \perp -MTC^\text{GEN}_t + \sum_{i \leq i} \theta^i \left( \sum_{p,s} NORM^G_{f,t,p,s} \lambda^{1,i}_{f,t,p,s} \right) + \lambda^2_{f,t} \geq 0, \forall f,t, \quad \text{(B.2)} \\
0 \leq \text{cap}^{\text{bid}}_f & \perp DR_t \times \kappa + \lambda^2_{f,t} \geq 0, \forall f,t, \quad \text{(B.3)} \\
0 \leq \text{cap}^{\text{bid}}_f & \perp -\text{cap}^{\text{bid}}_f + \text{CAP}_{f,t} + \text{inv}_{f,t} \geq 0, \forall f,t, \quad \text{(B.4)} \\
0 & = \sum_{f,t} DR_t \times \text{cap}^{\text{bid}}_f - \text{TARG}_{\text{ET}}, \quad (\kappa, \text{ free}), \quad \text{(B.5)} \\
0 & = \sum_{i \leq i} \theta^i - 1, \quad (\alpha, \text{ free}), \quad \text{(B.6)} \\
\end{align*}
\]

\[
0 \leq \theta^i \perp \alpha + \sum_{f,t,p,s} \left( PR_s \left( 2B^\text{GEN}_t \times gen^i_{f,t,p,s} \right) \times \sum_{i \leq i} \left( \theta^i \times gen^i_{f,t,p,s} \right) \right) \\
+ \sum_{f,t,p,s} \lambda^{1,i}_{f,t,p,s} \times \left( \text{CAP}_{f,t} + \text{inv}_{f,t} - \text{exit}_{f,t} \right) \times \text{NORM}^G_{f,t,p,s} \\
+ \sum_{k,p,s} \left( PR_s \times 2B_{k,p} \times \text{ls}^i_{k,p,s} \right) \times \sum_{i \leq i} \left( \theta^i \times \text{ls}^i_{k,p,s} \right) \\
+ \sum_{k,p,s} \left( PR_s \times 2B_{k,p} \times \text{micro}^i_{k,p,s} \right) \times \sum_{i \leq i} \left( \theta^i \times \text{micro}^i_{k,p,s} \right) \\
+ \sum_{k,p,s} \left( \mu^{1,i}_{k,p,s} \times \text{LSMAX}^i_{k} \right) + \sum_{k,p,s} \left( \text{FAC}^\text{STOR}_{k} \times \text{INT}^\text{STOR}_{k} \right) + \sum_{k,p,s} \left( \text{FAC}^\text{STOR}_{k} \times \text{INT}^\text{STOR}_{k} \right) \\
+ \sum_{k,p,s} \left( \mu^{2,i}_{k,p,s} \times \text{FAC}^\text{STOR}_{k} \times \text{INT}^\text{STOR}_{k} \right) + \sum_{k,p,s} \left( \text{NORM}^\text{PV}_{p,s} \times \text{INT}^\text{PV}_{k} \right) + \sum_{k,p,s} \left( \text{INT}^\text{STOR}_k \right) \\
+ \left( \mu^{3,i}_{k,p,s} \times \text{INT}^\text{STOR}_{k} \right) \geq 0, \forall i \leq i. \quad \text{(B.7)}
\]

**Appendix B.3. Convergence metric**

In this appendix the convergence metric for the Benders Decomposition algorithm is presented. It follows from the metric derived in [Gabriel and Fuller [2010] Note: each variable with an index $M$ is determined from its corresponding variables with $i$ indices as follows:

\[
x^{M,i} = \sum_{i \leq i} \theta^i x^i. \quad \text{(B.8)}
\]
The convergence metric is:

\[
TOL^i = \sum_{f,t,p,s} PR_s \left( 2B_t^{GEN} \times (\text{gen}^{f,t,p,s} - \text{gen}^{M,i}_{f,t,p,s}) \right) \times \text{gen}^{M,i}_{f,t,p,s} \\
+ \sum_{f,t,p,s} \left( \lambda^{1,i}_{f,t,p,s} - \lambda^{1,M,i}_{f,t,p,s} \right) \times (CAP_{f,t} + \text{int}_{f,t} - \text{exit}_{f,t}) \times NORM_{f,t,p,s}^i \\
+ \sum_{k,p,s} \left( PR_s \times 2B_{k,p}^{LS} \times (\text{g}_{k,p,s}^{k,i} - \text{g}_{k,p,s}^{M,i}) \right) \times \text{g}_{k,p,s}^{M,i} \\
+ \sum_{k,p,s} \left( PR_s \times 2B_{k,p}^{MICRO} \times (\text{g}_{k,p,s}^{\text{micro},i} - \text{g}_{k,p,s}^{\text{micro},M,i}) \right) \times \text{g}_{k,p,s}^{\text{micro},M,i} \\
+ \sum_{k,p,s} \left( \mu^{1,i}_{k,p,s} - \mu^{1,M,i}_{k,p,s} \right) \times G_{k,p}^{\text{MAX}} \times \text{IN}_{k,p}^{\text{STOR}} \\
+ \left( \mu^{3,i}_{k,p,s} - \mu^{3,M,i}_{k,p,s} \right) \times F_{k,p}^{\text{STOR}} \times \text{IN}_{k,p}^{\text{STOR}} + \left( \mu^{4,i}_{k,p,s} - \mu^{4,M,i}_{k,p,s} \right) \times \text{IN}_{k,p}^{\text{MICRO}} \\
+ \left( \mu^{5,i}_{k,p,s} - \mu^{5,M,i}_{k,p,s} \right) \times NORM_{p,s}^{PV} \times \text{IN}_{k,p}^{PV} + \left( \mu^{6,i}_{k,p,s} - \mu^{6,M,i}_{k,p,s} \right) \times \text{IN}_{k,p}^{\text{STOR}} \\
+ \left( \mu^{8,i}_{k,p,s} - \mu^{8,M,i}_{k,p,s} + \gamma^{i}_{p,s} - \gamma^{M,i}_{p,s} \right) \times D_{k,p}^{\text{REF}}. 
\] (B.9)

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<table>
<thead>
<tr>
<th>Year</th>
<th>Number</th>
<th>Title/Author(s)</th>
</tr>
</thead>
</table>
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